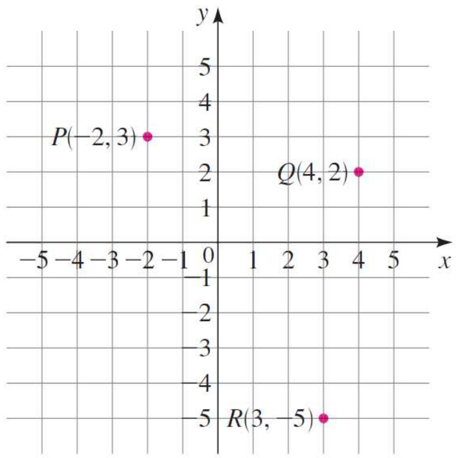
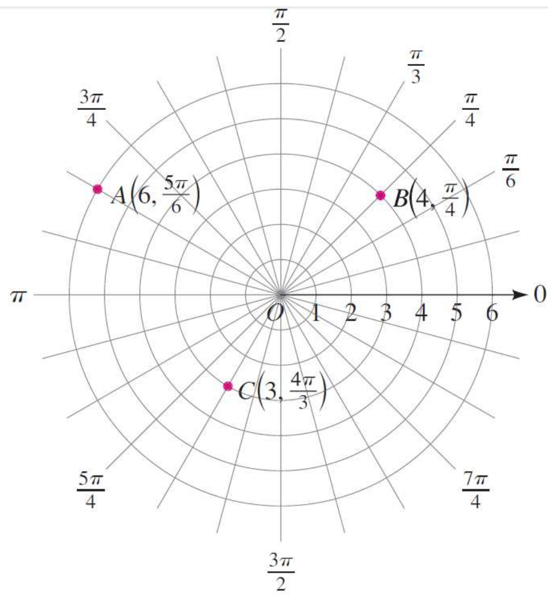
Section 8.2 Graphs of Polar Equations

Graphing Polar Equations

The **graph of a polar equation** $r = f(\theta)$, or more generally $F(r, \theta) = 0$, consists of all points P that have at least one polar representation (r, θ) whose coordinates satisfy the equation.



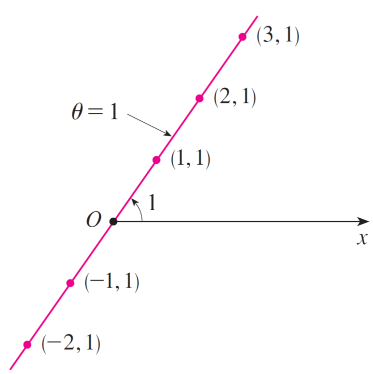
(a) Grid for rectangular coordinates



(b) Grid for polar coordinates

EXAMPLE: Sketch the polar curve $\theta = 1$.

Solution: This curve consists of all points (r, θ) such that the polar angle θ is 1 radian. It is the straight line that passes through O and makes an angle of 1 radian with the polar axis. Notice that the points $(r, 1)$ on the line with $r > 0$ are in the first quadrant, whereas those with $r < 0$ are in the third quadrant.



EXAMPLE: Sketch the following curves:

(a) $r = 2,\ 0 \le \theta \le 2\pi$.

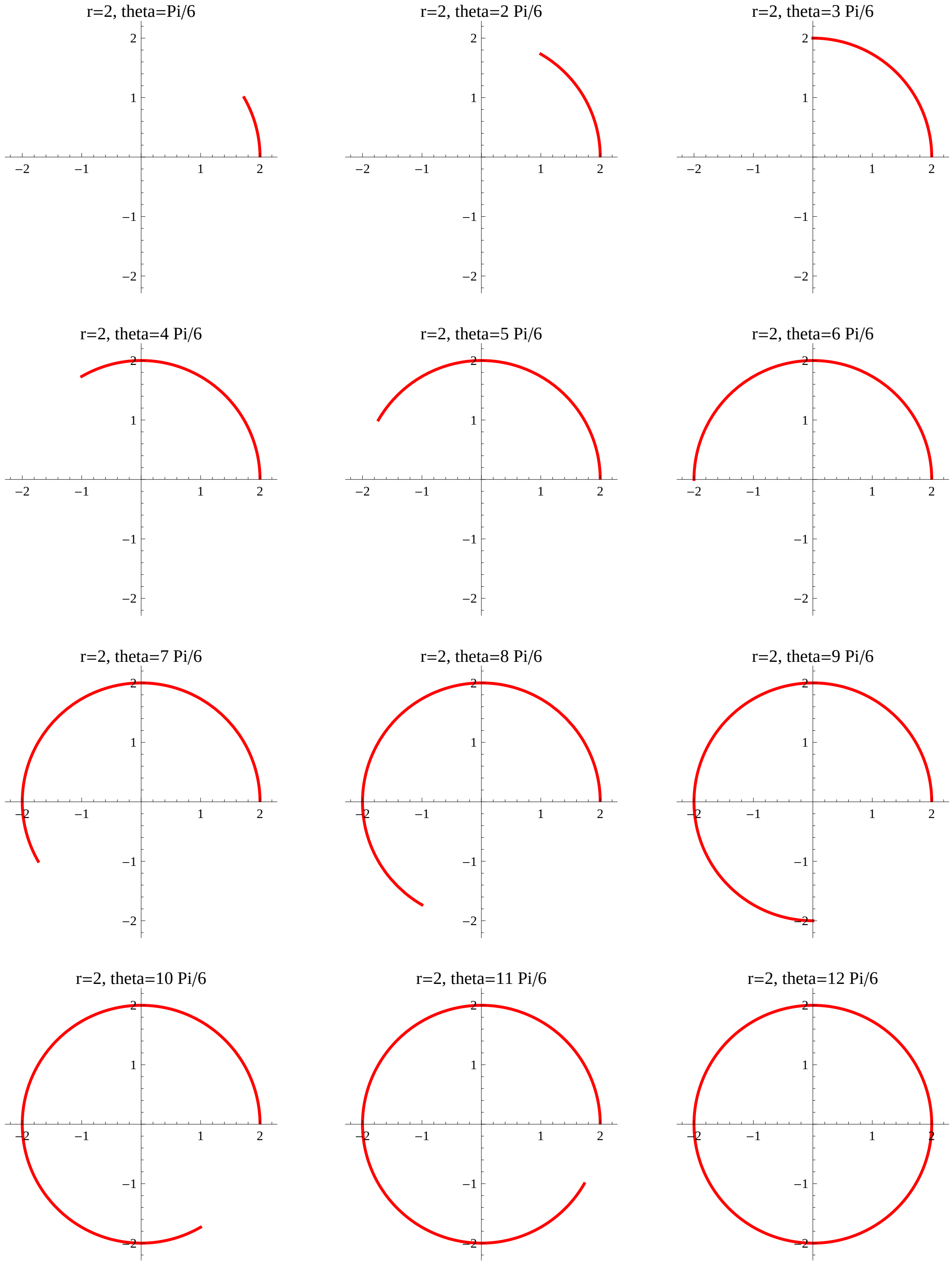
(b) $r = \theta,\ 0 \le \theta \le 4\pi$.

(c) $r = 2\cos\theta,\ 0 \le \theta \le \pi$.

EXAMPLE: Sketch the curve $r = 2,\ 0 \leq \theta \leq 2\pi$.

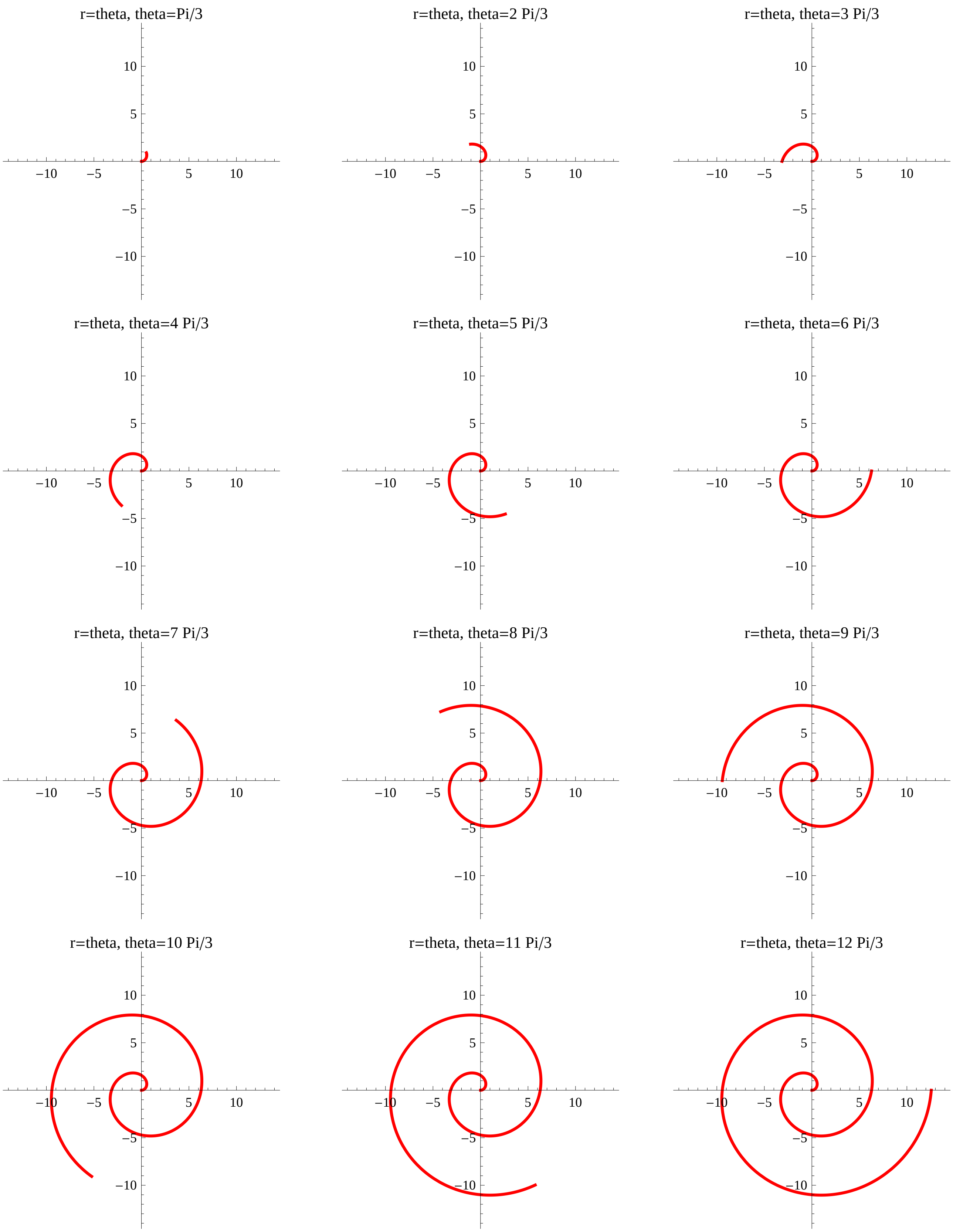
Solution 1: Since $r = 2$, it follows that $r^2 = 4$. But $r^2 = x^2 + y^2$, therefore $x^2 + y^2 = 4$ which is a circle of radius 2 with the center at the origin.

Solution 2: We have



EXAMPLE: Sketch the curve $r = \theta,\ 0 \le \theta \le 4\pi$.

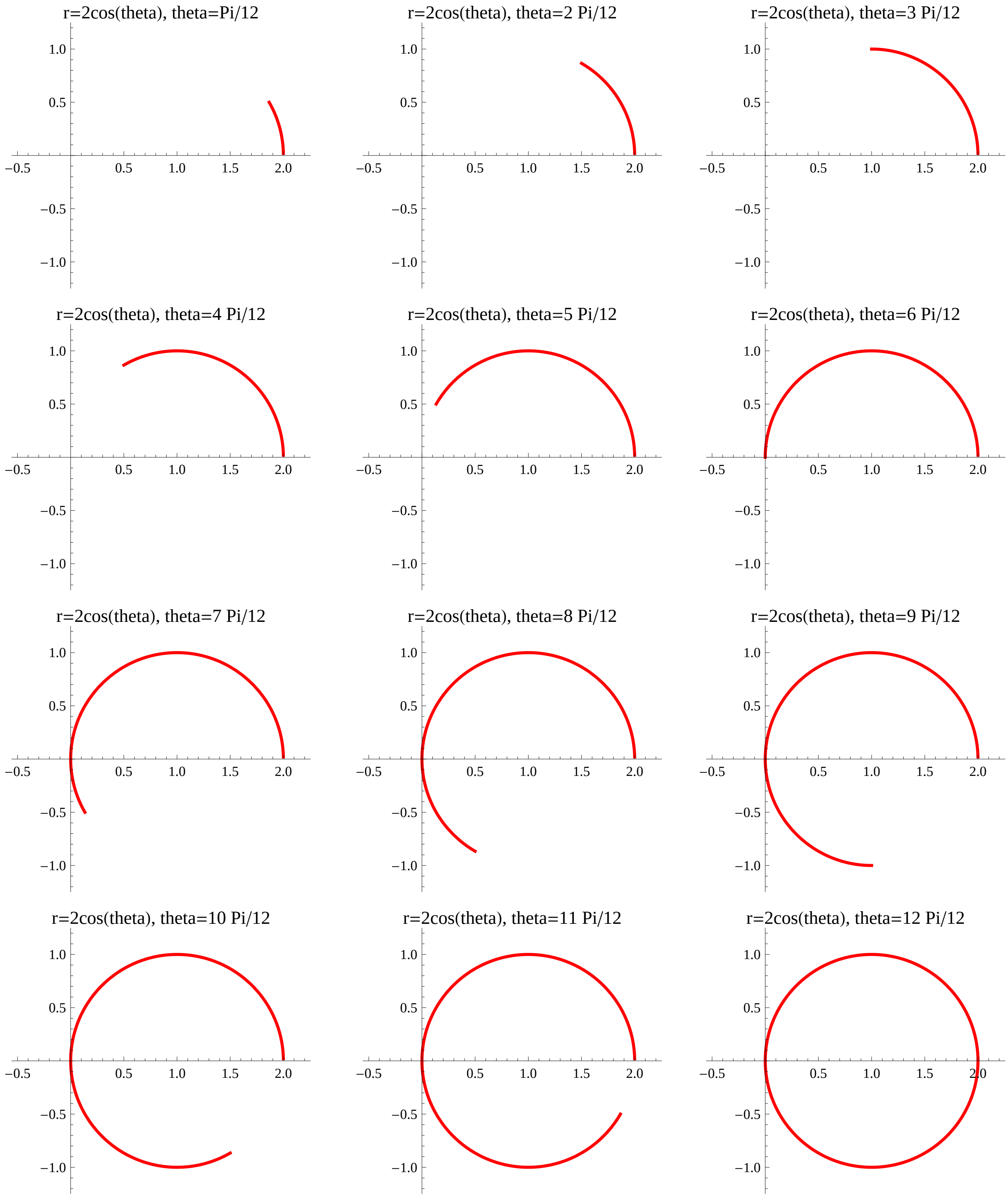
Solution: We have



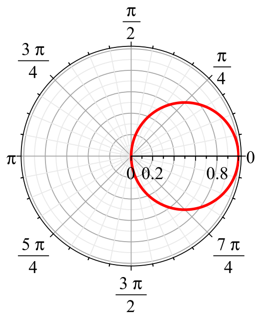
EXAMPLE: Sketch the curve $r = 2\cos\theta,\ 0 \le \theta \le \pi$.

Solution 1: Since $r = 2\cos\theta$, it follows that $r^2 = 2r\cos\theta$. But $r^2 = x^2 + y^2$ and $r\cos\theta = x$, therefore $x^2 + y^2 = 2x$. This can be rewritten as $(x-1)^2 + y^2 = 1$ which is a circle of radius 1 with the center at $(1, 0)$.

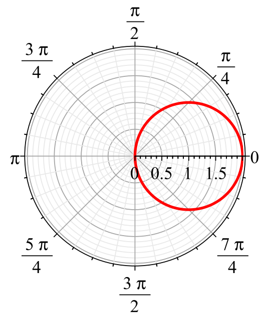
Solution 2: We have



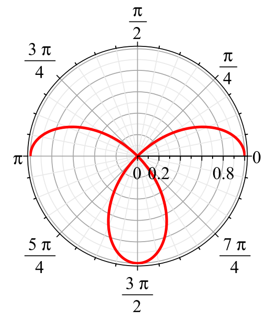
EXAMPLES:



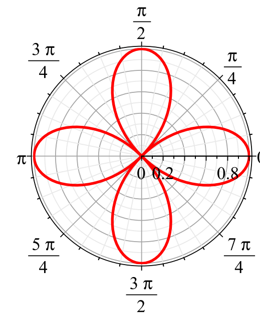
$r=\cos\theta,\ 0<\theta<\pi$



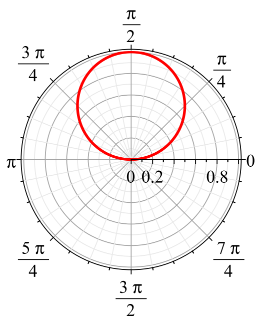
$r=2\cos\theta,\ 0<\theta<\pi$



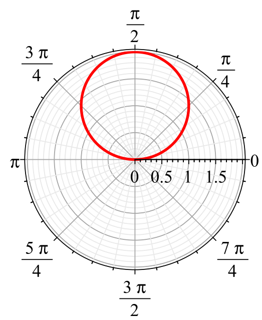
$r=\cos(2\theta),\ 0<\theta<\pi$



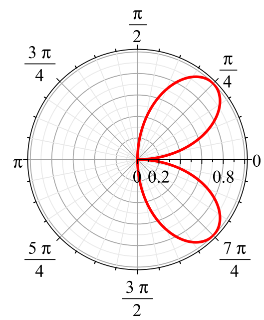
$r=\cos(2\theta),\ 0<\theta<2\pi$



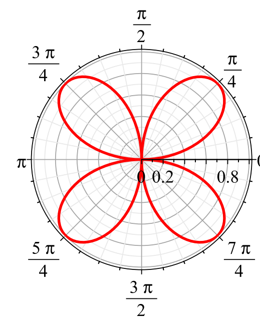
$r=\sin\theta,\ 0<\theta<\pi$



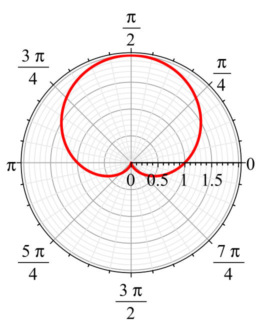
$r=2\sin\theta,\ 0<\theta<\pi$



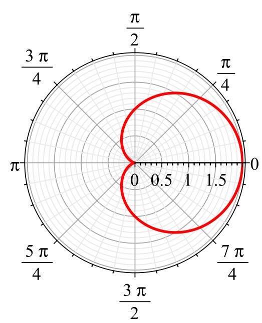
$r=\sin(2\theta),\ 0<\theta<\pi$



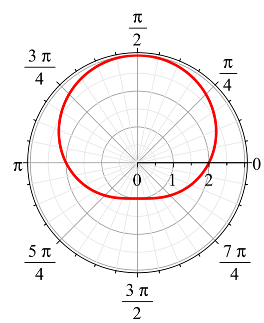
$r=\sin(2\theta),\ 0<\theta<2\pi$



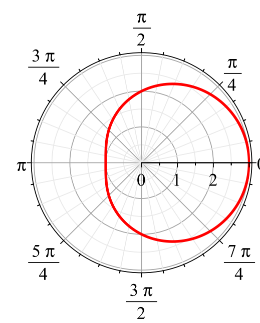
$r=1+\sin\theta,\ 0<\theta<2\pi$



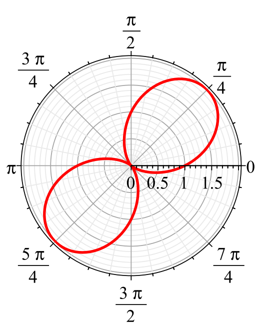
$r=1+\cos\theta,\ 0<\theta<2\pi$



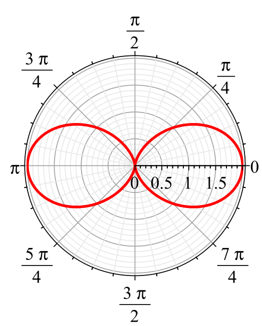
$r=2+\sin\theta,\ 0<\theta<2\pi$



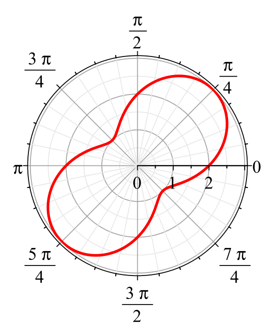
$r=2+\cos\theta,\ 0<\theta<2\pi$



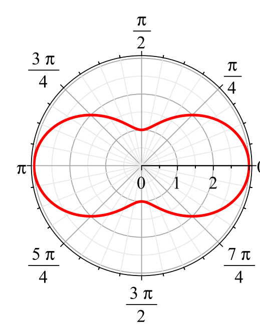
$r=1+\sin(2\theta),\ 0<\theta<2\pi$



$r=1+\cos(2\theta),\ 0<\theta<2\pi$



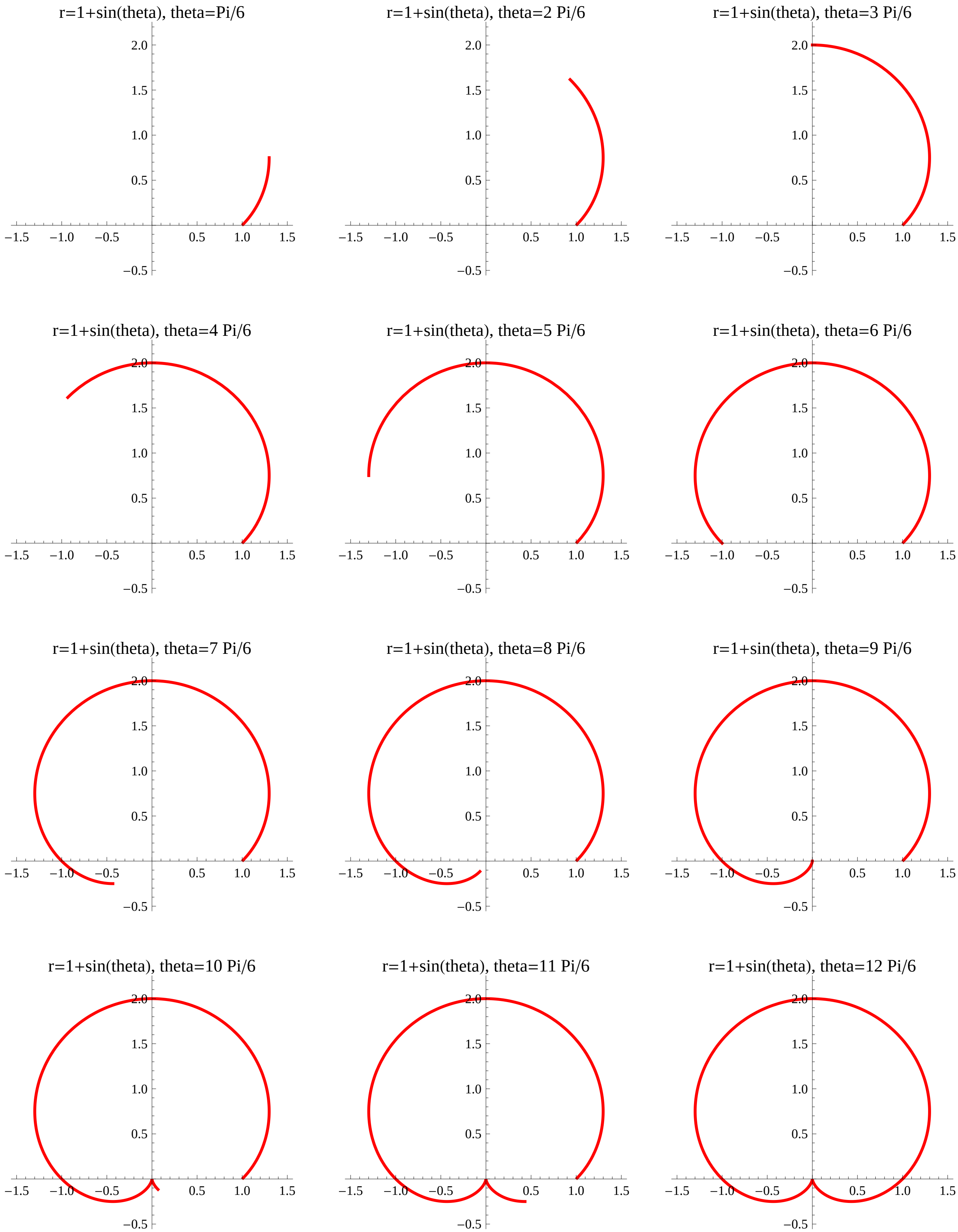
$r=2+\sin(2\theta),\ 0<\theta<2\pi$



$r=2+\cos(2\theta),\ 0<\theta<2\pi$

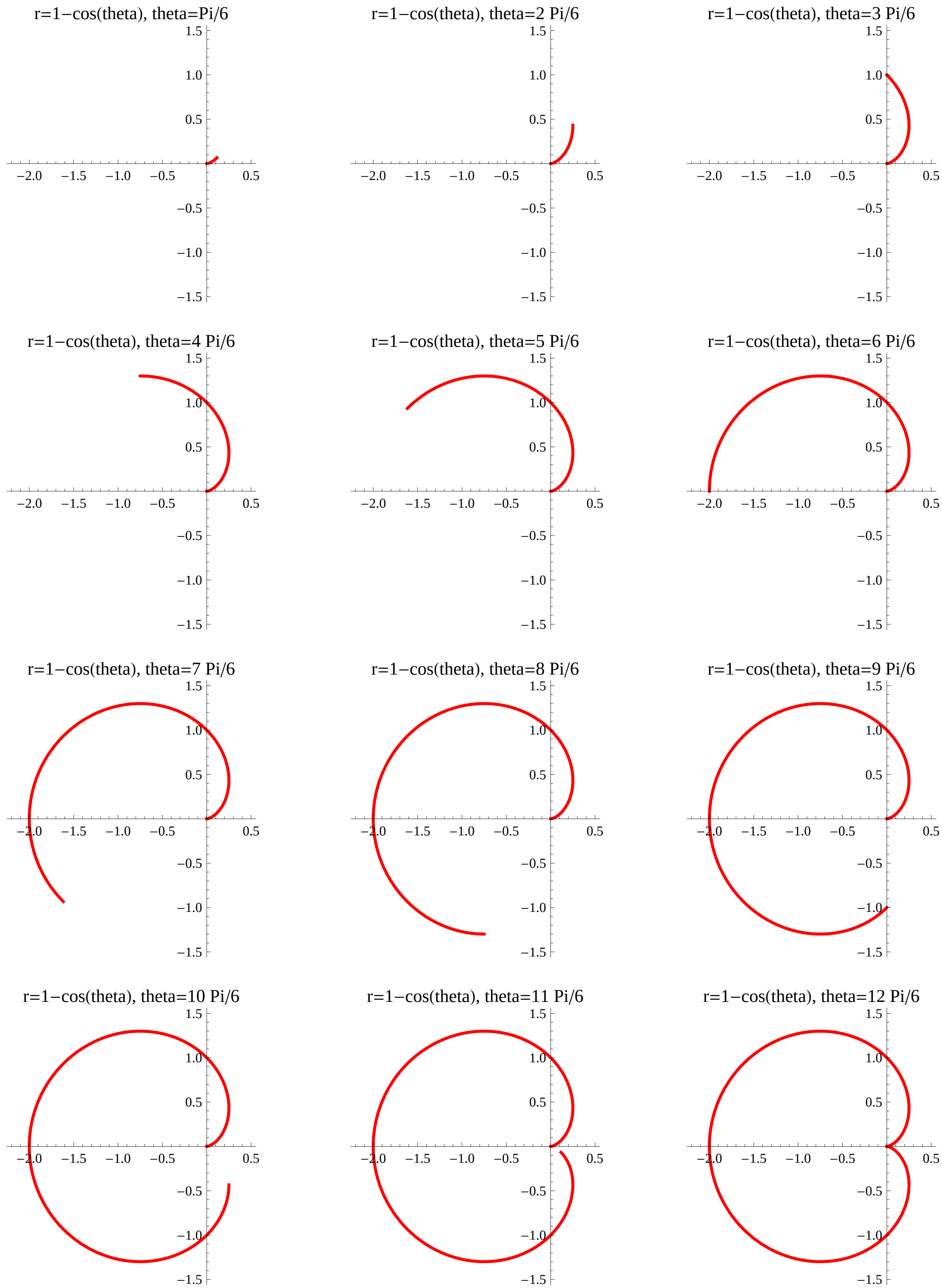
EXAMPLE: Sketch the curve $r = 1 + \sin\theta,\ 0 \le \theta \le 2\pi$ (cardioid).

Solution: We have



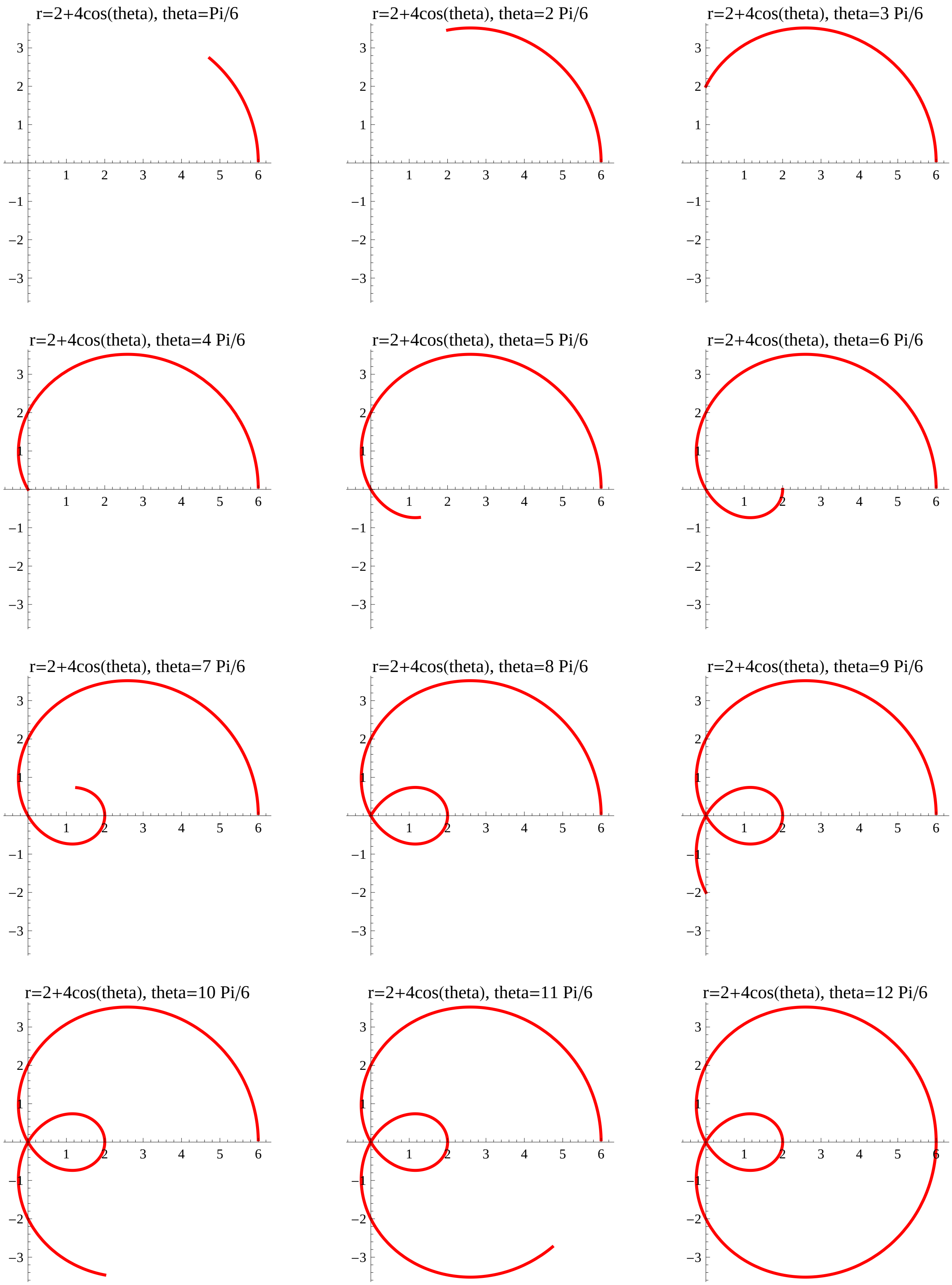
EXAMPLE: Sketch the curve $r = 1 - \cos\theta,\ 0 \le \theta \le 2\pi$ (cardioid).

Solution: We have



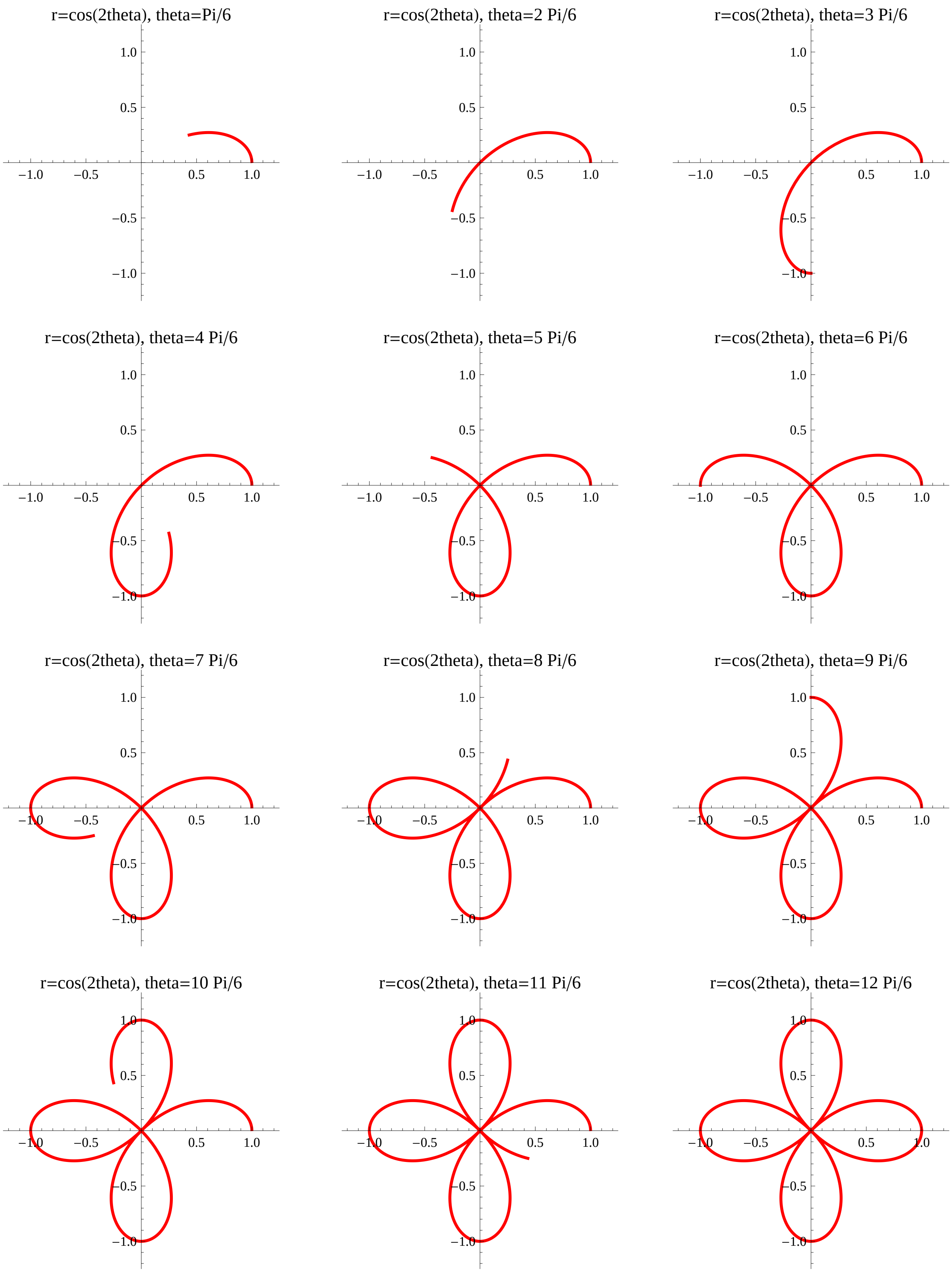
EXAMPLE: Sketch the curve $r = 2 + 4\cos\theta,\ 0 \leq \theta \leq 2\pi$.

Solution: We have



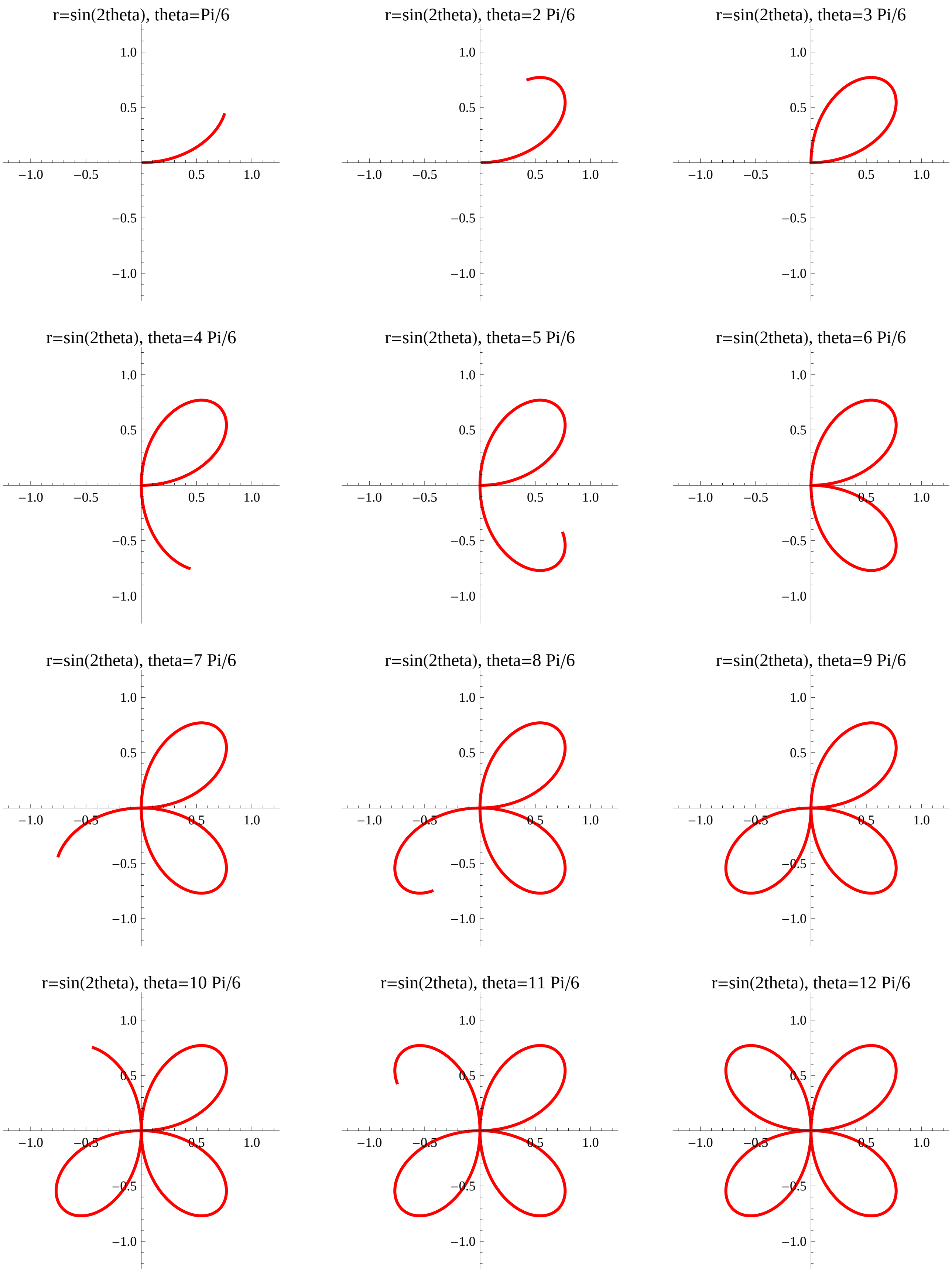
EXAMPLE: Sketch the curve $r = \cos(2\theta)$, $0 \leq \theta \leq 2\pi$ (four-leaved rose).

Solution: We have



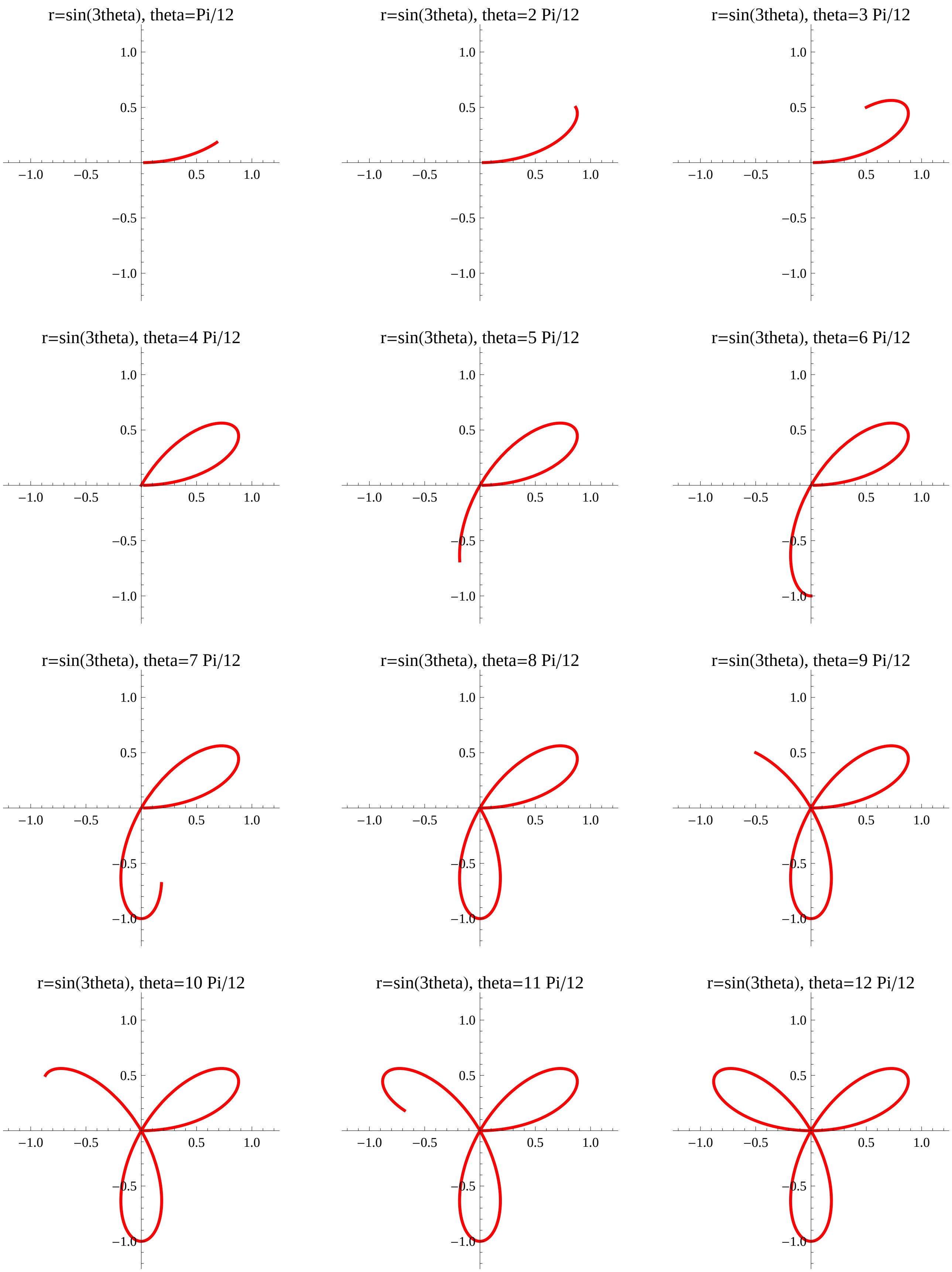
EXAMPLE: Sketch the curve $r = \sin(2\theta)$, $0 \le \theta \le 2\pi$ (four-leaved rose).

Solution: We have



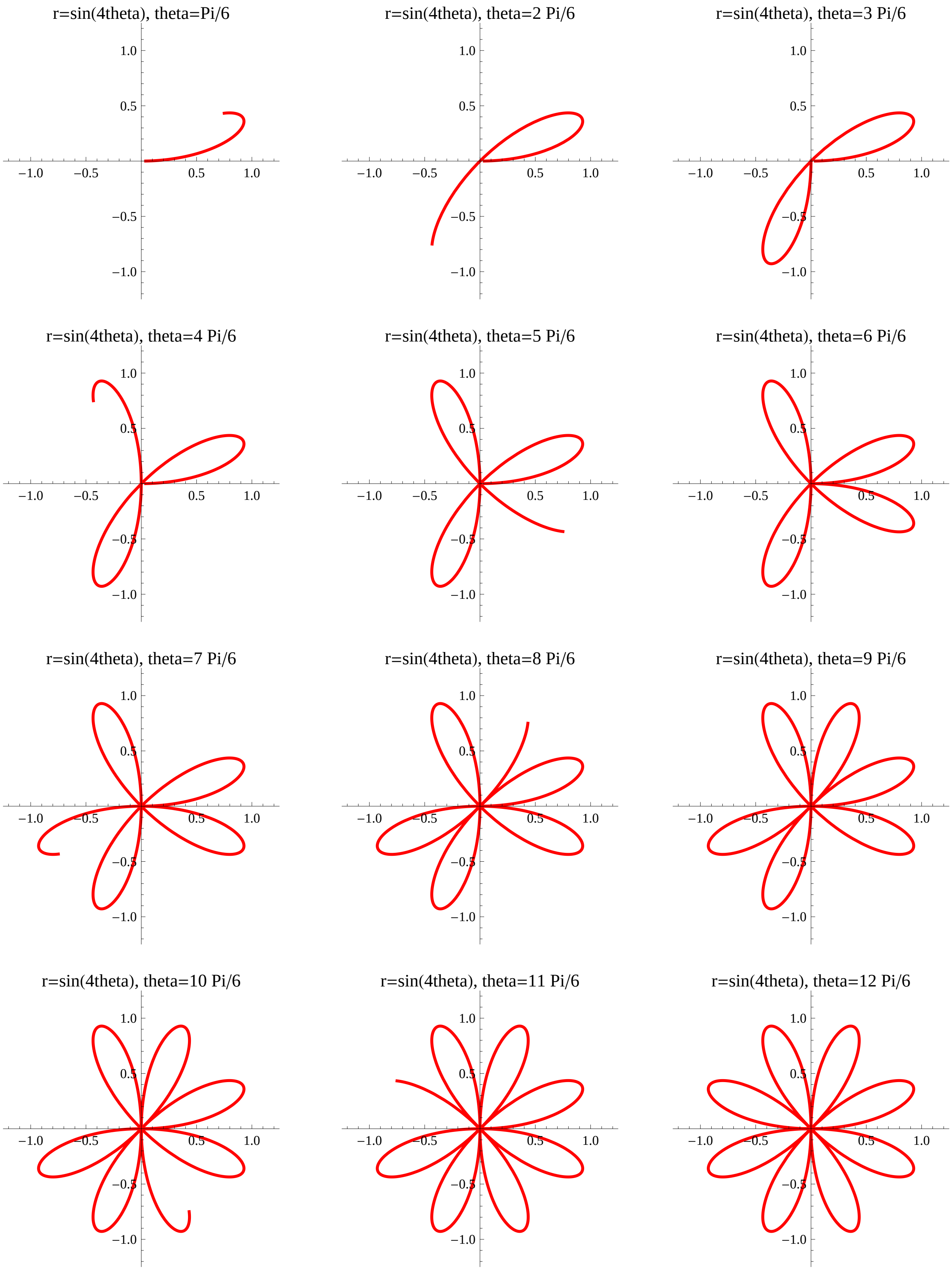
EXAMPLE: Sketch the curve $r = \sin(3\theta)$, $0 \le \theta \le \pi$ (three-leaved rose).

Solution: We have



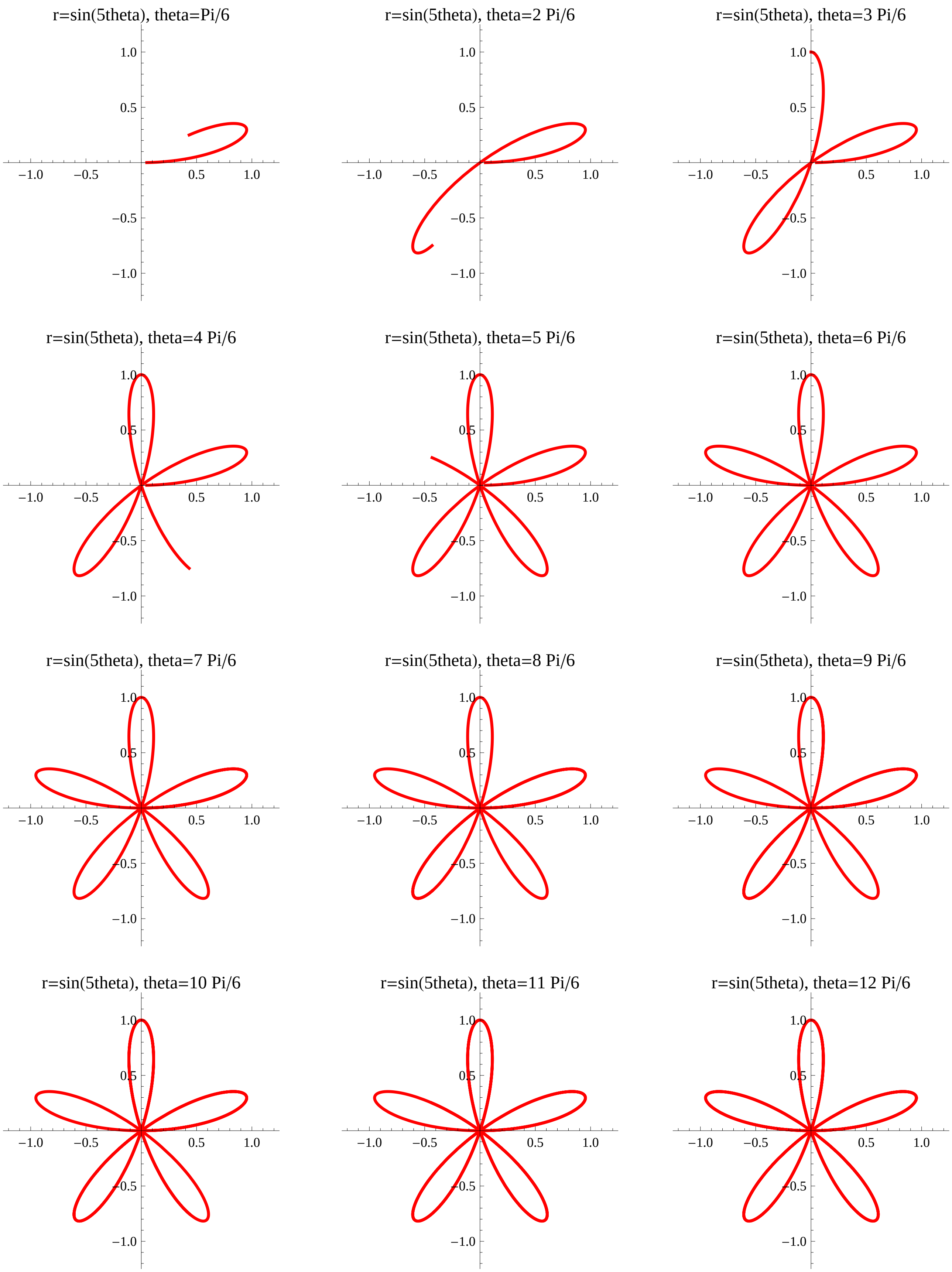
EXAMPLE: Sketch the curve $r = \sin(4\theta)$, $0 \le \theta \le 2\pi$ (eight-leaved rose).

Solution: We have



EXAMPLE: Sketch the curve $r = \sin(5\theta)$, $0 \le \theta \le 2\pi$ (five-leaved rose).

Solution: We have

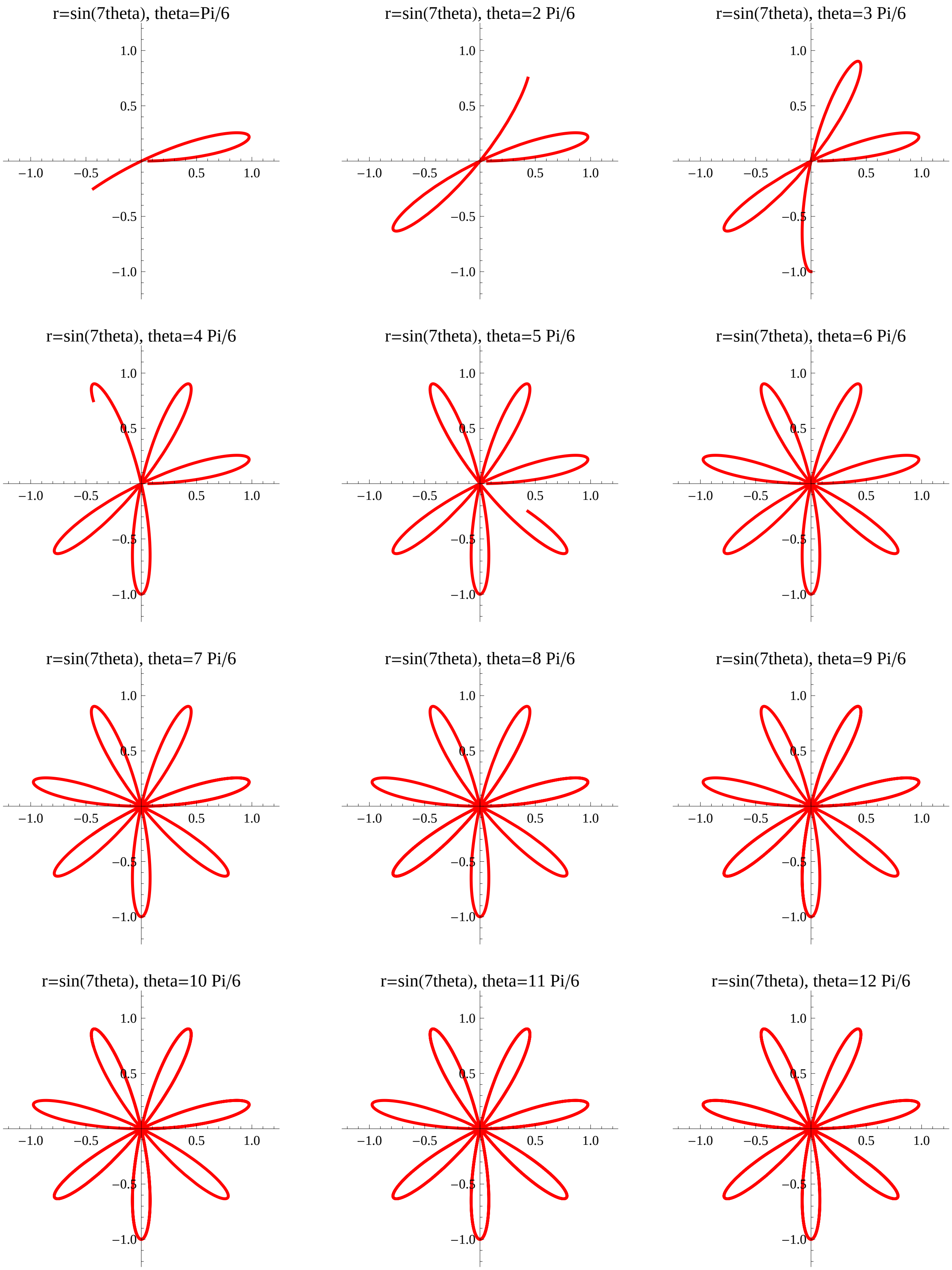


EXAMPLE: Sketch the curve $r = \sin(6\theta)$, $0 \le \theta \le 2\pi$ (twelve-leaved rose).

Solution: We have

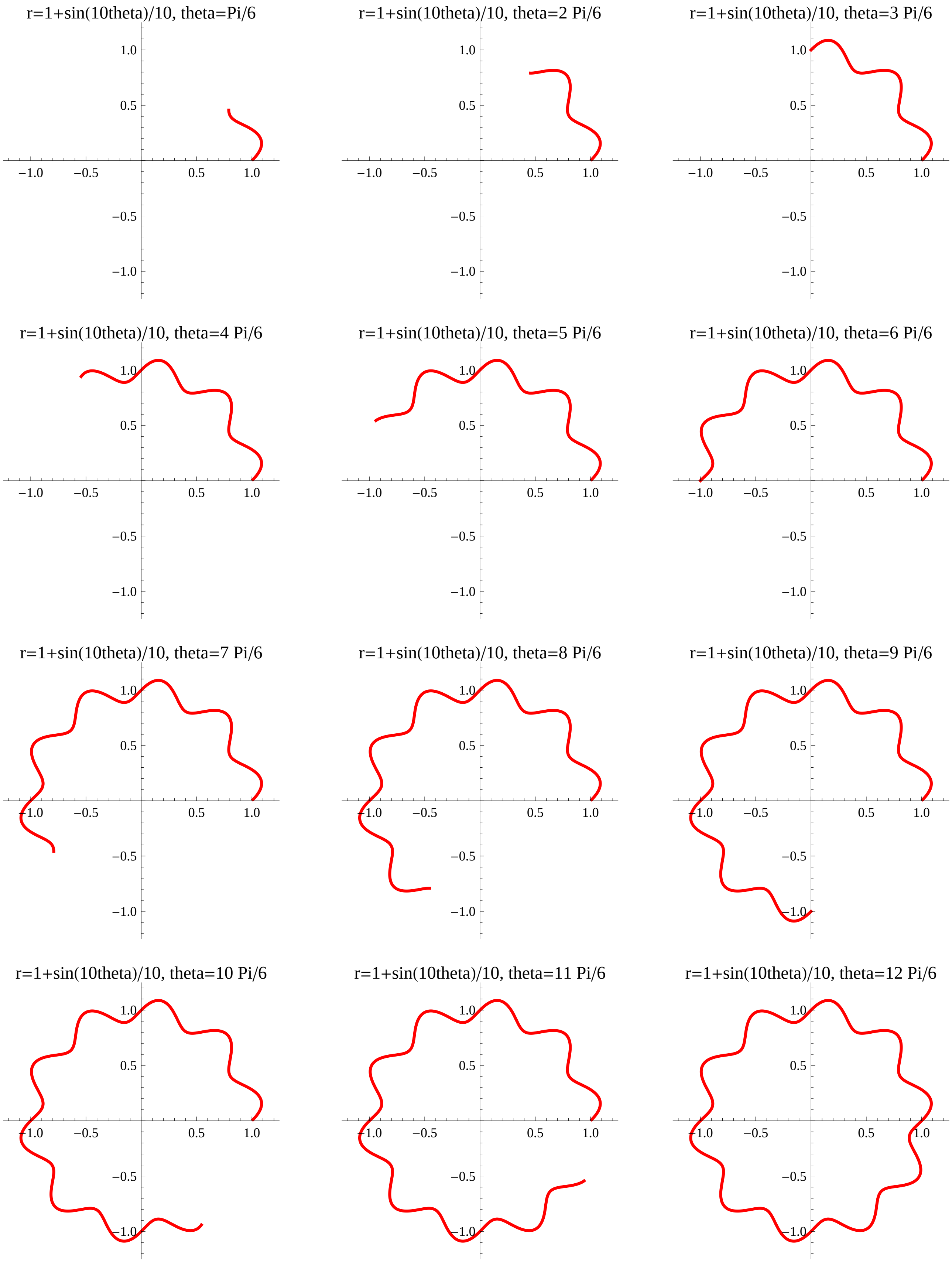
EXAMPLE: Sketch the curve $r = \sin(7\theta)$, $0 \le \theta \le 2\pi$ (seven-leaved rose).

Solution: We have



EXAMPLE: Sketch the curve $r = 1 + \dfrac{1}{10}\sin(10\theta),\ 0 \le \theta \le 2\pi.$

Solution: We have



EXAMPLE: Match the polar equations with the graphs labeled I-VI:

(a) $r = \sin(\theta/2)$ (b) $r = \sin(\theta/4)$

(c) $r = \sin\theta + \sin^3(5\theta/2)$ (d) $r = \theta\sin\theta$

(e) $r = 1 + 4\cos(5\theta)$ (f) $r = 1/\sqrt{\theta}$