Name: ID: Course: Group:

Notes:
1. The homework covers the chapters: From 1 to 6
2. Deadline: Thursday 11-11-2021   Time: 15:00
3. Write the answers on the external papers.
4. Send your answer either by e-mail or by placing it in the mailbox in the Mathematics department.

1 - 8 ■ Choose the correct answer. **(You have to show the details of your answer)**

1 The sum $\sum\limits_{k=1}^{n^2} (k-1)$ is equal to

(a) $\frac{n^2(n-1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n^2(n^2+1)}{2}$ (d) $\frac{n^2(n^2-1)}{2}$

2 If $\sum\limits_{k=1}^{4} (k+a) = 14$, then the value of a is equal to

(a) 1   (b) 4   (c) -4 (d) -1

3 The average value of the function $f(x) = \sqrt[3]{x+1}$ on $[-2,0]$ is equal to

(a) 3   (b) 0   (c) -1 (d) -3

4 The number z that satisfies the Mean Value Theorem for $f(x) = 1+x^2$ on $[-3,0]$ is

(a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

5 If $F(x) = \int_{1}^{x^3} \sqrt{5+t^2}\, dt$, then $F'(1)$ is equal to

(a) 0   (b) $3\sqrt{6}$ (c) $\sqrt{6}$ (d) $\frac{2}{\sqrt{6}}$

6 The average value of $f(x) = |x-1|$ on $[0,1]$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0   (d) $\frac{1}{2}$

7 If $f(x) = \log_2 \frac{x}{x-1} = 1$, then x is equal to

(a) 1   (b) 2   (c) $\frac{1}{2}$ (d) -1

8 If $f(x) = x^{x+1}$, then $f'(x)$ is equal to

(a) $(1+\frac{1}{x}+\ln x)x^{x+1}$ (b) $(\ln x + \frac{1}{x})x^{x+1}$ (c) $(1+\ln x)x^{x+1}$ (d) $(1+\frac{1}{x}+\ln x)x^x$

9 - 32 ■ Evaluate the integral.

9 $\int \frac{4^{-\ln x}}{x}\, dx$

10 $\int \frac{2x+3}{\sqrt{4-x^2}}\, dx$

11 $\int \frac{(\sin^{-1} x)^{-2}}{\sqrt{1-x^2}}\, dx$

12 $\int \frac{e^{\frac{x}{2}}}{7+e^x}\, dx$

13 $\int \frac{\sin x}{\sqrt{e^{\cos x}-1}}\, dx$

14 $\int \frac{dx}{e^{-x}\sqrt{e^{2x}-1}}$

15 $\int \sec^4 x \tan^7 x\, dx$

16 $\int \frac{dx}{(x+1)\frac{5}{6}-(x+1)^{\frac{1}{2}}}$

17 $\int \frac{dx}{2+\cos x}$

18 $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\, dx$

19 $\int (\ln x + 1)\, 3^{x\,\ln x}\, dx$

20 $\int \frac{\sec^2 x}{\sqrt{9-\tan^2 x}}\, dx$

21 $\int \frac{dx}{x\,\ln x\,\sqrt{1-(\ln x)^4}}$

22 $\int \frac{5^{\tan x}}{\cos^2 x}\, dx$

23 $\int \frac{dx}{\sqrt{x}\,(2+x)}$

24 $\int \frac{\ln x + 1}{\sqrt{16(x\ln x)^2-9}}\, dx$

25 $\int \frac{dx}{x\,\sqrt{x^5-4}}$

26 $\int \frac{2\,e^{-3x}}{1-e^{-6x}}\, dx$

27 $\int \frac{\tan x}{\sqrt{4-\cos^4 x}}\, dx$

28 $\int e^{2x} \sin x\, dx$

29 $\int \frac{6x^2+x+8}{x^3+4x}\, dx$

30 $\int \frac{dx}{x^3\,\sqrt{x^2-4}}$

31 $\int \frac{1}{x\,\sqrt{16x^4-1}}\, dx$

32 $\int \frac{x^2}{\sqrt{x^6-25}}\, dx$

33 - 34 ■ Approximate the definite integral.

33 $\int_{-1}^{1} \sqrt{x^2+1}\, dx$ Using the trapezoidal rule for the given $n = 4$.

34 $\int_{0}^{2\pi} \cos^2 x\, dx$ Use Simpson's rule for the given $n = 6$.

35 - 39 ■ Find the derivative of the following functions.

35 $y = e^{2x^2} (x-1)^{\frac{3}{2}}$

36 $y = 2^{\sin^2 x} + x^\pi \, \pi^x$

37 $F(x) = \ln(2x) \displaystyle\int_1^{4x^2} (1+t^2)^{10} \, dx$, then find $F'(\frac{1}{2})$

38 $F(x) = \cos x \displaystyle\int_0^{\tan x} \sqrt{1+t^2} \, dx$, then find $F'(0)$

39 $F(x) = \displaystyle\int_{\sin^{-1} x}^{\ln x} (1+t^3) \, dx$, then find $F'(1)$

40 - 43 ■ Find the limit if it exists.

40 $\displaystyle\lim_{x\to\infty} \frac{2^x - 1}{x}$

41 $\displaystyle\lim_{x\to1} \frac{\ln x}{x-1}$

42 $\displaystyle\lim_{x\to1} x^{1/(1-x)}$

43 $\displaystyle\lim_{x\to0} \frac{e^x - x - 1}{\sin x}$

44 - 49 ■ Determine whether the integral converges or diverges.

44 $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x^2-1}} \, dx$

46 $\displaystyle\int_0^3 \frac{1}{(x-2)^2} \, dx$

48 $\displaystyle\int_{-\infty}^\infty \frac{x}{(x^2+3)^2} \, dx$

45 $\displaystyle\int_{-\infty}^2 \frac{1}{5-2x} \, dx$

47 $\displaystyle\int_0^4 \frac{1}{x^2+x-6} \, dx$

49 $\displaystyle\int_0^9 \frac{1}{\sqrt{x}\,(x+9)} \, dx$