Example 3. 3

We have a time series of 40 random numbers shown in Table3-3

Table 3-3

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t |  | t |  | t |  | t |  |
| 1 | 343 | 11 | 946 | 21 | 704 | 31 | 555 |
| 2 | 574 | 12 | 142 | 22 | 291 | 32 | 476 |
| 3 | 879 | 13 | 477 | 23 | 43 | 33 | 612 |
| 4 | 728 | 14 | 452 | 24 | 118 | 34 | 574 |
| 5 | 37 | 15 | 727 | 25 | 682 | 35 | 518 |
| 6 | 227 | 16 | 147 | 26 | 577 | 36 | 296 |
| 7 | 613 | 17 | 199 | 27 | 834 | 37 | 970 |
| 8 | 157 | 18 | 744 | 28 | 981 | 38 | 204 |
| 9 | 571 | 19 | 627 | 29 | 263 | 39 | 616 |
| 10 | 72 | 20 | 122 | 30 | 424 | 40 | 97 |

This series is a sample selected randomly from a population (n=40). Because these data are random, autocorrelations for all time lags should be theoretically equal to zero. These 40 values are only one set of a large number of possible samples of size 40. Each sample will produce different autocorrelations.

Figure 3-6 Time Series Plot of 40 Random Numbers

Most of these samples will produce sample autocorrelation coefficients that are close to zero. Using the computer we can generate the first 10 individual autocorrelation coefficients and the first 10 autocorrelation coefficients as a group.

The question is: are the magnitudes of the first 10  (autocorrelation coefficients) as a group larger than one would expect under the hypothesis of no autocorrelation at any lag?

We will use portmanteau test, Q statistic, to see whether the test is significantly different from a set in which all values are zero.





The Q statistic has a chi-square distribution with degree of freedom (*df*) = 10.

|  |  |  |  |
| --- | --- | --- | --- |
| Lag (*k*) |  | t | LBQ  Q |
| 1 | -0.191171 | -1.21 | 1.57 |
| 2 | -0.006293 | -0.04 | 1.58 |
| 3 | -0.145224 | -0.89 | 2.53 |
| 4 | 0.104475 | 0.63 | 3.04 |
| 5 | -0.253756 | -1.50 | 6.13 |
| 6 | 0.028640 | 0.16 | 7.17 |
| 7 | 0.169071 | 0.95 | 7.63 |
| 8 | -0.027390 | -0.15 | 7.67 |
| 9 | -0.032551 | -0.18 | 7.73 |
| 10 | 0.021546 | 0.12 | 7.75 |

From the above table the value of Q (LBQ) for 10 time lags is 7.75.

Example: compute Q for k = 1,



compute Q for k = 2,

 compute Q for k = 3,

.....

.....

compute Q for k = 10,

 From CHI-SQUARE () VALUES Table the upper .05 point of a  distribution with 10 *df* is 18.3. Since 7.25<18.31, the null hypothesis () cannot be rejected at the 5% significance level.

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13. The number of marriages in the US is given in Table P-13.

**Table: p-13**

|  |  |  |  |
| --- | --- | --- | --- |
| Year | Marriages(1000) | Year | Marriages(1000) |
| 1985 | 2413 | 1995 | 2336 |
| 1986 | 2407 | 1996 | 2344 |
| 1987 | 2403 | 1997 | 2384 |
| 1988 | 2396 | 1998 | 2244 |
| 1989 | 2403 | 1999 | 2358 |
| 1990 | 2443 | 2000 | 2329 |
| 1991 | 2371 | 2001 | 2345 |
| 1992 | 2362 | 2002 | 2254 |
| 1993 | 2334 | 2003 | 2245 |
| 1994 | 2362 | 2004 | 2279 |

1. Compute the first differences for these data.

|  |  |  |  |
| --- | --- | --- | --- |
| Year | Marriages(1000) | Lag 1 |  |
| 1985 | 2413 |  |  |
| 1986 | 2407 | 2413 | -6 |
| 1987 | 2403 | 2407 | -4 |
| 1988 | 2396 | 2403 | -7 |
| 1989 | 2403 | 2396 | 7 |
| 1990 | 2443 | 2403 | 40 |
| 1991 | 2371 | 2443 | -72 |
| 1992 | 2362 | 2371 | -9 |
| 1993 | 2334 | 2362 | -28 |
| 1994 | 2362 | 2334 | 28 |
| 1995 | 2336 | 2362 | -26 |
| 1996 | 2344 | 2336 | 8 |
| 1997 | 2384 | 2344 | 40 |
| 1998 | 2244 | 2384 | -140 |
| 1999 | 2358 | 2244 | 114 |
| 2000 | 2329 | 2358 | -29 |
| 2001 | 2345 | 2329 | 16 |
| 2002 | 2254 | 2345 | -91 |
| 2003 | 2245 | 2254 | -9 |
| 2004 | 2279 | 2245 | 34 |

1. Plot the original data and the difference data as time series
2. Is there a trend in either of these series?
3. The general trend of number marriages over the entire 1985-2004 period is down. To test the trendlike behavior compute the first 10 autocorrelation coefficients using the following formula:



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

Then compute the magnitudes of the first 10  (autocorrelation coefficients). Use portmanteau test, Q statistic, to see whether the test is significantly different from a set in which all values are zero.



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

From CHI-SQUARE () VALUES Table the upper .05 point of a  distribution with 10 *df* is ......?...... Since ..........?........, the null hypothesis ................?............ at the 5% significance level.

**The final result:**

These data are:

Choice 1: correlated and exhibit trendlike behavior.

Choice 2: Uncorrelated at any time lag and do not exhibit trendlike behavior.

1. The general trend of first differences of number marriages over the entire 1985-2004 period is as follows:

|  |  |
| --- | --- |
| Year |  |
| 1985 |  |
| 1986 | -6 |
| 1987 | -4 |
| 1988 | -7 |
| 1989 | 7 |
| 1990 | 40 |
| 1991 | -72 |
| 1992 | -9 |
| 1993 | -28 |
| 1994 | 28 |
| 1995 | -26 |
| 1996 | 8 |
| 1997 | 40 |
| 1998 | -140 |
| 1999 | 114 |
| 2000 | -29 |
| 2001 | 16 |
| 2002 | -91 |
| 2003 | -9 |
| 2004 | 34 |

To test the trendlike behavior compute the first 10 autocorrelation coefficients using the following formula:



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

Then compute the magnitudes of the first 10  (autocorrelation coefficients). Use portmanteau test, Q statistic, to see whether the test is significantly different from a set in which all values are zero.



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

From CHI-SQUARE () VALUES Table the upper .05 point of a  distribution with 10 *df* is ......?...... Since ..........?........, the null hypothesis ................?............ at the 5% significance level.

**The final result:**

These data are:

Choice 1: correlated and exhibit trendlike behavior.

Choice 2: Uncorrelated at any time lag and do not exhibit trendlike behavior.