

## Homework 2

1. Problem 3 in Homework 1.
2. Determine the value  $c$  so that each of the following functions can serve as a probability distribution of the random variable  $X$ :

- (a)  $f(x) = c(x^2 + 1)$ , for  $x = 1, 3, 5$ .
- (b)  $f(x) = c(5 - x^2)$ , for  $0 \leq x \leq 2$ .
- (c)  $f(x) = c \binom{2}{x} \binom{3}{3-x}$ , for  $x = 0, 1, 2$ .
- (d)  $f(x) = c(0.3)^{x-1}$ , for  $x = 1, 2, 3, \dots$

Hint for (d): Geometric series:

$$\sum_{k=0}^{n-1} ar^k = a \frac{1 - r^n}{1 - r}.$$

Here  $a$  is the first term of the series, and  $r$  is the common ratio. As  $n$  goes to infinity and the absolute value of  $r$  less than one, then the series becomes

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r}.$$

3. Let  $X$  and  $Y$  be the random variables with joint probability distribution,  $f(x, y)$ , indicated in the following Table.

$f(x, y)$		$y$		
		-1	0	1
$x$	0	0.2	0.2	0.1
	1	0.1	$c$	0.2

- (a) Find the value of  $c$ .
- (b) Find the marginal probability density functions of  $X$  and  $Y$ , respectively, and determine if  $X$  and  $Y$  are independent?
- (c) Find the conditional probability  $Y < 1$  given  $X = 1$ , that is,  $P(Y < 1 | X = 1)$ .

4. Let  $X$  and  $Y$  have joint probability density function (p.d.f.)

$$f(x, y) = \begin{cases} 6xy^2, & \text{if } 0 < x < 1 \text{ and } 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal probability density functions of  $X$  and  $Y$ , respectively, and determine if  $X$  and  $Y$  are independent? Find the conditional probability  $P(0.5 < X < 1 | Y = 0.4)$ .

5. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let  $X$  be the number of months between successive payments. The cumulative distribution function  $X$  is

$$F(x) = \begin{cases} 0, & \text{if } x < 1; \\ 0.4, & \text{if } 1 \leq x < 3; \\ 0.6, & \text{if } 3 \leq x < 5; \\ 0.8, & \text{if } 5 \leq x < 7; \\ 1.0, & \text{if } x \geq 7. \end{cases}$$

- (a) What is the probability mass function of  $X$ ?
- (b) Compute  $P(1 < X \leq 5)$ .
6.  $X$  is a continuous random variable with probability density function given by  $f(x) = cx$  for  $0 < x < 1$  where  $c$  is a constant. Find  $c$  and the cumulative distribution function (c.d.f.) of  $X$ .