KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS

QUANTUM MECHANICS H.W №1

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PROBLEM (1)

Given the operators $\hat{X} = x$ and $\hat{P} = d/dx$, show that they do not commute. I.e $XP \neq PX$.

PROBLEM (2)

If two operators do not commute, show that we cannot simultaneously diagonalise them .

PROBLEM (3)

A particle of a mass m is described by the normalised wave function:

$$\psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$$

1. Find A
2. For what potential energy function $V(x)$ does ψ satisfy the Scrödinger equation ?
3. Calculate the expected value for x,x^2, p and p^2.
4. find σ_x and σ_p, and their product $\sigma_x\sigma_p$.

PROBLEM (4)

Show that if $\psi(x)$ is real, then $\langle\hat{p}\rangle = 0$.

PROBLEM (5)

It is known that multiplying ψ with a constant phase $e^{i\phi}$ does not affect the physical system. Show, however, if $\phi = \phi(x)$ a function of x, then this is no longer true.

PROBLEM (6)

For $|\psi\rangle = \begin{pmatrix} i \\ -2 \\ 1 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} -1 \\ 3i \\ \sqrt{2} \end{pmatrix}$.

1. Calculate $4|\psi\rangle - i|\phi\rangle$

2. Find $\langle\phi|\psi\rangle$ and $\langle\psi|\phi\rangle$, what do you observe ?

3. Express the vector $|\psi\rangle$ in terms of the basis:
   $|\varepsilon_1\rangle = \begin{pmatrix} 1 \\ -2i \\ 1 \end{pmatrix}$, $|\varepsilon_2\rangle = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $|\varepsilon_3\rangle = \begin{pmatrix} -i \\ 1 \\ -i \end{pmatrix}$.

4. Normalise the vector $|\phi\rangle$.

PROBLEM (7)

Evaluate the following integrals :

1. $\int_0^\infty [\sin(3x) + 2] \frac{d}{dx} \delta(x - \pi) dx$.
2. $\int_{-1}^{+1} e^{|x|+3} \delta(x - 2) dx$.
3. $\int_{-\infty}^{+\infty} \Gamma(x) \delta(x - \frac{1}{2}) dx$.

Use the property of the delta function :

$$f(x)\delta(x-a) = f(a)$$

$$\int_a^b \delta(x-c)dx = \begin{cases} 1 & \text{if } c \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$

and its derivative:

$$\int dx \delta'(x-a) f(x) = -\int dx \delta(x-a) f'(x)$$

Problem (8)

Given the operator :

$$\hat{A} = \begin{pmatrix} .8 & .3 \\ .2 & .7 \end{pmatrix}$$

Find its eigenvalues and eigenvectors.

Problem (9)

Guess a solution to the differential equation based on your study to the eigenvalue problem:

$$\left(\frac{d^2}{dx^2} - \lambda^2\right) f(x) = 0$$

Problem (10)

An electron can take 3 possible energy states $E_1 = 0.5eV$, $E_2 = 1.2eV$ and $E_3 = 1.6eV$. With probabilitie: $P_1 = 0.8$, $P_2 = 0.13$ and $P_3 = 0.07$.
a) Write the Hamiltonian in matrix form.
b) Write the normalised eigenbasis .
c) Find $\langle E \rangle$ and $\sigma(E)$.
d) What is the state ket after measuring the system and finding it taking the second energy state ?
e) Show that $\langle \psi | \psi \rangle = 1$

Problem (11)

Let the state $|\psi\rangle = N|\psi_1\rangle + 2iN|\psi_2\rangle + iN|\psi_3\rangle$, where the kets $|\psi_1\rangle, |\psi_2\rangle$ and $|\psi_3\rangle$ are orthonormal. Find N such that so $|\psi\rangle$ is normalised.

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