

## QUANTUM MECHANICS H.W N<sup>o</sup>4

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### PROBLEM (1)

An electron with kinetic energy  $E = 5.00eV$  is incident on a barrier with thickness  $L = 0.1$  nm and height  $V_0 = 7eV$ . Calculate the tunneling probability. If you know the reflection probability is given by the formula:

$$R = \frac{V_0^2 \sinh^2(\alpha L)}{V_0^2 \sinh^2(\alpha L) + 4(V_0 - E)E}$$

Where  $\alpha$  is the absorption coefficient.

### PROBLEM (2)

Using the above formula for reflection of particle with energy less than the potential height. Derive  $R$  and  $T$  for a particle with energy higher than the potential height.

*Hint:* You might want to use the identity :

$$\sinh ix = \sin x$$

### PROBLEM (3)

Proton and electron with same kinetic energy passing through an potential barrier with height higher than their kinetic energy, which of them has higher probability of tunneling ?

### PROBLEM (4)

Show that for a particle **bounded** in a finite well has a wavefunction that extends outside the well. What does that mean ?

### PROBLEM (5)

We define the quantum revival time, by the time in which the system evolving in time returns to its initial configuration.

Show that the quantum revival time for a particle in an infinite well in the ground state is given by:

$$T = \frac{4mL^2}{\pi\hbar}$$

### PROBLEM (6)

Consider a particle of a potential energy  $V(x) = \lambda x$  where  $\lambda$  is a positive constant. ( $V$  arises, for example from a force field - e.g a uniform electric field).

Write Ehrenfest theorem for  $\langle \hat{x} \rangle$  and  $\langle \hat{p} \rangle$ . Integrate these equations and compare the results to the classical results.

### PROBLEM (7)

Consider a particle described by the normalised wavefunction:

$$\psi(x) = N \frac{e^{ip_0x/\hbar}}{\sqrt{x^2 + a^2}}$$

1. Find  $N$  - the normalisation constant.
2. Compute  $P(-a/\sqrt{3} \leq x \leq +a/\sqrt{3})$
3. Calculate  $\langle \hat{p} \rangle$ .

### PROBLEM (8)

If a quantum SHO was charged by a charge  $q$  and put in a uniform electric field of magnitude  $\epsilon$ . Show that the energy spectrum of this harmonic oscillator is shifted by the quantity:

$$E'_n = E_n + \frac{q^2 \epsilon^2}{2m\omega^2}$$