Chapter(10)
One-Sample Tests of Hypothesis
(Examples)

Types of Tests:

According the alternative hypothesis the hypothesis tests can be one or two tailed.

Case1: One- tailed test (Right-tailed)

For Example:

$$H_0 : \pi \le \pi_0 \quad \textbf{Or} \quad H_0 : \mu \le \mu_0$$
$$H_1 : \pi > \pi_0 \quad\quad H_1 : \mu > \mu_0$$

Case2: One- tailed test (left-tailed)

For Example

$$H_0 : \pi \ge \pi_0 \quad \textbf{Or} \quad H_0 : \mu \ge \mu_0$$
$$H_1 : \pi < \pi_0 \quad\quad H_1 : \mu < \mu_0$$

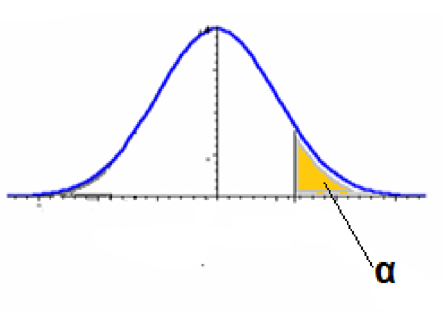
Case3: Two-tailed test

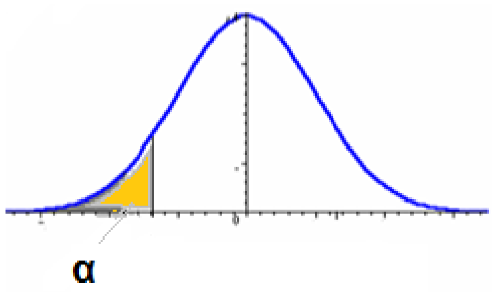
For Example:

$$H_0 : \pi = \pi_0 \quad \textbf{Or} \quad H_0 : \mu = \mu_0$$
$$H_1 : \pi \ne \pi_0 \quad\quad H_1 : \mu \ne \mu_0$$

(Diagram 1)

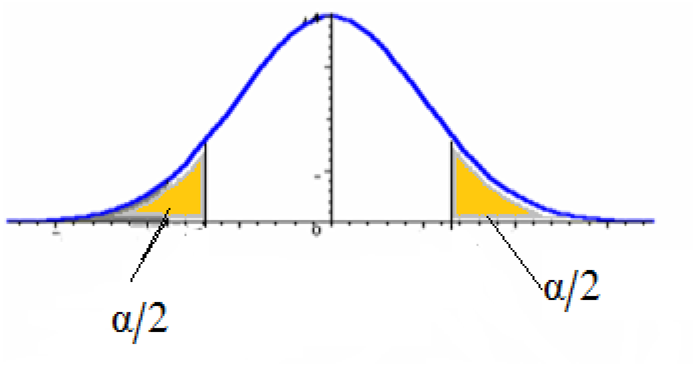
Right tailed **Left tailed**





Case3: If the alternative hypothesis is two- tailed test (α) appear in the left & right sides of the curve.

We divide (α) by 2 (α/2) (diagram 3)



- **Results of a Hypothesis test :**

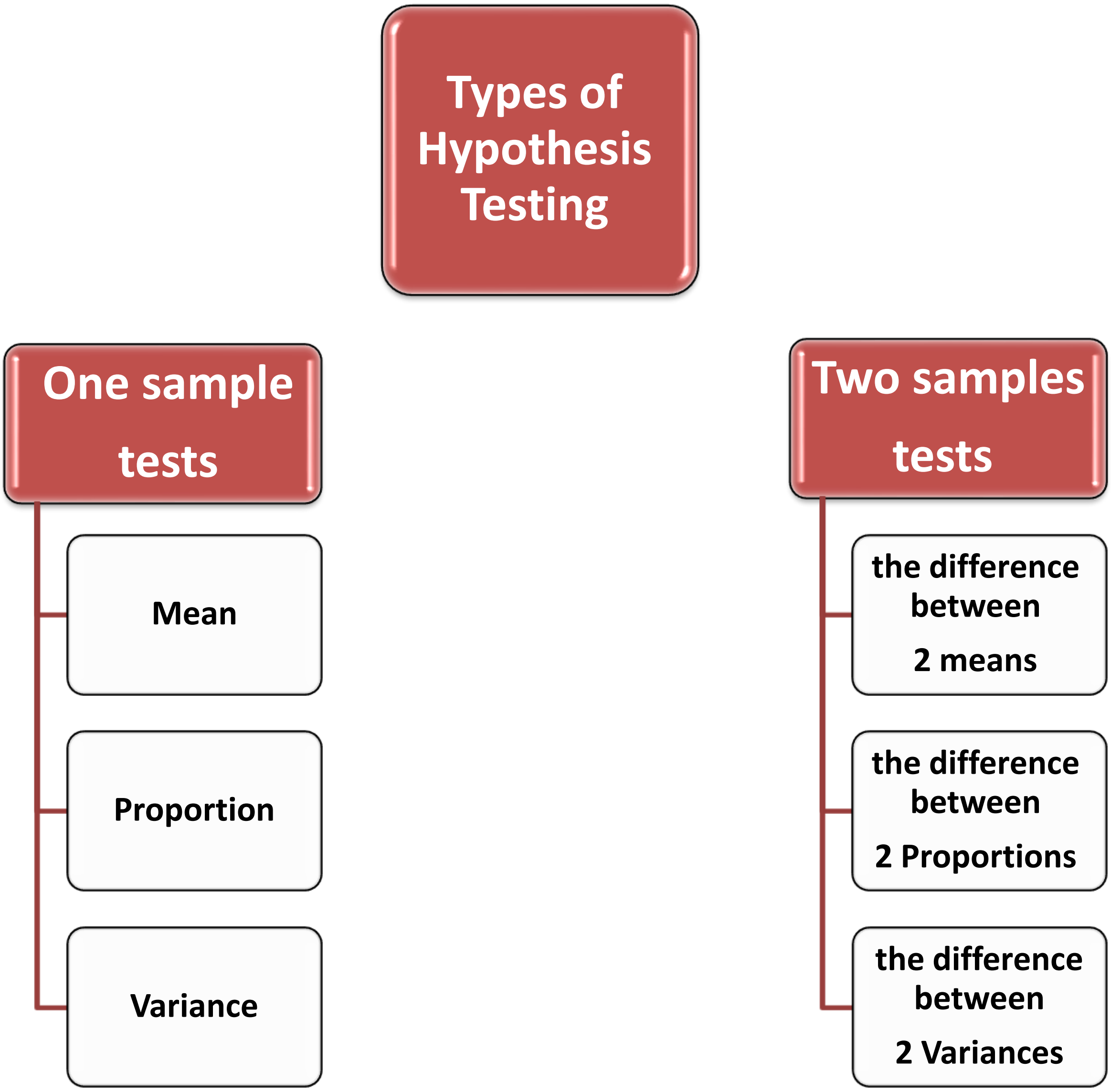
Either: Accept the null hypothesis (H0) as reasonable possibility	It is A week conclusion ; not significant result means the difference only cause of sampling error
Or: Reject the null hypothesis and Accept the researcher hypothesis (H1)	It is A strong conclusion ;a significant result (means there is difference)

Correct decision($1-\alpha$):
Dose not rejecting the null hypothesis, H_0, when It is true

Correct decision(Power) ($1-\beta$):
Rejecting the null hypothesis, H_0, when It is false

Type I error:
Rejecting the null hypothesis, H_0, when It is true(α)

Type II error:
Do not rejecting the null hypothesis, H_0, when It is false (β)


Types of Hypothesis Testing
One sample tests
Mean
Proportion
Variance
Two samples tests
the difference between 2 means
the difference between 2 Proportions
the difference between 2 Variances

Testing for apopulation mean

Step (1): State the null (H_0) and alternate (H_1) hypothesis

$H_0 : \mu \le \mu_0$ **or** $H_0 : \mu \ge \mu_0$ **or** $H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$ $\quad$ $H_1 : \mu < \mu_0$ $\quad$ $H_1 : \mu \ne \mu_0$

Step (2): Select a level of significance.

Step (3): Select the Test Statistic (computed value)

- **Known population Standard Deviation**

$$Z_c = \frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- **unknown population Standard Deviation($n \ge 30$)**

$$Z_c = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

- **unknown population Standard Deviation($n < 30$)**

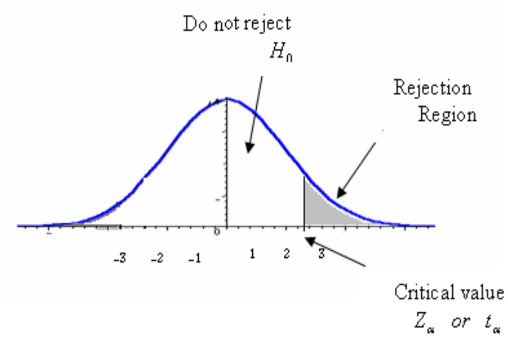
$$t_c = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

Step (4): Selected the Critical value

Types of test	Z	t
The one – tailed test (Right)	Z_{α}	$t_{(\alpha, n-1)}$
The one – tailed test (left)	$-Z_{\alpha}$	$t_{(-\alpha, n-1)}$
The two – tailed test	$\pm Z_{\alpha/2}$	$\pm t_{\left(\frac{\alpha}{2}, n-1\right)}$

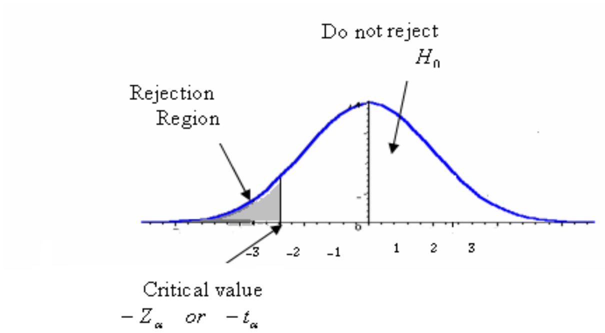
Step (5): Formulate the Decision Rule and Make a Decision

Case1: Reject H_0 if $Z_c > Z_{\alpha}$, $t_c > t_{v,\alpha}$



Case2: Reject H_0 **if** $|Z_c| > Z_{\alpha}$, $|t_c| > t_{v,\alpha}$

This means $Z_c < -Z_\alpha \qquad , t_c < -t_{v,\alpha}$

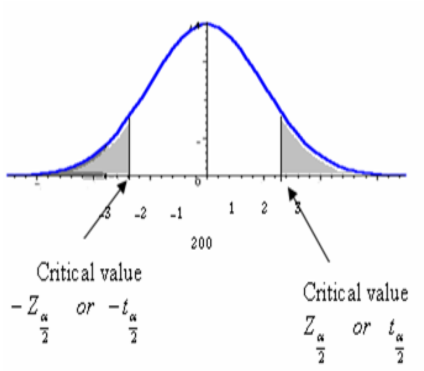


Case3: Reject H_0 if $|Z_c| > Z_{\frac{\alpha}{2}}$, $|t_c| > t_{v,\frac{\alpha}{2}}$ that is:

$$Z_c > Z_{\frac{\alpha}{2}} \qquad , t_c > t_{v,\frac{\alpha}{2}}$$

or

$$Z_c < -Z_{\frac{\alpha}{2}} \qquad , \ t_c < -t_{v,\frac{\alpha}{2}}$$



Known population Standard Deviation
Example (1)

Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in western New YorkState. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, because of market expansion, new production methods have been introduced and new employees hired. The vice president of manufacturing would like to investigate whether there has been a change in the weekly production of the Model A325 desk. Is the mean number of desks produced at Fredonia Plants different from 200 at the 0.01 significance level? IF the vice president takes sample of 50 weeks and he finds the mean is 203.5, use the statistical hypothesis testing procedure to investigate whether the production rate has changed from 200 per week.

Solution:

Step (1): State the null hypothesis and the alternate hypothesis.

H_0: $\mu = 200$

H_1: $\mu \neq 200$

This is Two-tailed test

(Note: keyword in the problem "has changed")

Step (2): Select the level of significance.

α = 0.01 as stated in the problem

Step (3): Select the test statistic

Use Z-distribution since **σ** is known

$\mu_0 = 200 \quad , \quad \bar{X} = 203.5 \quad , \quad \sigma = 16 \quad , \quad n = 50$

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{203.5 - 200}{\frac{16}{\sqrt{50}}} = \frac{3.5}{2.2627} = 1.55$$

Step (4): Formulate the decision rule (Critical value)

$Z_{\frac{\alpha}{2}} = Z_{\frac{0.01}{2}} = Z_{0.005}$ 0.5 – 0.005 =0.4950

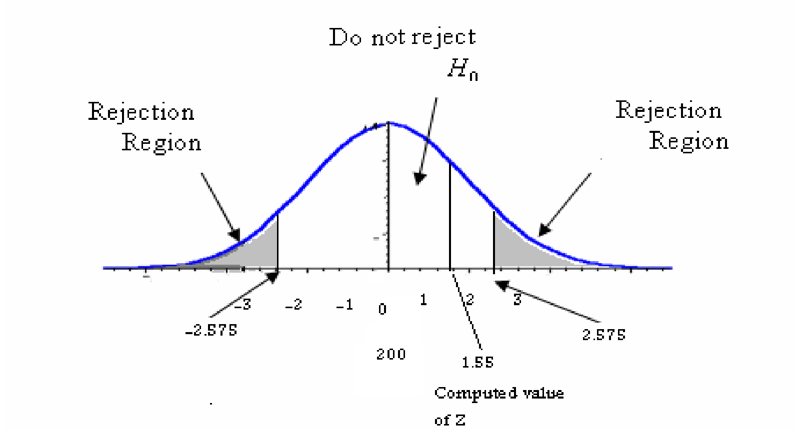
$Z_{0.005} = \pm 2.575$

Reject H_0 if $Z_c > 2.575$ or $Z_c < -2.575$

Step (5): Make a decision and interpret the result.

Reject H_0 if $Z_c < -Z_{\frac{\alpha}{2}}$ or $Z_c > Z_{\frac{\alpha}{2}}$

$Z_c = 1.55 < 2.575$



Because 1.55 does not fall in the rejection region, H0 is not rejected at significance level 0.01. We conclude that the population mean is not different from 200. So we would report to the vice president of manufacturing that the sample evidence does not show that the production rate at the Fredonia Plant has changed from 200 per week. The difference of 3.5 units between the historical weekly production rate and that last year can reasonably be attributed to sampling error.

Example (2)

Suppose you are buyer of large supplies of light bulbs. You want to test at the 5% significant level, the manufacturer's claim that his bulbs last more than 800 hours, you test 36 bulbs and find that the sample mean is 816 hours , with standard deviations 70 hours .should you accept the claim?

Solution:

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu \le 800$

$H_1 : \mu > 800$

This is one-tailed test (right),

(Note: keyword in the problem "more than")

Step 2: Select the level of significance.

α = 0.05 as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since σ is known

$\mu_0 = 800 \quad , \quad \overline{X} = 816 \quad , \quad S = 70 \quad , \quad n = 36$

$$Z = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{816 - 800}{\frac{70}{\sqrt{36}}} = \frac{16}{11.6667} = 1.37$$

Step 4: Formulate the decision rule.

Reject H_0 if $Z_c > Z_\alpha$

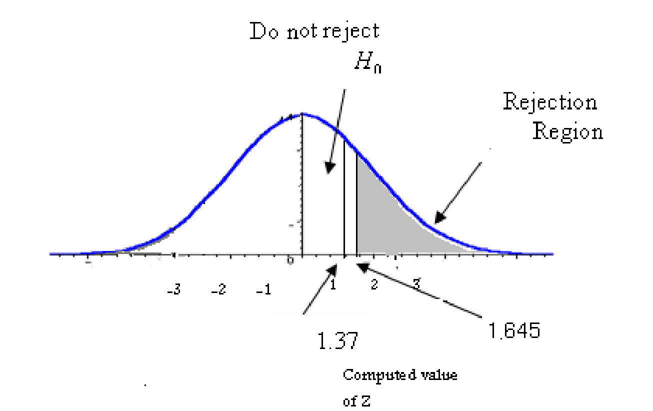
$Z_\alpha = Z_{0.05} = 0.5 - 0.05 = 0.4500$

$Z_{0.05} = 1.645$

Step 5: Make a decision and interpret the result.

Reject H_0 if $Z_c > 1.645$

$Z_c = 1.37 < 1.645$



Because 1.37 does not fall in the rejection region, H_0 is not rejected. We conclude that the population mean is less than or equal 800. The decision that we don't reject the null hypothesis at significant level 0.05, but we reject the alternative hypothesis the manufacturer's claim that μ>800.

The difference of 16 hours between sample and production can reasonably be attributed to sampling error.

Testing for apopulation mean

Unknown population Standard Deviation ($n < 30$)

$$t_c = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

Example (3)

The McFarland Insurance Company Claims Department reports the mean cost to process a claim is \$60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of 26 claims processed last month. The sample information is reported below.

$n = 26$, $\bar{X} = 56.42$, $S = 10.04$

At the .01 significance level is it reasonable a claim is now less than \$60?

Solution:

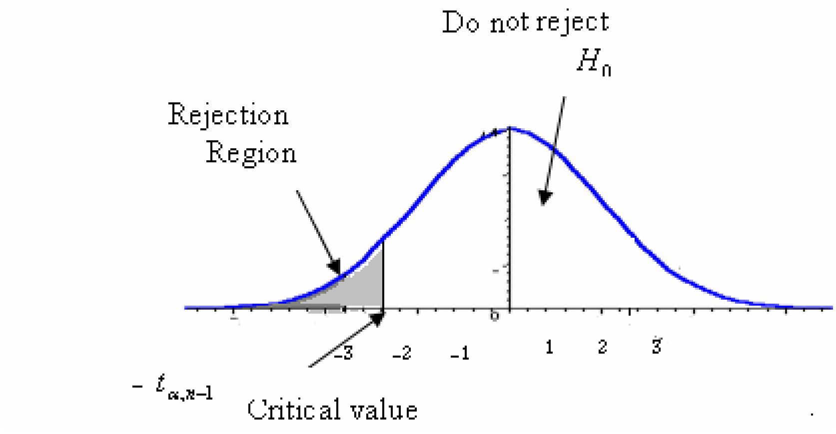
Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu \geq \$60$

$H_1 : \mu < \$60$

This is one-tailed test (Left)

(Note: keyword in the problem “now less than”)



Step 2: Select the level of significance.

α = 0.01 as stated in the problem

Step 3: Select the test statistic.

Use t-distribution since σ is unknown

$\mu_0 = 60$, $\overline{X} = 56.42$, $S = 10.04$, $n = 26$

$$t_c = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{56.42 - 60}{\frac{10.04}{\sqrt{26}}} = \frac{-3.58}{1.9690} = -1.82$$

Step 4: Formulate the decision rule.

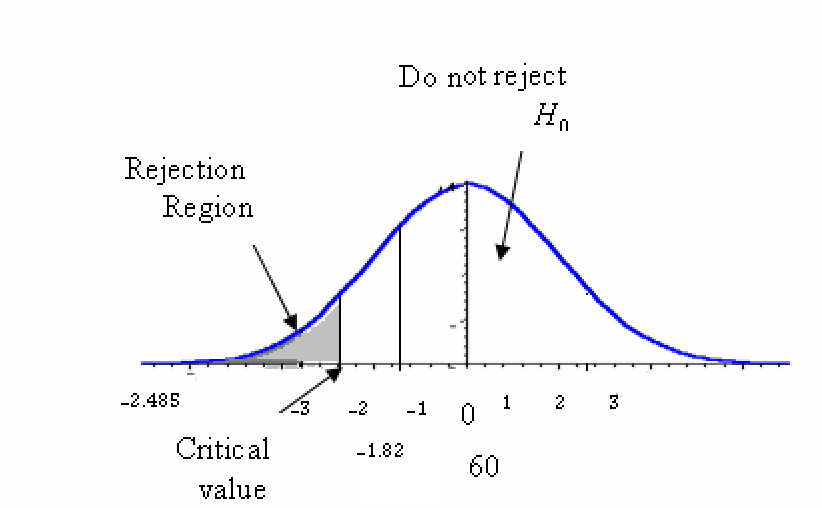
Reject H_0 if $t_c < -t_{\alpha,n-1}$

$$-t_{\alpha,n-1} = -t_{0.01,25} = -2.485$$

Step 5: Make a decision and interpret the result.

Reject H_0 if $t_c < -2.485$

$t_c = -1.82 > -2.485$



Because -1.82 does not fall in the rejection region, H_0 is not rejected at the .01 significance level. We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than \$60. The difference of \$3.58 (\$56.42 - \$60) between the sample mean and the population mean could be due to sampling error.

P-VALUE

The p-value or observed of significance level of a statistical test is the smallest value of α for which H_0 can be rejected. It is the actual risk of committing a type I error, if H_0 is rejected based on the observed value of the test statistic. The p-value measures the strength of the evidence against H_0.
The probability of observing samples data by chance under the Null hypothesis (i.e. null hypothesis is true).

In testing a hypothesis, we can also compare the *p*-value to with the significance level (α).

- **If the *p*-value < significance level(α) , H_0 is rejected (means significant result)**
- **If the *p*-value $\geq$ significance level (α), H_0 is not rejected. (means not significant result).**

Compute the P- value
(Only we used the Z distribution)

Case 1 : If the one – tailed test (Right)

$$\boxed{p-value = P(Z > z_c) = 0.5 - \Phi(z_c)}$$

Case 2: If the one – tailed test (left)

$$\boxed{p-value = P(Z < -z_c) = 0.5 - \Phi(z_c)}$$

Case3: If the two – tailed test

$$p-value = P(Z > z_c) + P(Z < -z_c) = [0.5 - \Phi(z_c)] + [0.5 - \Phi(z_c)] = 1 - 2\,\Phi(z_c)$$

Or

$$\boxed{p-value = 2P(Z > |z_c|) = 2[0.5 - \Phi(z_c)]}$$

Example (4)

Refer to example (1)

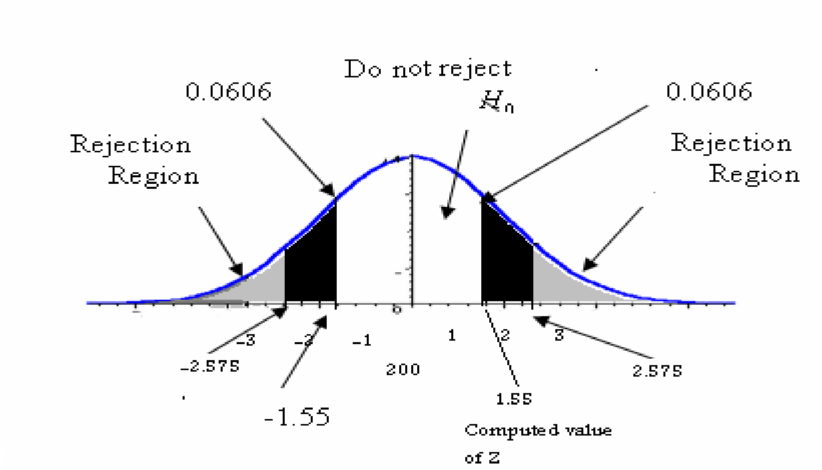
$H_0 : \mu = 200 \;\; H_1 : \mu \neq 200$

$\mu_0 = 200 \quad , \quad \overline{X} = 203.5 \quad , \quad \sigma = 16 \quad , \quad n = 50 \quad , \alpha = 0.01$

$$p-value = 2P(Z > |z_c|) = 2[0.5 - \Phi(z_c)]$$
$$= 2P(Z > 1.55) = 2[0.5 - \Phi(1.55)] = 2[0.5 - 0.4394] = 2 \times 0.0606 = 0.1212$$

$P-value = 0.1212 > \alpha = 0.01$

$\therefore Do \;\; not \;\; reject \; H_0$



Test of hypothesis concerning a population proportion

- A Proportion is the fraction or percentage that indicates the part of the population or sample having a particular trait of interest.
- The sample proportion is denoted by p and is found by $\frac{X}{n}$

Step (1) : State the Null(H_0) and alternate(H_1) hypothesis

$H_0 : \pi \le \pi_0$, $H_1 : \pi > \pi_0$ or $H_0 : \pi \ge \pi_0$, $H_1 : \pi < \pi_0$ or $H_0 : \pi = \pi_0$, $H_1 : \pi \ne \pi_0$

Step (2): Select a level of significance:

Step (3): Select the Test Statistic (computed value)

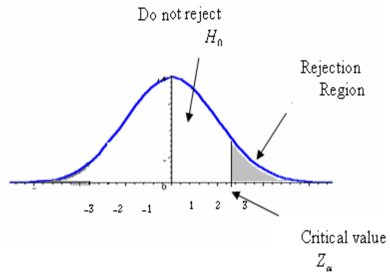
$$Z_c = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

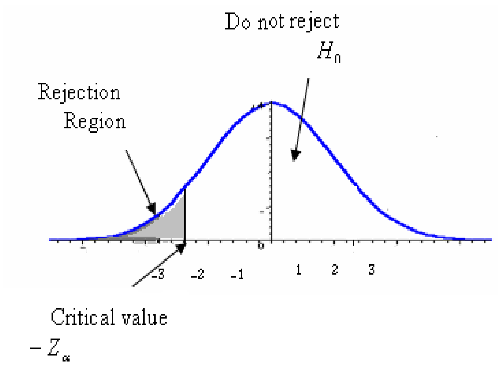
Step (4): Selected the Critical value

The one – tailed test (Right)	Z_{α}
The one – tailed test (left)	$-Z_{\alpha}$
The two – tailed test	$\pm Z_{\alpha/2}$

Step (5): Formulate the Decision Rule and Make a Decision

Case1: Reject H_0 if $Z_c > Z_\alpha$

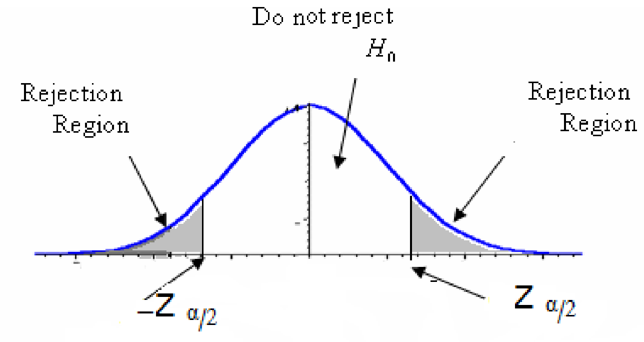




Case2: Reject H_0 if $|Z_c| > Z_\alpha$ or $Z_c < -Z_\alpha$

Case3: Reject H_0 if

$$Z_c > Z_{\frac{\alpha}{2}} \quad or Z_c < -Z_{\frac{\alpha}{2}}$$



Example (5)

Suppose prior elections in a certain state indicated it is necessary for a candidate for governor to receive at least 80 percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office. A sample survey of 2,000 registered voters in the northern section of the state revealed that 1540 planned to vote for the incumbent governor. Using the hypothesis-testing procedure, assess the governor's chances of reelection (the levelofsignificance is 0.05).

Solution:

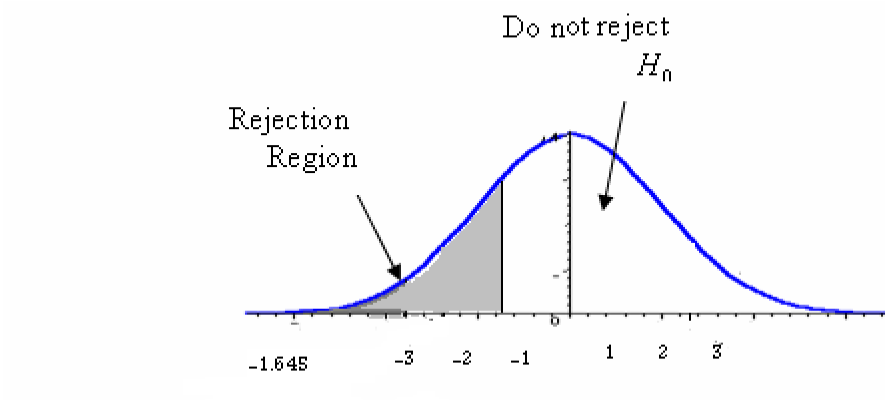
Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \pi \geq 0.80$

$H_1 : \pi < 0.80$

This is one-tailed test (Left)

(Note: keyword in the problem "at least")



Step 2: Select the level of significance.

α = 0.05 as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since the assumptions are met

and $n\pi$ and $n(1-\pi) \geq 5$

$$\pi_0 = 0.80 \ , \quad p = \frac{1540}{2000} = 0.77 \quad , \quad n = 2000$$

$$Z_c = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.78 - 0.80}{\sqrt{\frac{0.8 \times 0.20}{2000}}} = \frac{-0.03}{0.0089} = -3.37$$

Step 4: Formulate the decision rule.

Reject H_0 if $Z_c < -Z_\alpha$

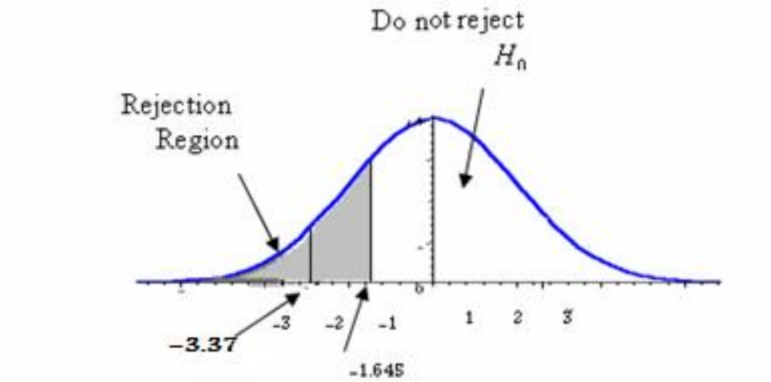
$Z_{0.05} = -1.645$

$0.5 - 0.05 = 0.4500$

Step 5: Make a decision and interpret the result.

Reject H_0 if $Z_c < -1.645$

$Z_c = -3.37 < -1.645$

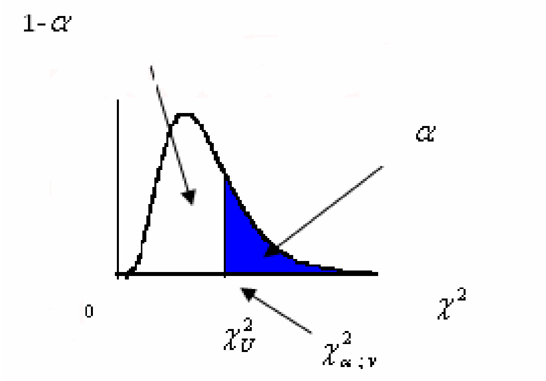


Reject H_0

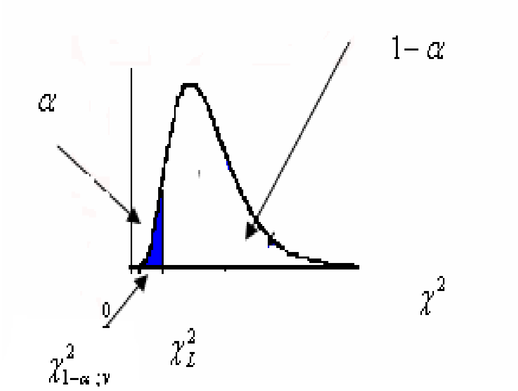
The computed value of (-3.37) is in the rejection region, so the null hypothesis is rejected at the .05 level. The difference of 2.5 percentage points between the sample percent (77 percent) and the hypothesized population percent (80) is statistically significant. The evidence at this point does not support the claim that the incumbent governor will return to the governor's mansion for another four years.

Test of hypothesis concerning apopulation variance

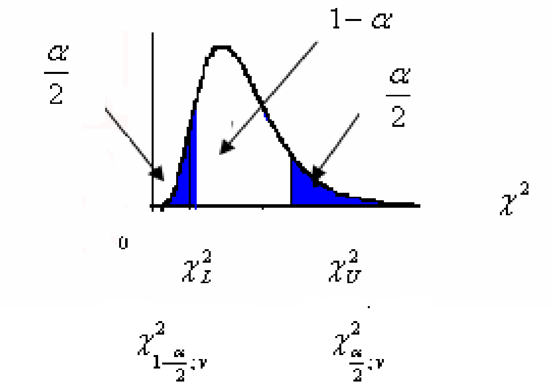
Step (1) : State the Null (H_0) and alternate (H_1) hypothesis



Case 1: $H_0 : \sigma^2 \le \sigma_0^2$
$H_1 : \sigma^2 > \sigma_0^2$



Case 2: $H_0 : \sigma^2 \ge \sigma_0^2$
$H_1 : \sigma^2 < \sigma_0^2$



Case 3: $H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 \ne \sigma_0^2$

Step (2): Select a level of significance

Step (3): Select the Test Statistic (computed value)

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

Step (4): Selected the Critical value

The one – tailed test (Right)	$\chi^2_{(\alpha,n-1)}$
The one – tailed test (left)	$\chi^2_{(1-\alpha,n-1)}$
The two – tailed test	$\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ & $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$

Step (5): Formulate the Decision Rule and Make a Decision

Case1: The one – tailed test (Right)

Reject H_0 if $\chi^2_{(\alpha,n-1)}$

Case2: The one – tailed test (left)

Reject H_0 if $\boldsymbol{\chi^2_c < \chi^2_{(1-\alpha,n-1)}}$

Case3: The two – tailed test; rejectH_0 if

$$\boldsymbol{\chi^2_c > \chi^2_{\left(\frac{\alpha}{2},n-1\right)}}$$

Or

$$\boldsymbol{\chi^2_c < \chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}$$

Example (6)

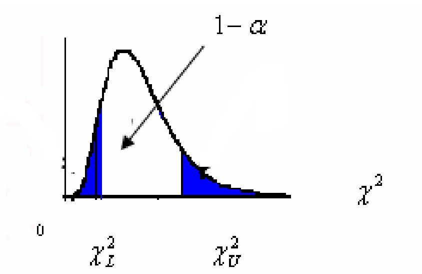
A sample of size 10 produced a variance of 14.Is this sufficient to reject the null hypothesis that σ^2 is equal to 6

When tested using a 0.05 level of significance?

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \sigma^2 = 6$

$H_1 : \sigma^2 \neq 6$



This is two-tailed test

(Note: keyword in the problem "that σ^2 is equal to 6")

Step 2: Select the level of significance.

$\alpha = 0.05$ as stated in the problem

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

$$1 - \frac{\alpha}{2} = 1 - \frac{0.05}{2} = 0.975$$

Step 3: Select the test statistic.

$$\chi_c^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(10-1)14}{6} = \frac{126}{6} = 21$$

Step 4: Formulate the decision rule (Critical value)

$$\chi^2_{0.025\,;.9} = 19.022 \qquad , \qquad \chi_{0.975\,;\,9} = 2.7003$$

Right tail areas for the Chi-square Distribution					
V df\area	Q				
	0.250	0.100	0.050	0.025	0.010
1	1.3233	2.7055	3.8415	5.0239	6.6349
2	2.7726	4.6052	5.9915	7.3778	9.2104
3	4.1083	6.2514	7.8147	9.3484	11.3449
4	5.3853	7.7794	9.4877	11.1433	13.2767
5	6.6257	9.2363	11.0705	12.8325	15.0863
6	7.8408	10.6446	12.5916	14.4494	16.8119
7	9.0371	12.0170	14.0671	16.0128	18.4753
8	10.2189	13.3616	15.5073	17.5345	20.0902
9	11.3887	14.6837	16.9190	19.0228	21.6660
10	12.5489	15.9872	18.3070	20.4832	23.2093

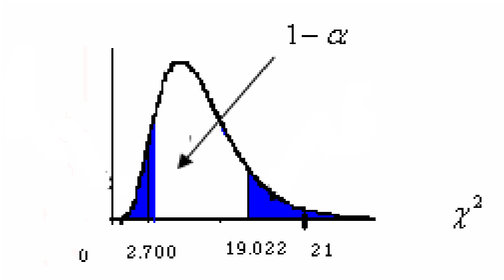
Right tail areas for the Chi-square Distribution					
V df\area	Q				
	0.750	0.900	0.950	0.975	0.990
1	0.101531	0.015791	0.003932	0.000982	0.000157
2	0.575364	0.210721	0.102586	0.050636	0.020100
3	1.212532	0.584375	0.351846	0.215795	0.114832
4	1.922568	1.063624	0.710724	0.484419	0.297107
5	2.674604	1.610309	1.145477	0.831209	0.554297
6	3.454598	2.204130	1.635380	1.237342	0.872083
7	4.254852	2.833105	2.167349	1.689864	1.239032
8	5.070642	3.489537	2.732633	2.179725	1.646506
9	5.898823	4.168156	3.325115	2.700389	2.087889
10	6.737199	4.865178	3.940295	3.246963	2.558199
11	7.584145	5.577788	4.574809	3.815742	3.053496

Step 5: Make a decision and interpret the result.

Reject H_0 if $\chi_c^2 > \chi^2_{\frac{\alpha}{2},v} = \chi^2_{0.025\,;.9} = 19.022$,

Or $\chi_c^2 < \chi^2_{1-\frac{\alpha}{2},v} = \chi_{0.975\,;\,9} = 2.7003$

The decision is to reject the null hypothesis, because the computed χ^2 Value (21) is larger than the critical value (19.022).
We conclude that there is a difference



The Types of errors

Null Hypothesis	Does Not Reject H_0	Rejects H_0
H_0 **is true**	**Do not rejecting The null hypothesis,** H_0 **,when It is true(**$1-\alpha$**)** **{Correct decision}**	**rejecting The null hypothesis,** H_0 **,when It is true(**α**)** **{Type I error}**
H_0 **is false**	**Do not rejecting The null hypothesis,** H_0 **,when It is false(**β**)** **{Type II error}**	**rejecting The null hypothesis,** H_0 **,when It is false (Power) (**$1-\beta$**)** **{Correct decision}**

Note:

The quantity $(1-\beta)$ is called the power of the test because it measures the probability of taking the action that we wish to have occur-that is ,rejecting the H0 when it is false and H1 is true.

$(1-\beta)$ **= P(rejecting the H0 when it is false)**

Ideally, you would like (α) to be small and the power $(1-\beta)$ to be large.

Example (7)

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is greater than 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: "Take a sample of 100 steel bars, at the .05 significance level.

Suppose the unknown population mean of an incoming lot, designated μ_1 is really 10120 psi. Find

a .The type I error (Rejecting the null hypothesis, H_0, when It is true (α)).

b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$).

c .The type II error (Do not rejecting the null hypothesis, H_0, when It is false (β)).

d .The correct decision (Rejecting The null hypothesis, H_0, when It is false ($1-\beta$)).

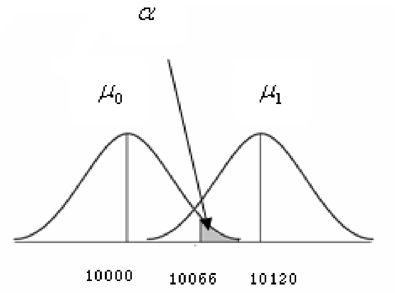
Solution:

$$H_0 : \mu \le 10000 \quad Z = 1.645$$
$$H_1 : \mu > 10000$$

$$\mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 10000 + 1.645 \frac{400}{10} = 10000 + 65.8 = 10066$$

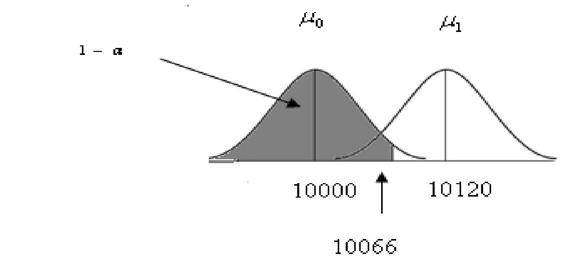
a .The type I error (where rejecting the null hypothesis, H_0, when it is true (α)).

$\alpha = 0.05$



b. The correct decision (Do not rejecting The null hypothesis, H_0, when It is true ($1-\alpha$).

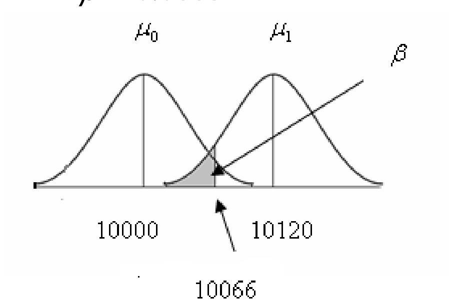
$1-\alpha = 1-.05 = 0.95$



c .The type II error (where accepting the null hypothesis, H_0, when it is false (β)).

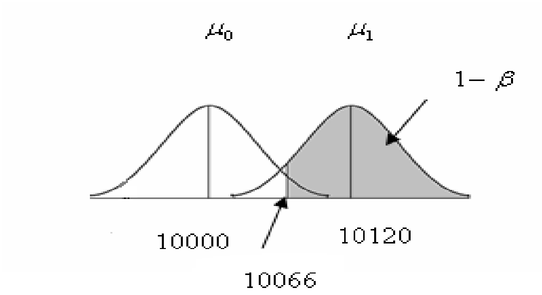
$$P\left(Z < \frac{\overline{X}_c - \mu_1}{\frac{\sigma}{\sqrt{n}}} \right) = P\left(Z < \frac{10066-10120}{\frac{400}{\sqrt{100}}} \right) = P\left(Z < \frac{-54}{40} \right) = 0.5 - P(-1.35 < Z < 0)$$

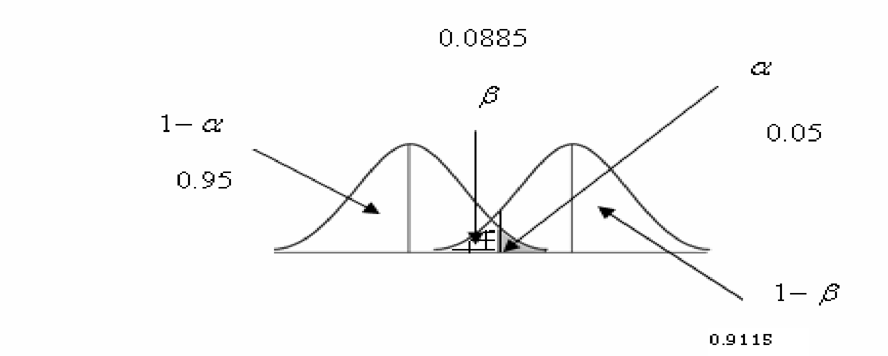
$0.5 - \Phi(1.35) = 0.5 - 0.4115 = 0.0885 \therefore \beta = 0.0885$



d .The correct decision (where H_0 is false and reject it ($1-\beta$)).

$1-\beta = 1-0.0885 = 0.9115$





Example (8)

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is less than 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: "Take a sample of 100 steel bars, at the .05 significance level.

Suppose the unknown population mean of an incoming lot, designated μ_1 is really 9900psi.find:

a .The type I error (Rejecting the null hypothesis, H_0, when It is true (α)).

b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$)).

c .The type II error (Do not rejecting the null hypothesis, H_0, when It is false (β)).

d .The correct decision (Rejecting the null hypothesis, H_0, when It is false ($1-\beta$)).

Solution:

$H_0 : \mu \geq 10000$

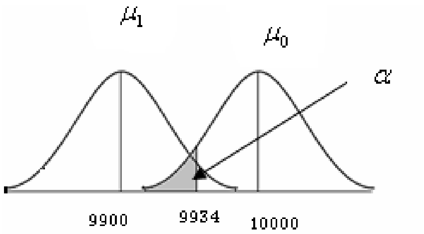
$H_1 : \mu. < 10000$

$Z = 1.645$

$$\mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}} = 10000 - 1.645\frac{400}{10} = 10000 - 65.8 = 9934$$

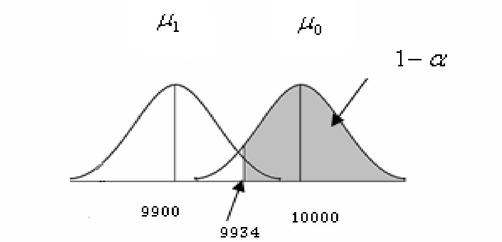
a .The type I error (where rejecting the null hypothesis, H_0, when it is true (α)).

$\alpha = 0.05$



b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$)).

$1-\alpha = 1-0.05 = 0.95$



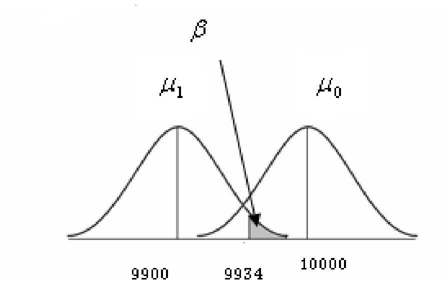
c .The type II error (where accepting the null hypothesis, H_0, when it is false (β)).

$$P\left(Z > \frac{\overline{X}_c - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(Z > \frac{9934-9900}{\frac{400}{\sqrt{100}}}\right) = P\left(Z > \frac{34}{40}\right) = 0.5 - P(0 < Z < 0.85)$$

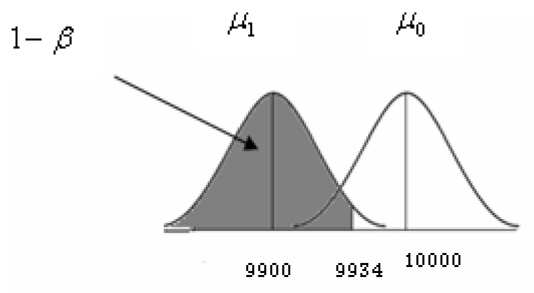
$0.5 - \Phi(0.85) = 0.5 - 0.3023 = 0.1977$

$\therefore \beta = 0.1977$



d .The correct decision (where H_0 is false and reject it ($1-\beta$)).

$1-\beta = 1-0.1977 = 0.8023$

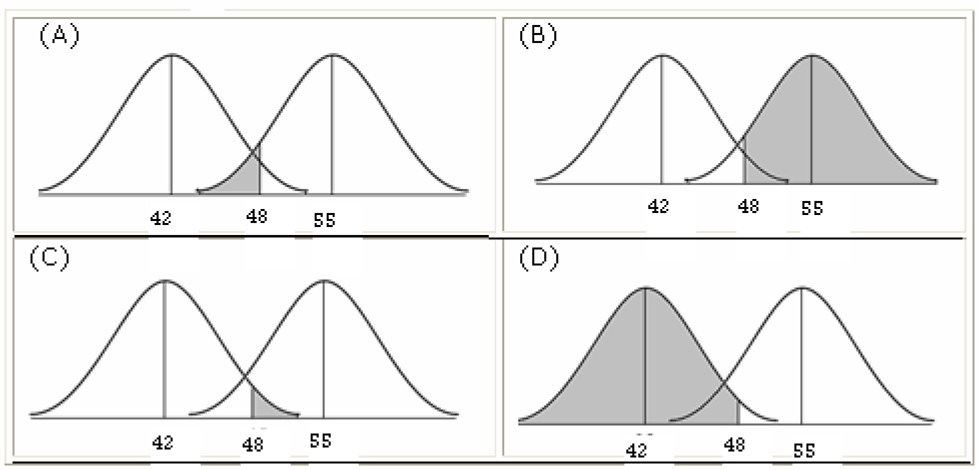


Extra examples

Example (9)

If

$\mu_1 = 55$, $H_0 : \mu \leq 42$
$H_1 : \mu > 42$



Complete the following statements:

a. The probability of not rejecting the null hypothesis when it is true is the shaded area in diagram..........

b. The probability of Type I error is the shaded area in diagram…….

c. The probability of Type II error is the shaded area in diagram…….

d .The probability of rejecting the null hypothesis when it is false is the shaded area in. diagram..........

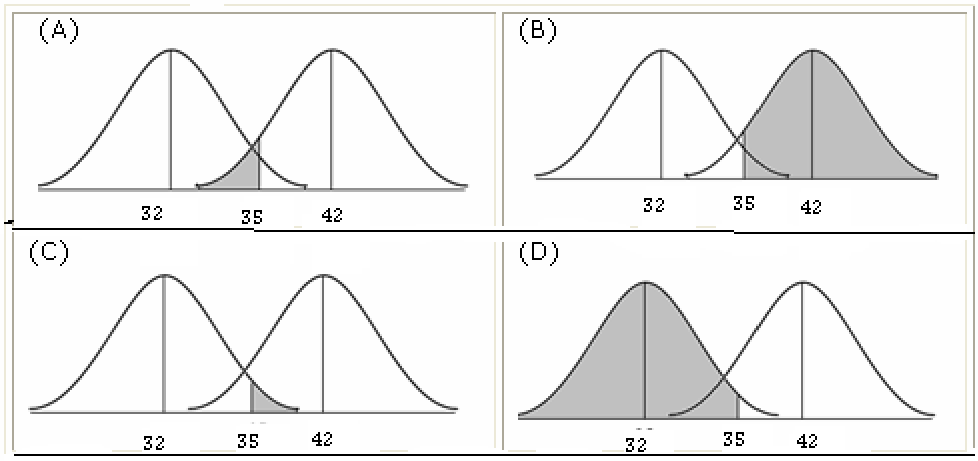
Solution:

a. Diagram D
b. Diagram C
c. Diagram A
d. Diagram B

Example (10)

If

$\mu_1 = 32$, $H_0 : \mu \geq 42$
$H_1 : \mu < 42$



Complete the following statements:

a. The probability of not rejecting the null hypothesis when it is true is the shaded area in diagram..........

b. The probability of Type I error is the shaded area in diagram…….

c. The probability of Type II error is the shaded area in diagram…….

d .The probability of rejecting the null hypothesis when it is false is the shaded area in diagram..........

Solution:

a. Diagram B
b. Diagram A
c. Diagram C
d. Diagram D

Example (11)

Null Hypothesis	Does Not Reject H_0	Rejects H_0
H_0 is true	Do not rejecting The null hypothesis, H_0 ,when It is true($1-\alpha$) Example √ $H_0 : \mu \geq 60$ x $H_1 : \mu < 60$ If the decision is Do not reject H_0 Let $\mu_1 = 80$ $\therefore$ The decision is correct Do not rejecting The null hypothesis, H_0 ,when It is true($1-\alpha$)	rejecting The null hypothesis, H_0 ,when It is true(α) Example x $H_0 : \mu \geq 60$ √ $H_1 : \mu < 60$ If the decision is reject H_0 Let $\mu_1 = 80$ $\therefore$ The decision is incorrect rejecting The null hypothesis, H_0 ,when It is true(α)
H_0 is false	Do not rejecting The null hypothesis, H_0 ,when It is false(β) Example √ $H_0 : \mu \geq 60$ x $H_1 : \mu < 60$ If the decision is Do not reject H_0 Let $\mu_1 = 50$ $\therefore$ The decision is incorrect Do not rejecting The null hypothesis, H_0 ,when It is false(β)	rejecting The null hypothesis, H_0 ,when It is false (Power)($1-\beta$) Example x $H_0 : \mu \geq 60$ √ $H_1 : \mu < 60$ If the decision is reject H_0 Let $\mu_1 = 50$ $\therefore$ The decision is correct rejecting The null hypothesis, H_0 ,when It is false (Power)($1-\beta$)

Note about P_c :

$H_0 : \pi \leq \pi_0$ $H_1 : \pi > \pi_0$,	$\pi_0 + Z_\alpha \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$,	$\beta = P\left(Z < \frac{P_c - \pi_1}{\sqrt{\frac{\pi_1(1-\pi_1)}{n}}} \right)$	

$H_0 : \pi \geq \pi_0$ $H_1 : \pi < \pi_0$,	$\pi_0 - Z_\alpha \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$,	$\beta = P\left(Z > \frac{P_c - \pi_1}{\sqrt{\frac{\pi_1(1-\pi_1)}{n}}} \right)$	