لا يكتب ف
الهام

(2) Given
$x_n = \frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n}$ prove that (x_n) is convergent?

- (x_n) decresing sequense because

$$x_{n+1} - x_n = \left(\cancel{\frac{1}{n+1}} + \cancel{\frac{1}{n+2}} + \dots + \cancel{\frac{1}{2n}} + \frac{1}{2n+1} + \frac{1}{2n+2}\right) - \left(\frac{1}{n} + \cancel{\frac{1}{n+1}} + \cancel{\dots} + \cancel{\frac{1}{2n}}\right)$$

$$= \left(\frac{1}{2n+1} + \frac{1}{2n+2}\right) - \frac{1}{n}$$

$$= \frac{(2n+2) + (2n+1)}{(2n+1)(2n+2)} - \frac{1}{n}$$

$$= \frac{(2n+2) + (2n+1) - (2n+1)(2n+2)}{n(2n+1)(2n+2)}$$

$$= \frac{\boxed{-3n-2}}{n(2n+1)(2n+2)} = -\frac{(\ )}{\ } < 0$$

$\therefore x_{n+1} \leq x_n, \ \forall n \in \mathbb{N}$

$$x_n = \frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n} \geq \underbrace{\frac{1}{n+n} + \frac{1}{n+n} + \dots + \frac{1}{n+n}}_{n+1 \text{ times}}$$

$$= \frac{n+1}{n+n} \geq \frac{n}{2n} = \frac{1}{2}$$

(x_n) bounded below by $\frac{1}{2}$

$\therefore (x_n)$ is convergent