Blackhole Entropy and Quantum Spacetime

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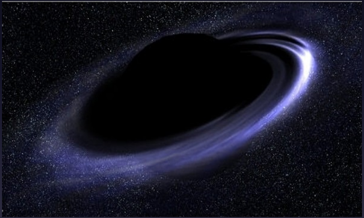
Notions in Blackhole Physics

Carl Schwarzchild, was first to predict the existence of blackholes from solving Einstein equations.



The solution was so strange, even Einstein himself considered it unphysical. Later, it was found to be a suitable solution for the end of the life a massive star.

Schwarzchild blackholes

Schwarzchild blackholes, are the solution for star collapse, without rotation nor charge.

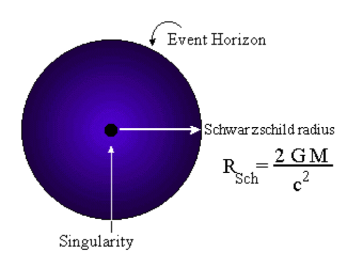
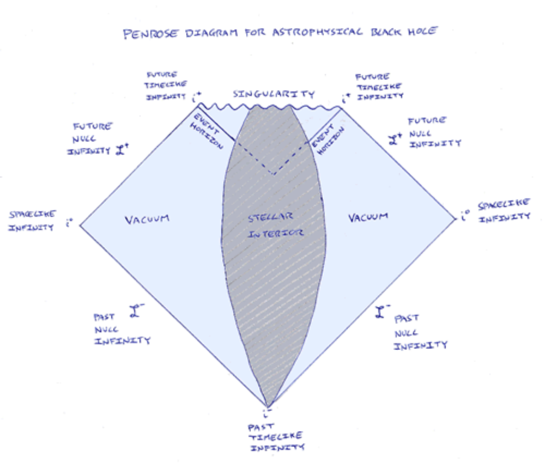


Figure: The structure of a Schwarzchild blackhole

Figure: A drawn Penrose diagram showing a stellar collapse into a blackhole, and formation of a singularity and event horizon



When the blackhole has a non-vanishing angular momentum $J \neq 0$. They are called *Kerr Blackholes*

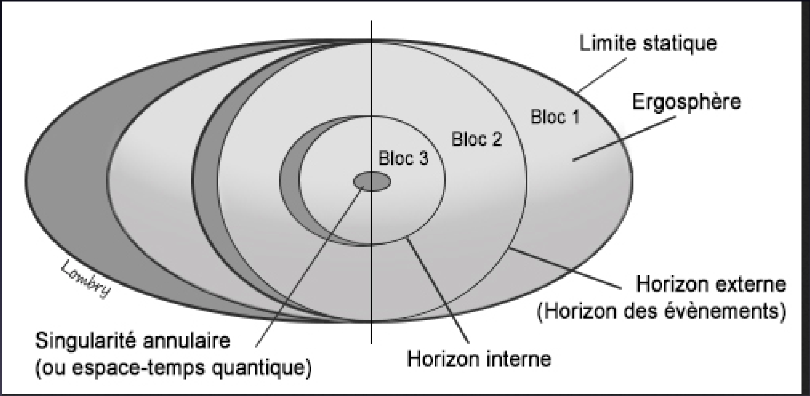
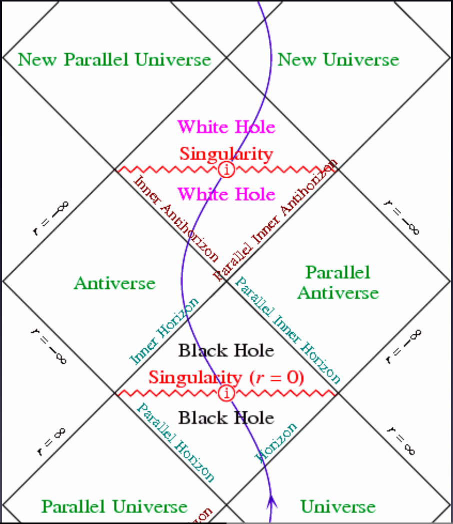
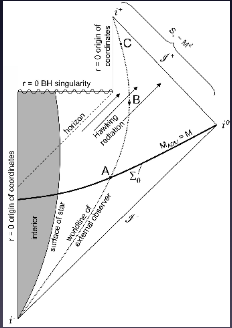


Figure: A Penrose diagram for the maximal extension of Kerr solution



Blackhole Evapouration

In the early 1970's S, Hawking had observed that the surface gravity of the blackhole would affect the quantum fields (that lay beneath). Casing the inertial observers to detect radiation coming out from the blackhole's event horizon.



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Many paradoxes appeared from this observation. Like **Information paradox** and **Firewall paradox**

Laws of Blackhole mechanics

Since blackholes radiate with Plankian spectrum. This implies they have temperature and finite entropy. This leads us to write laws for Blackhole 'mechanics'in parallel to the ones of thermodynamics

- **The zeroth law:** The surface gravity of the blackhole event horizon is constant.

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- **The first law:**

$$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ.$$

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Compare this to the first law of thermodynamics :

$$dE = T\, dS + F\, dD + \mu\, dN.$$

We conclude that:

- The temperature of the blackhole is proportional to its surface gravity

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- The entropy of the blackhole is proportional to its surface area. It was found by Hawking and Bekenstein that:

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Observe how **thermodynamic** quantities are directly linked to **geometric** ones.

Blackholes have no hair!

We can also, read-off the **no-hair theorem,** which states that the only types information that one may obtain about what is inside the blackhole are the mass, the electric ($U(1)$) charge and the angular momentum.

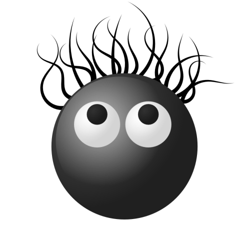

Figure: Sadly, blackholes don't have hair ! ☹

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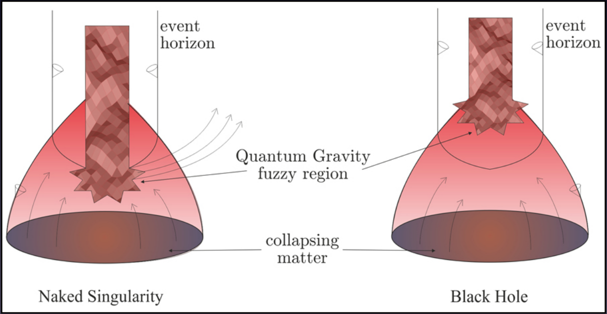
  This however applies only for isolated blackhole. A generalised second law - to allow Hawking radiation- is :

$$\frac{dS_{tot}}{dt} \geq 0$$

For S_{tot} the entropy of the blackhole and its surroundings.

- **The third law:**
  There are no blackholes with vanishing surface gravity. In other words, there are no naked singularities. All blackholes should have an event horizons that shield the universe from their singularities.



What Are the Microstates ??

Recall the formula:

$$S_{BH} = k_b \, \frac{A}{4\ell_p^2}$$

If the blackhole has 10 times the Solar mass, then the entropy would be of order $\sim 10^{80}$. This is huge. What would be the microstates for the blackhole giving it such an enormous entropy ?

In fact, this question is one of the deepest questions in blackhole physics, and answering it is directly connected to the theories of quantum gravity. And the structure of spacetime.

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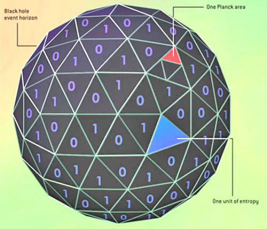

Figure: Information about the 3D world is enconded in the 2D boundary

Blackhole Entropy in LQG

Loop quantum gravity predicts that the spacetime is separated into ’3-space’ foliated into ’4-spacetime’ by ’time’, this is known as the ADM-formalism.

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Each 3-space is made from **spin-networks**, a type of (directed) graph . The spin network states are the quantum states of the 3-space.

The area in LQG is a quantum observable, its expected value for a 2-subsurface Σ is given by:

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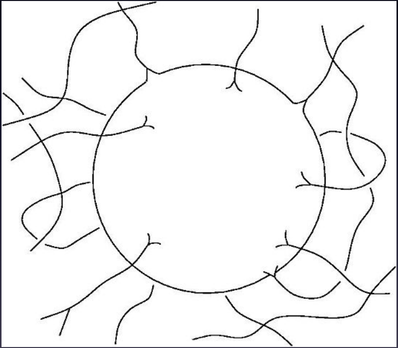
Now we ask, how many SN-states can be found 'puncturing the horizon of the blackhole, i.e. how many $|T_s\rangle$ such that :

$$\langle T_s | \hat{A}r | T_s \rangle \in [A - \ell_p^2, A + \ell_p^2]$$

Module-away the guage motions of $|T_s\rangle$ generated by the constraints of gravity (like diffeomorphsim constraint)
We obtain the number of microstates of the blackhole surface.



Our work

Because of quantum mechanics and gravity, one cannot define distances less than ℓ_p. due to an uncertainty relation for spacetime and radius of curvature:

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Hence, we conclude that the spacetime should admit a foam-like structure at this scale.
This is first mentioned by Wheeler, he called it the **quantum foam.**

In 1995, Hawking has studied this quantum foam more rigorously, he showed it is possible that the spacetime at the Plank scale is a sea of virtual blackholes that pop-out and disappear in **pairs**

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Nevertheless, he insisted that the spacetime is simply-connected, and have a topology of $S^2 \times S^2$.
Extra hidden dimensions could be enclosed by changing the topology to different manifold products. keeping the simple-connectedness.

ER=EPR

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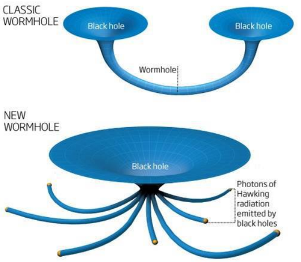
It was conjectured by Susskind and Maldacena in order to resolve the firewall paradox.



This was later proven for a TQFT that two entangeled particles are connected by a wormhole by Beaz. Using category theory.

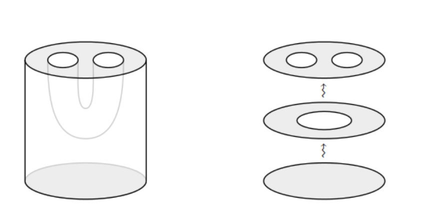


Figure: In category theory, the creation of particle-antiparticle pair, being entangled implies the formation of a wormhole between them

What we aim to do, is to generalise Beaz proof to include 4-D relativistic TQFT, not just for 3-D case.
This can be done by adapting $B - F$ theory for gravity. In a very similar fashion used in the SN formalism in LQG.
However, here the particles of the TQFT are the virtual blackholes (bubbles) of Hawking's 1995 paper.

The spacetime at the quantum scale is conjectured to be made from entangled bubbles, forming wormholes.

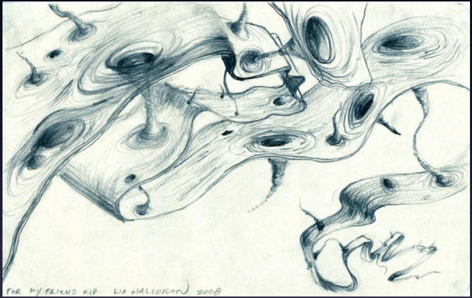


Figure: An artistic vision of a multiply-connected spacetime

Take a 2-D boundary, defined by the blackhole event horizon. These entangled bubbles could define ’paths’for making ’loops’enclosing the inner and outer region. These loops cannot be shrunk to a point.

Take a 2-D boundary, defined by the blackhole event horizon. These entangled bubbles could define ’paths’for making ’loops’enclosing the inner and outer region. These loops cannot be shrunk to a point.
Hence, the event horizon has a non-trivial topology. characterised by its first fundamental group .

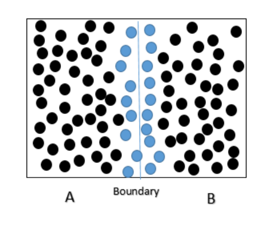


Figure: At the boundary, the nearest bubbles are entangled forming different paths to enter via this boundary

This is very similar to the number of microstates defined in LQG. If one carries the path integral of the theory over the event horizon, defining the result is related to the first fundamental group.

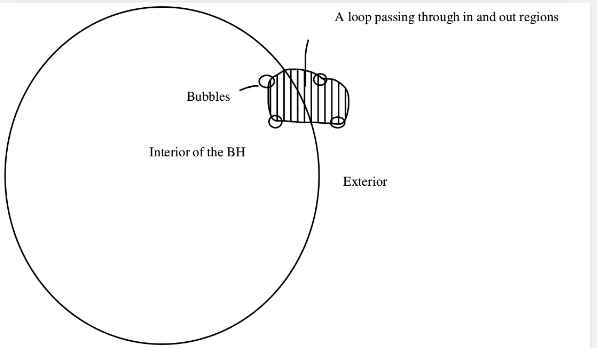


Figure: Calculating the first fundamental group of the event horizon surface

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- The spacetime therefore is multiply-connected. This can be felt by the 2-surfaces. In particular the BH event horizons.
- The topology of the event horizon is directly connected to the entropy of the blackhole.

Thank You !

End of lecture …

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