King Saud University
College of Science
Mathematics Department

Program of the degree of Master of Science in Mathematics (Courses Option)

College of Science
Department of Mathematics

Introduction:

The Department of Mathematics is amongst the oldest and largest departments of the College of Science. It came into existence in 1958 (1378 H), the year of establishment of the College of Science.

The present faculty is fifty strong; of these twelve are full professors, twenty two are associate professors and sixteen are assistant professors. The faculty is augmented by fourteen demonstrators and lecturers. The female branch of the department first offered a B.Sc. program in 1993 (1413 H).

The Department of Mathematics is one of the first departments to offer graduate programs in the University. Its M.Sc. program (by courses and thesis) was passed in 1979 (1400 H). Since then, many students have graduated from the program. Some have continued their studies for the Ph. D. degree in mathematics and have since strengthened the faculty pool of the Department of Mathematics in King Saud University, and that of such departments in the various other universities of the Kingdom. Other graduates shoulder the responsibilities of leading many government departments, particularly in the Ministry of Education.

To meet the demands of a rising number of students, particularly female students, for further high education, the Department instituted a Ph.D. program in 1993 (1414 H). So far, nine students have successfully obtained the degree, while others are either preparing their theses or preparing for the comprehensive examination.

Notwithstanding all the success, and in spite of the significant increase in faculty mass, the Department has failed to satisfy the wishes of an ever increasing number of students for enrollment in its M. Sc. program. This failure is attributable to the unusually high load imposed on the Department by the service courses it offers throughout the University, a situation that depletes the pool of the faculty available for theses supervision. To circumvent the problem, a parallel M.Sc. program (by courses) is now established; it is hoped that this would greatly increase the number of students admitted to the M.Sc. program without significantly increasing the Department's load commitment.

It is worth noting here that the new program does not fall short of the aims of the old program; it offers the same especialities: algebra, analysis, geometry and topology, computing and discrete mathematics, and applied mathematics. It is further hoped that, by presenting a strong broad background of courses with considerable depth, the new program may be better suited to equip the enrolled students for meeting the demands of the Ph.D. program (such as the comprehensive examination).

Degree Title

Master of Science in Mathematics

Program Objectives

The new program is not for a new degree but rather a scheme designed to renew standards and course contents while reducing the role enjoyed by the thesis in the old program. Therefore, the general aims of the previous program are retained. Moreover this process of reward is expected to add the following benefits:

1) Provision of a broad spectrum of mathematical knowledge suitable for students whose graduate study ends with an M.Sc. degree, in particular for those working in the field of education.
2) In the presence of a successful Ph.D. program the breadth of mathematical knowledge furnished by the new program should provide students with the required background for successfully meeting the demands of the Ph.D. program (such as the comprehensive examination).
3) Enrollment of a bigger number of applicants without significantly increasing the Department's load by supervision commitments.
4) The new program offers some recent specialities (such as discrete mathematics and computational mathematics) which bestow on the graduates a professional aura that should prove useful for employment in some applied fields like computer systems and digital communication.

Admission Requirements

1) The applicant must satisfy all the admission conditions stipulated in the unified graduate studies regulations for Saudi universities.

(article 15 of the unified law organizing the graduate studies in Saudi universities.)

2) The applicant must hold a B.Sc. degree in mathematics from King Saud University or an equivalent degree.
3) The applicant must pass the written admission examination held by the Department.

Degree Requirements

The student must successfully complete 42 units of M. Sc. courses in four semesters; these are divided as follows:

1) 35 units of compulsory courses.
2) 4 units from optional courses.
3) 3 units for a research project.

Program Structure

42 credit hours including a research project of three credit hours:

Course code	Number & Type of Courses	Credit Hours
--	9 Core Courses	35
--	1 Elective Course	4
Math 599-1	Research Project	3
Total		42

Program Plan:

1. **Core Courses:**

Course Code	Course Title
Math 510-1	Ordinary Differential Equations
Math 530-1	Introduction to Discrete Structures
Math 539-1	Selected Topics in Mathematics
Math 540-1	Group Theory and Field Extensions
Math 541-1	Introduction to Rings and Modules
Math 550-1	Numerical Analysis
Math 570-1	Geometry and Topology
Math 580-1	Measure Theory
Math 581-1	Functional Analysis

2. **Elective Courses:**

Course Code	Course Title
Math 512-1	Partial Differential Equations
Math 514-1	Quantum Mechanics
Math 531-1	Combinatorics
Math 532-1	Ordered Sets
Math 542-1	Rings and Modules
Math 543-1	Algebraic Number Theory
Math 551-1	Numerical Linear Algebra
Math 552-1	Numerical Solutions of Ordinary Differential Equations
Math 571-1	Algebraic Topology
Math 572-1	Differential Geometry
Math 582-1	Complex Analysis
Math 583-1	Advanced Functional Analysis

Program Schedule:

First Semester

Course Code	Course Title	Credit Hours
Math 540-1	Group Theory and Field Extensions	4
Math 570-1	Geometry and Topology	4
Math 580-1	Measure Theory	4
Total		12

Second Semester

Course Code	Course Title	Credit Hours
Math 541-1	Introduction to Rings and Modules	4
Math 550-1	Numerical Analysis	4
Math 581-1	Functional Analysis	4
Total		12

Third Semester

Course Code	Course Title	Credit Hours
Math 510-1	Ordinary Differential Equations	4
Math 530-1	Introduction to Discrete Structures	4
Math ???-1	Elective Course*	4
Total		12

Fourth Semester

Course Code	Course Title	Credit Hours
Math 539-1	Selected Topics in Mathematics	**3**
Math 599-1	Research Project	**3**
Total		**6**

* The student chooses a 4 unit course from a list of 4 optional courses offered by the Department subject to its capabilities.

Courses Description

Math 510-1 Ordinary Differential Equations **4(3+1)**
Existence and uniqueness of solutions of linear systems. Stability Theory, Liapunov method. Two-dimensional autonomous systems, Poincare-Bendixson Theory. Second order linear differential equations: properties and zeros of the solutions. Sturm-Liouville theory: Linear differential operators in L^2 space, eigen-functions and eigen-values of self-adjoint linear operators, orthogonal polynomials and eigen-function expansions.

References :
1-Ordinary Differential Equations, By Birkhoff Rota, J. Wiley.
2- Functions of Mathematical Physics, By Spain and Smith.

Math 512-1 Partial Differential Equations **4(3+1)**
Elementary theory of distributions. Types of linear partial differential equations. Fundamental solutions of some linear differential operators such as the Laplacian, the heat, and the wave operators. Applications to solving non-homogeneous problems. Green's function and its application to boundary-value problems. Elliptic equations: Boundary-value problems for the Laplacian operators in n-dimensions, harmonic functions, the Dirichlet problem, existence and uniqness of solutions. Generalizations and applications. Solutions of parabolic and hyperbolic equations in Sobolev spaces. Generalized mixed problems.

References :
1- An Introduction to Partial Differential Equations, By Michael Renardy and Robert C. Rogers.
2- Introduction to Partial Differential Equations and Boundary Value Problems, By Dennemeyer.

Math 514-1 Quantum Mechanics **4(3+1)**
Foundation of quantum mechanics and its mathematical tools. Energy spectra for some molecules. Dirac formulation of quantum Mechanics (fundamental concepts), elementary quantum systems (application of wave mechanics and uncertainity principle). Symmetries in quantum mechanics (groups matrix and its generators), operator algebra. Quantum theory of radiation. Quantum theory of damping-Langevin approach.

References :
1- Application of Quantum Mechanics, By Amnon Yariv.
2- Statistical Properties of Radiation, By W. H. Louisell.

Math 530-1 Introduction to Discrete Structures **4(3+1)**

Graphs, Directed graphs, Basic definitions, Isomorphism of graphs, Subgraphs, Paths and cycles, Matrix representation of graphs, Connectedness, Bridges, Cut-vertices. Trees, Spanning trees, Weighted graphs and minimum spanning trees, Shortest paths, Eulerian circuits, Hamiltonian cycles, Tournaments, Applications. Planar graphs, Euler's Formula, Kuratowski's theorem. Graph coloring, Vertex coloring, Edge coloring, Map coloring, Chromatic polynomials. Ordered sets, Comparability and covering graphs, Dilworth theorem, Block designs, Latin squares, Orthogonal Latin squares, Finite geometries, Basic definitions and properties.

References :

1- Introduction to Graph Theory, By Douglas B. West, 2^{nd} Edition, Prentice Hall, 2001.

2- A Course in Combinatorics, By J. H. Van Lint and R. M. Wilson, Cambridge University Press, 1992.

Math 531-1 Combinatorics **4(3+1)**

General counting methods, The inclusion-exclusion principle, Ordinary generating functions, Exponential generating functions, Recurrence relations, Linear recurrence relations, Homogeneous recurrence relations, Nonhomogeneous recurrence relations, Polya counting theory, Equivalence relations, Permutation groups, Burnside's Lemma, Inequivalent colorings, Cycle index, Polya's enumeration formula. Mobius inversion formula, Techniques of computing Mobius functions, Mobius functions for special lattices. The Pigeon-hole principle and its generalizations, Ramsey numbers, Ramsey theorem .

Prerequisite : Math 530-1

References :

A Course in Combinatorics, By J. H. Van Lint and R. M. Wilson, Cambridge University Press, 1992.

Math 532-1 Ordered Sets **4(3+1)**

Basic definitions, Fundamental theorems, Chain decomposition, Linear extensions, Fixed points, Algorithmic aspects of chain decomposition, Cutsets, Fibers, Cutest and fiber decomposition, Drawing , The diagram , Algorithmic and structural aspects of linear extensions, Dimension, Jump number, Sorting, Linear extensions and probability, Single machine scheduling, Many machine scheduling, Order preserving maps, Structure and classification, Lattics, Free Lattices, Distributive Lattices, Planar Lattices.

Prerequisite : Math 530-1

Reference :

Introducing Ordered Sets, Lectures on Ordered Sets, By Ivan Rival, 1996.

Math 539-1 Selected Topics in Mathematics **3(3+0)**

The course covers selected topics in mathematics suggested by the student's supervisor.

Math 540-1 Group Theory and Field Extensions **4(3+1)**

Group action on a set, Series of groups, Solvable groups, Supersolvable groups, Polycyclic groups and nilpotent groups, Semi-direct product and group extensions, Free groups, group presentations, Finite and algebraic field extensions, Normal and separable extensions, Galois extensions, Galois group and Artin's Theorem.

Reference :

Algebra, By Thomas W. Hungerford; Holt, Rinehart and Winston Inc, 1974

Math 541-1 Introduction to Rings and Modules **4(3+1)**

Modules, Module homomorphisms, Exact sequences, External direct product, Internal direct product, Complete direct sum, Direct sum, Free modules, Projective and injective modules, Modules over principal ideal domains, Algebras, Tensor products, Localization, Primary decomposition, Integraly closed domains, Chain conditions, Noetherian and Artinian rings.

Prerequisite : Math 540-1

Reference :

Algebra, By Thomas W. Hungerford; Holt, Rinehart and Winston Inc, 1974

Math 542-1 Rings and Modules **4(3+1)**

Ring Extensions, Dedekind domains, Hilbert's Nullstellensatz. Simple and primitive rings, The Jacobson radical of a ring, Semi-simple rings, Wedderburn-Artin theorem for semi-simple Artinian rings, Essential and small submodules, Singular submodules, Radical of a module, Primitive rings and density theorem, prime ideals and lower nilradical.

Prerequisite : Math 540-1 and Math 541-1

Reference :

Algebra, By Thomas W. Hungerford; Holt, Rinehart and Winston Inc, 1974

Math 543-1 Algebraic Number Theory **4(3+1)**

Number fields, Solvable and radical extensions, Abel's theorem, Kummer's theorem, The ring of algebraic integers, Trace and norm, Discriminant and integral basis, Prime factorization of ideals, Norm of ideals, Quadratic and cyclotomic fields. Transcendence bases, Linear disjointness and separability.

Prerequisite : Math 540-1 and Math 541-1

Reference :

1- A Classical Introduction to Modern Number Theory (Graduate Texts in Mathematics) (v. 84) by Kenneth Ireland and Michael Rosen (Hardcover) - Aug 1, 1998
2- Algebraic Number Theory (Graduate Texts in Mathematics) by Serge Lang (Hardcover) - Jul 19, 2000

Math 550-1 Numerical Analysis **4(3+1)**

Floating point arithmetic and rounding errors, well-posed computation and convergence. Numerical methods for solving nonlinear equations with one variable: bisection, regula-falsi, functional iterative, Newton, secant and Aitken Δ^2. Error and convergence analysis for these methods. Special numerical methods for solving polynomials: evaluation of polynomials and their derivatives. Matrix and vector norms, convergence of vectors. Method for solving system of nonlinear equations: Fixed point, Newton, finite difference Newton, quasi-Newton, steepest descent. Error and convergence analysis for these methods

References :

1-Numerical Methods for Nonlinear Equations and Optimization, By J. E. Dennis and R. B. Schnabel, Prentice-Hall Englewood Cliffs, 1983.

2- Iterative Methods for Linear and Nonlinear Equations, By C. T. Kelly, SIAM Philadelphia, 1995.

Math 551-1 Numerical Linear Algebra **4(3+1)**

Eigenvalues and eigenvectors. Special matrices. Direct methods for solving systems of linear equations. Analyzing the errors involved using these methods. Iterative refinement method. Iterative methods for solving systems of linear equations: Jacobi, Gauss-Seidel and SOR. Error and convergence analysis for these methods. Various methods for solving least square problems along with analytical and computational discussion. Numerical methods for the matrix eigenvalue problems: power and inverse power iteration, Jacobi, Givens, Householder, LR and QR. Singular value decomposition. Applications.

References :

1- Matrix Computation, By G. H. Golub and Ch. F. Vanloan, Johns Hopkins University Press, 1977.

2- Introduction to Matrix Computations, By G. Stewart, 1973.

Math 552-1 Numerical Solutions of Ordinary Differential Equations **4(3+1)**

Multi-step methods for solving initial value problems in ODE: Euler, midpoint, trapezoidal, Simpson, Adam-Multon and other. Derivation these methods using Taylor expansion, integration and interpolation techniques. Error and convergence analysis for these methods: local an global errors, consistency and stability. Predictor corrector methods, error (Milne's device) and stability. Various Runge-Kutta methods: derivation some of these methods. Error and stability of Runge-Kutta methods. Numerical solutions for solving system of first order ODE. Finite difference and shooting methods for solving linear and nonlinear boundary value problems in ODE. Error and convergence analysis for these methods. Applications.

References :

1- Computational Methods in Ordinary Differential Equations, By J. D. Lambert, John Wiley, 1991.

2- Numerical Methods for Two-Point Boundary Value Problems, By H. B. Keller, 1968

Math 570-1 Geometry and Topology **4(3+1)**

Connected spaces, Path connected spaces, Connected components, Locally connected spaces, Quotient spaces, The separation axioms (Hausdorff, Regular, Normal). Differentiable manifolds, Submanifolds of $\mathbb{R}^n$ and Classical Lie groups, Tangent spaces, Differentiable mappings between manifolds, Inverse and Implicit function theorems on manifolds.

References :

1- Topology, By James R. Munkres, 2nd Edition, Upper Saddle River, NJ, Prentice Hall, 2000.
2- Calculus on Manifolds, A modern Approach to Classical Theorems of Advanced Calculus, W. A. Benjamin Inc, 1965.
3- Differentiable Manifolds, By Yozo Matsushima, Translated by E. T. Kobayashi, Marcel Dekker Inc, 1972.

Math 571-1 Algebraic Topology **4(3+1)**

Homotopy of paths, the fundamental group. The fundamental group of the circle, the punctured plane, $\mathbf{S^n}$ and surfaces. Covering spaces, lifting properties. The classification of covering spaces, universal cover and deck transformations. Chain complexes, simplicial homology, Homotopy invariance, Excision, Mayer-Vietoris sequence, Cellular homology.

References :

1- Homology Theory, An Introduction to Algebraic Topology, By James W. Vick, 2nd Edition, Springer-Verlag, 1994.
2- Singular Homology Theory, By William S. Massey, Springer-Verlag, 1980.

Math 572-1 Differential Geometry **4(3+1)**

Definition and examples of manifolds, submanifolds, Immersions and submersions, Lie groups, Equivalence classes of curves and derivations, Tangent vectors, The tangent bundle of a manifold, Vector fields and flows, Lie derivatives and bracket, Differential forms, Integration on manifolds.

Reference :

Differentiable Manifolds, By Yozo Matsushima, Translated by E. T. Kobayashi, Marcel Dekker Inc, 1972.

Math 580-1 Measure Theory **4(3+1)**

Rings, Algebra, σ-algebra, Monotone classes, Measure, elementary properties, outer measure, extension, completion and approximation theorems, Lebesgue's measure, Lebesgue-Stieltje's measure, measurable functions, integration with respect to a measure, the main theorems, the convergence of measurable functions, Radon-Nikodym theorem (absolutely continuous functions), Fubini-Tonelli theorem, L^p spaces: Holder and Minkowski inequalities, completeness of L^p spaces, L^p space as a Banach space, the dual of L^p space.

References :

1-Measure Theory, By Donald L. Cohn, Birkhauser Boston Inc., Boston, MA, 1993

2-A Course in Functional Analysis, By J.B. Conway, Springer-Verlag, New-York, 1990

Math 581-1 Functional Analysis **4(3+1)**

Banach spaces: Basic properties and examples, convex sets, subspaces and quotient spaces, linear functionals and the dual spaces, Hahn-Banach theorem, the uniform boundedness principle, the open mapping theorem and closed graph theorem. Hilbert spaces: the Riesz representation theorem, orthonormal bases, isomorphic Hilbert spaces. Operators on Hilbert spaces: Basic properties and examples, adjoints, projection, invariant and reducing subspaces, positive operators and the polar decomposition, self-adjoint operators, normal operators, isometric and unitary operators, the spectrum and the numerical range of an operator.

References :

1-Kreyszig, E., A Introductory functional analysis with applications. John Wiley & Sons 1978.

2-Simmons, G.F. , Introduction to topology and modern analysis. Krieger Publishing Company 1963.

3-Rudin, W., Functional analysis. McGraw-Hill 1973.

Math 582-1 Complex Analysis **4(3+1)**

Holomorphic functions, Cauchy-Riemann equations, power series, logarithmic function, Cauchy integral formula (general form), Analytic functions, zeros of holomorphic functions. Maximum principle, Liouville's theorem, fundamental theorem of algebra, open mapping theorem, Schwarz lemma, Mobius transformations, Rouche's theorem, Conformal mappings and Riemann's theorem. Topology on the space of holomorphic functions. Montel's theorem, Harmonic and subharmonic functions. Weirstrass's theorem, Mittag Leffler theorem, Introduction to several complex variables.

References :

1- J. V. Ahlfors, Complex analysis, McGraw-hill (1978)

2- J. B Conway, Functions of one complex variable

3- Greene, R.E. and Krantz, S. Function theory of one complex variable. John Wiley & Sons 1997.

Math 583-1 Advanced Functional Analysis **4(3+1)**

Spectrum of an operator, compact linear operators and their spectral properties, spectral properties of bounded self-adjoint operators, positive operators, product of positive operators, square root of positive operator, projection operators: Theorem (positivity, norm), Theorem (partial order), Theorem (product of projections), Theorem (sum of projections), Theorem (difference of projections), Spectral family of a bounded self-adjoint operator, Banach algebras, Gelfand' mapping, spectral theorem for normal operators.

References :

1-Kreyszig, E., A Introductory functional analysis with applications. John Wiley & Sons 1978.

2-Rudin, W., Functional analysis. McGraw-Hill 1973.

3-Douglas, R.G., Banach algebra techniques in operator theory. 2nd edition, Springer. Academic Press 1972.

Math 599-1 Research Project **3(3+0)**

The student undertakes supervised independent study and review of current research papers in an active branch of Mathematics .