Curves
Math 473
Introduction to Differential Geometry
Lecture 1

Dr. Nasser Bin Turki

King Saud University
Department of Mathematics

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Curves

How can we describe a curve? There are different ways to describe a curve.

1. **By geometric properties**: The set of all points in $\mathbb{R}^2$ at the distance 1 from the origin $(0, 0)$ is the unit circle with centre at the origin.

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2. **By an equation:** as the set of all points $(x, y) \in \mathbb{R}^2$ which satisfy the equation $x^2 + y^2 = 1$:

$$\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 = 1\}.$$

3. **By a parametrisation**: as a path of a moving object, see definitions in the following slide.

Defnation (1):

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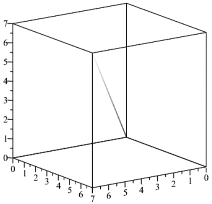
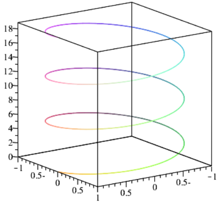
Examples:

(1): $\alpha : I \mapsto \mathbb{R}^3, \alpha(t) = (t, t, t)$ is a parametrisation of a straight line.



Examples: (2): $\alpha : I \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, t)$ is a Helix



Defnation (2):

A parametrised curve $\alpha : I \mapsto \mathbb{R}^3$ is **regular** if the map α can be differentiated infinitely many times and

$$\alpha'(t) \neq (0,0,0), \quad \forall t \in I.$$

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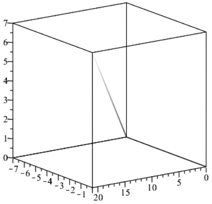
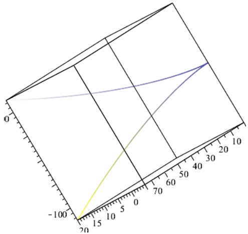
Example: Determine which of the following Curve α is regular:

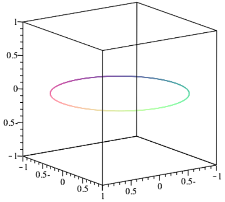
(1): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (3t, t-7, t)$



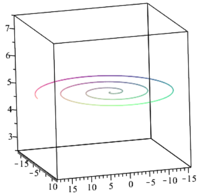
(2): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (3t^2, t^2 - 5, t^3)$



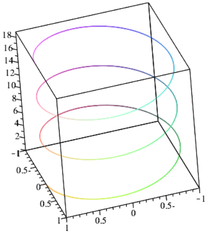
(3): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, 0)$



(4): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t\cos t, t\sin t, 5)$



(5): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, t)$



Thanks for listening.