# Curves
# Math 473
# Introducation to Differential Geometry
# Lecture 1

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How can we describe a curve? There are different ways to describe a curve.

1. **By geometric properties**: The set of all points in $\mathbb{R}^2$ at the distance 1 from the origin $(0,0)$ is the unit circle with centre at the origin.

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2. **By an equation:** as as the set of all points $(x,y) \in \mathbb{R}^2$ which satisfy the equation $x^2 + y^2 = 1$:

$$\{(x,y) \in \mathbb{R}^2 | x^2 + y^2 = 1\}.$$

3. **By a parametrisation**: as a path of a moving object, see definitions in the following slide.

Defnation (1): We define a **parametrised curve in** $\mathbb{R}^3$ as a map $\alpha : I \to \mathbb{R}^3$, where $I$ is an interval in $\mathbb{R}^3$.

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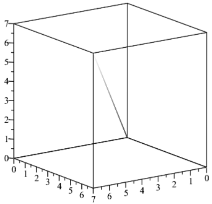
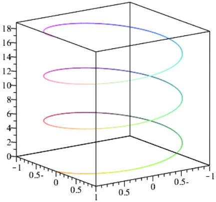
Examples:

(1): $\alpha : I \mapsto \mathbb{R}^3, \alpha(t) = (t, t, t)$ is a parametrisation of a straight line.



Examples: (2): $\alpha : I \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, t)$ is a Helix

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# Regular Curve

Defnation (2): A parametrised curve $\alpha : I \mapsto \mathbb{R}^3$ is **regular** if the map $\alpha$ can be differentiated infinitely many times and

$$\alpha'(t) \neq (0,0,0), \quad \forall t \in I.$$

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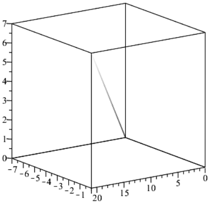
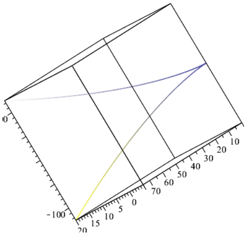
$$\alpha'(t) \neq (0,0,0), \quad \forall t \in I.$$

Example: Determine which of the following Curve $\alpha$ is regular:

(1): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (3t, t-7, t)$



(2): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (3t^2, t^2 - 5, t^3)$

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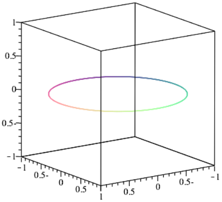
(3): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, 0)$

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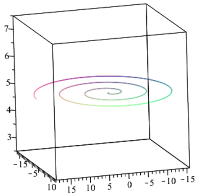
(4): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t\cos t, t\sin t, 5)$

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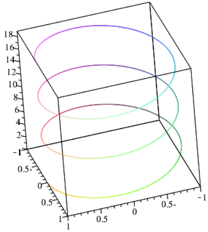
(5): $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, t)$

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*Thanks for listening.*