Curves
Math 473
Introducation to Differential Geometry
Lecture 2

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$$|\alpha'(t)| = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}.$$

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$$T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$$

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Thanks for listening.