PHY331
Magnetism

Lecture 4

Last week…

- Discussed Langevin’s theory of diamagnetism.
- Use angular momentum of precessing electron in magnetic field to derive the magnetization of a sample and thus diamagnetic susceptibility.
- Set the scene for the calculation of paramagnetic susceptibility.

This week…

- Will calculate the paramagnetic susceptibility using an entirely classical approach.
- We will obtain Curie's Law for a paramagnet.

$$\chi = \frac{M}{H} = \frac{\mu_0 N m^2}{3kT} = \frac{C}{T}$$

- We will then start to put quantum framework together to obtain a quantum theory of paramagnetism.

The calculation of the magnetisation M

Recall the energy of a dipole in a field $$E = -\underline{m} \cdot \underline{B}$$

$$E = -mB\cos\theta \tag{3}$$

$$dE = +mB\sin\theta \, d\theta \tag{4}$$

The number of dipoles with ***energy*** between E and $E + dE$ will be,

$$dn = c \, \exp(-E/kT) \, dE$$

If we substitute (3) and (4), this is also the number of dipoles with ***orientation*** between θ and $\theta + d\theta$

$$M = \sum \text{[1] Resolved component of a dipole in field direction} \times \text{[2] Number of dipoles with this orientation}$$

which is (2) in the equation above.

The ***resolved component*** of the dipoles with this orientation which is (1) in the equation above, is clearly just,

$$m\cos\theta \qquad (1)$$

therefore the total magnetisation is,

$$M = \int_0^\infty m\cos\theta \; dn \qquad (5)$$

for convenience, we can write,

$$M = N\langle m \rangle \qquad (6)$$

Where N = the total number of dipoles
and $<m>$ = their ***average component*** in the field direction

hence using (5) and (6)

$$\frac{\langle m \rangle}{m} = \frac{\int_0^N \cos\theta \, dn}{N} = \frac{\int_0^N \cos\theta \, dn}{\int_0^N dn}$$

so substituting we get,

$$\frac{\langle m \rangle}{m} = \frac{\int_0^\pi \cos\theta \quad c\, mB \sin\theta \quad \exp\left(mB \cos\theta / kT \right) d\theta}{\int_0^\pi c\, mB \sin\theta \quad \exp\left(mB \cos\theta / kT \right) d\theta}$$

***note**: the term inside the expo nential is positive, be cause it h as one negative sign from the - E and another from the - mBcosθ*

also, this equation is one of the general type $\langle x \rangle = \int x dm \Big/ \int dm$

to simplify and solve the above, substitute,

$$x = \frac{mB\cos\theta}{kT} = a\cos\theta$$

which gives,

$$\frac{\langle m \rangle}{m} = \frac{\frac{1}{a}\int_{-a}^{a} x\, e^{x}\, dx}{\int_{-a}^{a} e^{x}\, dx}$$

whose solution is

$$\frac{\langle m \rangle}{m} = \frac{e^{a} + e^{-a}}{e^{a} - e^{-a}} - \frac{1}{a}$$

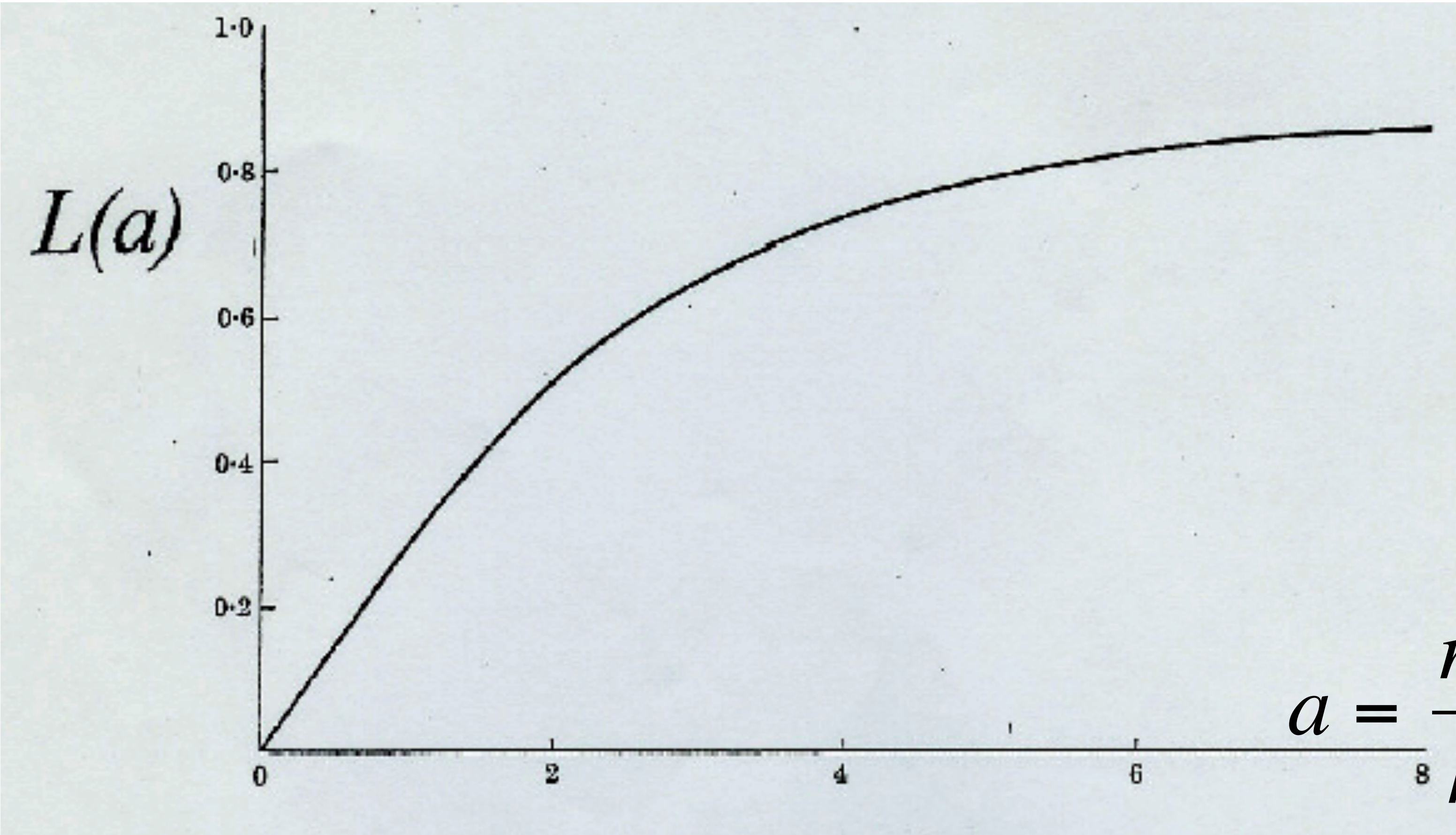
$$\frac{\langle m \rangle}{m} = L(a)$$

$$\langle m \rangle = mL(a)$$

Langevin function *L(a)*

Langevin Function

Plot as a function of $$a = \frac{mB}{kT}$$



$$a = \frac{mB}{kT}$$

Now $$M = N\langle m\rangle = NmL(a)$$

where $$a = \frac{mB}{kT}$$

Can see that Magnetization increases as the applied field *(B)* ***increases*** or, it ***decreases*** as the temperature *T increases. This* confirms that there is "competition" between these two factors,

$$k_B T \approx -\underline{m}.\underline{B}$$

Calculating magnetic susceptibility

We have, $$M = N\langle m\rangle$$

or $$M = NmL(a)$$

When a is small the Langevin function $L(a)$ can be expanded as a power series.

When a is small $mB << kT$

Thus this approximation works in the ***limit of small applied fields and high temperatures***.

$$a = \frac{mB}{kT}$$

$$L(a) = \frac{a}{3} + \frac{a^2}{45} + \frac{2a^5}{945} + \ldots\ldots\ldots \approx \frac{a}{3}$$

so that,

$$M \approx Nm\frac{a}{3}$$

or

$$M \approx \frac{Nm^2 B}{3kT} \approx \frac{Nm^2 \mu_0 H}{3kT}$$

and the paramagnetic susceptibility is then,

$$\chi = \frac{M}{H} = \frac{\mu_0 N m^2}{3kT} = \frac{C}{T}$$

- This is Curie's Law and C is Curie's constant

Quantum theory of paramagnetism

What changes do we need to make to Langevin's classical theory ?

i) detailed changes to the way in which

the magnetic dipole moment is defined,

The general definition is $\underline{m} = \underline{m}(\underline{L})$

Using QM, the angular momentum $\underline{L}$ is quantised, and ***every electron*** is now specified by ***four*** quantum numbers

$$(n \quad l \quad m_l \quad s)$$

n, ***principal quantum number*** (energy of the orbit)

$$n = 1, 2, 3,$$

l, ***angular momentum quantum number***

(elliptical orbits of different eccentricity)

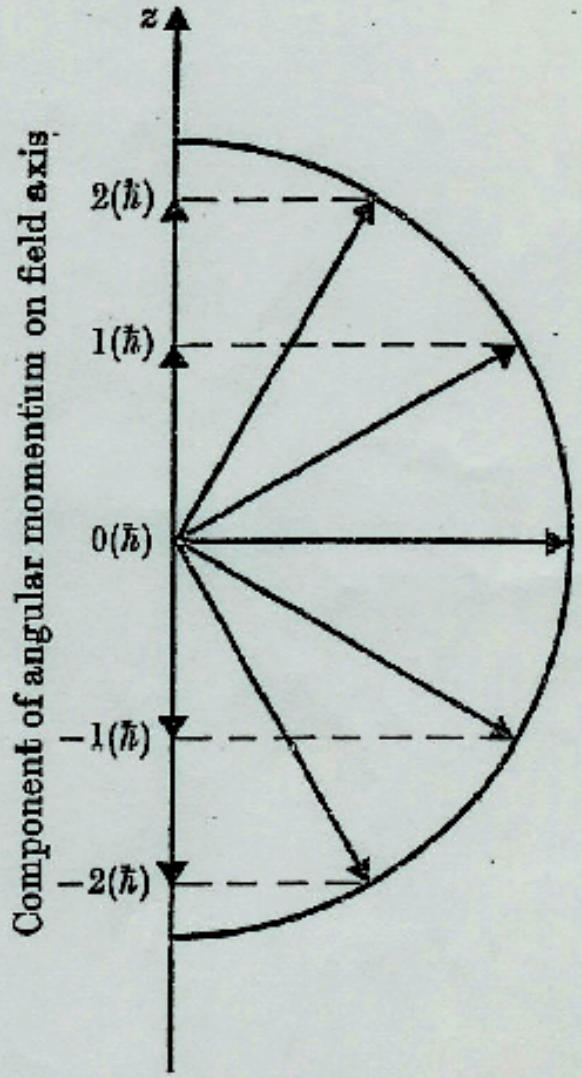
$$l = 0, 1, \ldots\ldots(n\text{-}1)$$

m_l, ***magnetic quantum number*** (describes the orientation of l in B and gives the magnitudes of the aligned components of l)

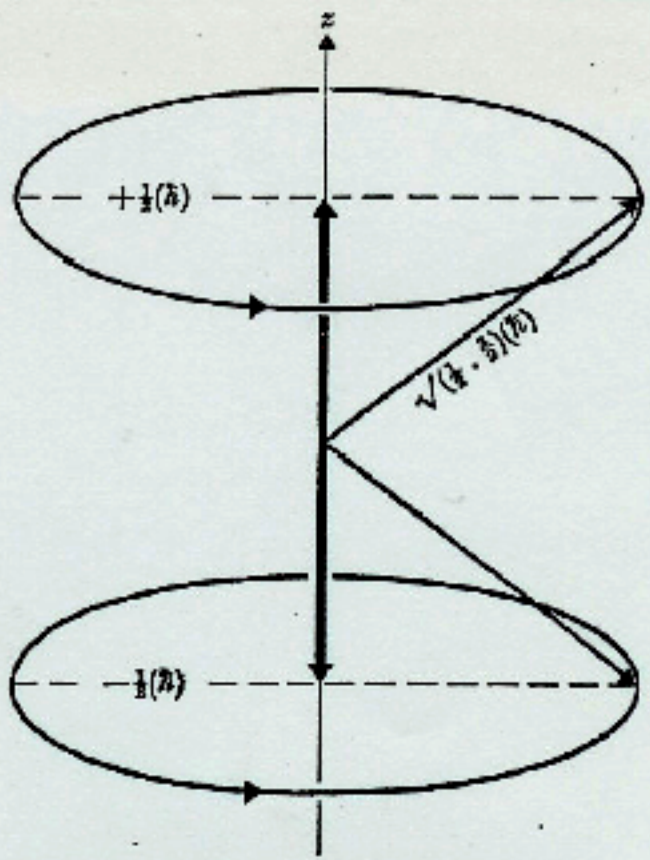
$$-l < m_l < +l$$

s, ***spin quantum number*** (electron spin)

$$s = \pm ½$$



Quantization of orbital angular momentum. The figure is drawn for the case of $l = 2$.



Quantization of spin angular momentum $s = \frac{1}{2}$.

Quantization….

Now, the ***angular momentum*** is quantised in terms of the ***quantum mechanical eigenvalue***

$$\hbar\sqrt{l(l+1)}$$

The ***spin motion*** is also quantised in terms of its ***quantum mechanical eigenvalue***

$$\hbar\sqrt{s(s+1)}$$

Will find that both l and s contribute to magnetic dipole moment…

Next week

- We will see how we can add angular momentum and spin together to define the *total* angular momentum of an atom (J).
- This will then be used to obtain the magnetic dipole moment of an atom and thus define paramagnetic susceptibility.
- Result will be similar to classical derivation!

Summary

- We calculated paramagnetic susceptibility using an entirely classical approach and obtained Curie’s Law for a paramagnet.
- We saw that quantization of angular momentum and spin will be important components of a quantum theory of paramagnetism.