Sphere Curves
Math 473
Introducation to Differential Geometry
Lecture 11

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Definition (1): We say that A **Sphere Curve** is a regular parametrised space curve $\alpha : I \mapsto \mathbb{R}^3$ if there is a point $m \in \mathbb{R}^3$ and $r \in \mathbb{R}$ such that

$$|\alpha(t) - m|^2 = r^2.$$

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$$|\alpha(t) - m|^2 = r^2.$$

In other words,let
$\alpha(t) = (x(t), y(t), z(t)), \quad m = (a, b, c).$
Then, α is a sphere curve if

$$|\alpha(t) - m| = \sqrt{(x(t) - a)^2 + (y(t) - b)^2, (z(t) - c)^2} = r$$

Hence,

$$(x(t)-a)^2+(y(t)-b)^2,(z(t)-c)^2=r^2$$

So, this is an equation of sphere in $\mathbb{R}^3$ with center $m=(a,b,c)$ and radius $r>0$.

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We denoted the sphere whose center is m and radius r by

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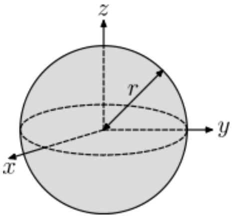
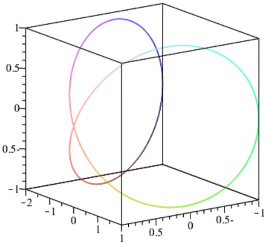


The curve $\alpha(t) = (-\cos 2t, -2\cos t, \sin 2t)$ is a sphere curve.



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If α is a sphere curve that lies on the sphere $S(m, r)$, then

$$\alpha(t) = m - \rho(t)N(t) - \rho'(t)\sigma(t)B(t),$$

where $\rho(t) = \frac{1}{\kappa(t)}$, $\sigma(t) = \frac{1}{\tau(t)}$.

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In particular,

$$\rho^2(t) + \big(\rho'(t)\sigma(t)\big)^2 = r^2.$$

Proof:

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Proof:

Thanks for listening.