Helix Curve
Math 473
Introduction to Differential Geometry
Lecture 12

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Definition (1): A general **Helix (cylindrical helix)** is a regular parametrised space curve $\alpha : I \mapsto \mathbb{R}^3$ such that for a constant unit vector $\vec{u}$, we have

$$T(t) \bullet u = \cos\theta.$$

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i.e. the tangent $T(t)$ makes a constant angle θ with $\vec{u}$ for all t.

Example(1): Show that every plane curve is a Helix.

Definition (2): A **circular helix**, (i.e. one with constant radius) has constant band curvature and constant torsion. The Circular Helix has the form

$$\alpha(t) = (a\cos t, a\sin t, bt),$$

where $\alpha : \mathbb{R} \mapsto \mathbb{R}^3$, a, b are constant and $a > 0$.

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The Circular refers to the fact that the projection of α in the plane is a circle.

Theorem(1):(Lancret, 1802)
Let $\alpha : I \mapsto \mathbb{R}^3$ be a regular parametrised space curve with $\kappa(t) \neq 0$, for all $t \in I$. Then, α is a Helix if and only if $\frac{\tau(t)}{\kappa(t)} = c$, where c is constant.

Proof:

Thanks for listening.