Bertrand Curves
Math 473
Introduction to Differential Geometry
Lecture 15

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Theorem(1):
Let $\alpha : I \mapsto \mathbb{R}^3$ be unit speed curve. Let the curvature $\kappa(t) > 0$ and the torsion $\tau(t) \neq 0$ for all $t \in I$.

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Let $\alpha : I \mapsto \mathbb{R}^3$ be unit speed curve. Let the curvature $\kappa(t) > 0$ and the torsion $\tau(t) \neq 0$ for all $t \in I$.Then, the Curve α is Bertrand curve if and only if

$$a\kappa(t) + b\tau(t) = 1,$$

holds for all $t \in I$ and $a, b \in \mathbb{R}$.

Proof:

Example(1) Let $\alpha : I \mapsto \mathbb{R}^3$ be unit speed curve whose curvature $\kappa(t) = e^t$ and the torsion $\tau(t) = e^{-t}$. Show that the curve α is **Not** Bertrand curve.

Example(2) Let $\alpha(t) = (\cos\frac{t}{\sqrt{2}}, \sin\frac{t}{\sqrt{2}}, \frac{t}{\sqrt{2}})$. Show that the curve α is Bertrand curve then find its Bertrand mate curve.

Question In Example (2), we have shown that every circular helix is a Bertrand curve. Hence, is every cylindrical helix is a Bertrand curve?

Example(3) Let $\alpha : I \mapsto \mathbb{R}^3$ be unit speed curve whose curvature $\kappa(t) = c$, where c is constant. Show the curve α is Bertrand curve.

Thanks for listening.