Tangent Surface, Involutes and Evolutes
Math 473
Introduction to Differential Geometry
Lecture 16

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October 21, 2018

In this Lecture, we are going to show that a given space curve $\alpha : I \mapsto \mathbb{R}^3$ determines two infinite systems of curves which are *involutes* and *evolutes* of α.

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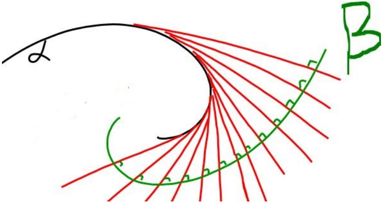
Definition (1):
Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit speed curve. The **Tangent Surface** of a curve α is the surface generated by lines tangent to α.

Definition (2):
Let α and β be two regular curves defined on an interval I. The curve β is an **involute** of α if β lies on the tangent surface ($\beta(t_0)$ lies on the tangent line to α at $\alpha(t_0)$) and the tangents to α and β at $\alpha(t_0)$ and $\beta(t_0)$ are perpendicular.



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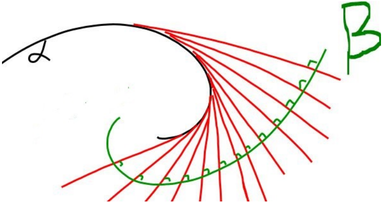


We have $\alpha' \bullet \beta' = 0, \qquad i.e. \;\; \alpha' \perp \beta'$

Lemma (1):
The formula of the curve β which is involute of α is

$$\beta(s) = \alpha(s) + (c - s)\,T_{\alpha}(s),$$

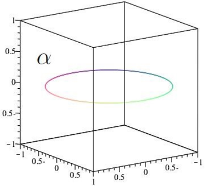
where $\alpha(s)$ is a unit speed curve and s is the arc-length. (Note that this formula for α is unit speed curve).

Proof:

Lemma (2):
If α is a regular curve (not a unit speed curve), then the formula of the curve β which is involute of α is

$$\beta(t) = \alpha(t) + (c - S(t))\frac{\alpha'(t)}{|\alpha'(t)|}.$$

Example(1) Let $\alpha(t) = (\cos t, \sin t, 0)$. Find the involute curve of α.



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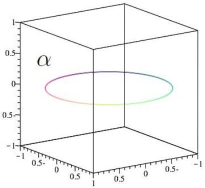
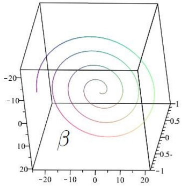
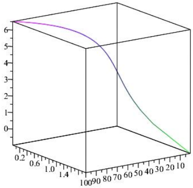




Example(2) Let $\alpha(t) = (t, \frac{1}{t}, \sqrt{2}\ln(t))$, where $t \in (0, \infty)$. Find the involute curve of α.



Thanks for listening.