Curvature of Space Curves
Math 473
Introducation to Differential Geometry
Lecture 4

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Defnation (1):
Let $\alpha : I \mapsto \mathbb{R}^3$ be a regular parametrised space curve with the unit tangent vector T. The curvature $\kappa(t)$ is the non-negative real number given by

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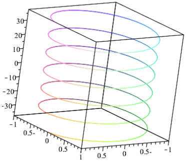
$$\kappa(t) = \frac{|T'(t)|}{|\alpha'(t)|}.$$

Example(1): Find the Curvature of the curve
$\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t, t, t)$.

Example(2): Find the Curvature of helix
$\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, 2t)$.



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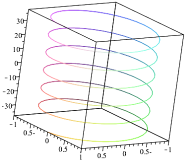
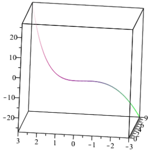


The curvature of the helix is constant. This is in line with the observation that the helix 'looks the same' at each point.

Example(3): Find the Curvature of twisted cubic $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t, t^2, t^3)$.



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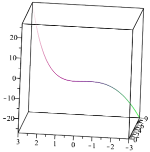

To compute the curvature according to the definition we would need to differentiate $T(t)$. Instead, we will now discuss a formula that will allow us to compute the curvature of complicated curves more efficiently.

Proposition (1):
For a regular parametrised space curve $\alpha : I \mapsto \mathbb{R}^3$ the curvature κ can be computed as

$$\kappa = \frac{|\alpha' \times \alpha''|}{|\alpha'|^3}.$$

Back to Example 3 The Curvature of twisted cubic $\alpha(t) = (t, t^2, t^3)$ can be computed as

Thanks for listening.