# Curvature of Space Curves
# Math 473
# Introduction to Differential Geometry
# Lecture 5

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September 15, 2018

**Defnation (1)**:
Let $\alpha : I \mapsto \mathbb{R}^3$ be a regular parametrised space curve with the unit tangent vector $T$. The curvature $\kappa(t)$ is the non-negative real number given by

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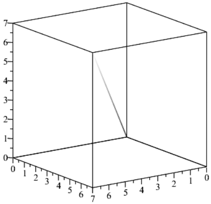
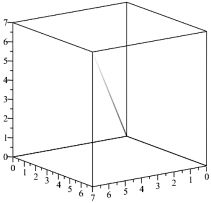
Let $\alpha : I \mapsto \mathbb{R}^3$ be a regular parametrised space curve with the unit tangent vector $T$. The curvature $\kappa(t)$ is the non-negative real number given by

$$\kappa(t) = \frac{|T'(t)|}{|\alpha'(t)|}.$$

**Example(1)**: Find the Curvature of the curve
$\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t, t, t)$.

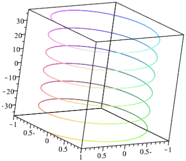


**Example(1)**: Find the Curvature of the curve $\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t, t, t)$.




The curvature of straight line is zero.

**Example(2)**: Find the Curvature of helix
$\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (\cos t, \sin t, 2t)$.



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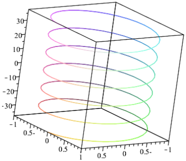
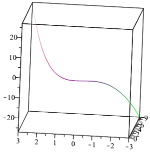



The curvature of the helix is constant. This is in line with the observation that the helix 'looks the same' at each point.

**Example(3)**: Find the Curvature of twisted cubic
$\alpha : \mathbb{R} \mapsto \mathbb{R}^3, \alpha(t) = (t, t^2, t^3)$.



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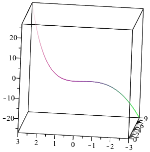


To compute the curvature according to the definition we would need to differentiate $T(t)$. Instead, we will now discuss a formula that will allow us to compute the curvature of complicated curves more efficiently.

# A formule of the curvature

**Proposition (1)**:
For a regular parametrised space curve $\alpha : I \mapsto \mathbb{R}^3$ the curvature $\kappa$ can be computed as

$$\kappa = \frac{|\alpha' \times \alpha''|}{|\alpha'|^3}.$$

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**Proof** Proof of this Proposition will be given later.

**Back to Example 3** The Curvature of twisted cubic $\alpha(t) = (t, t^2, t^3)$ can be computed as

*Thanks for listening.*