Serret-Frenet Basis for Space Curves and Torsion of Space Curves
Math 473
Introducation to Differential Geometry
Lecture 6

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Defnation (1):
For a regular parametrised space curve $\alpha : I \mapsto \mathbb{R}^3$ at a point $t \in I$ with $\kappa(t) \neq 0$ the **torsion** $\tau(t)$ is a real number such that

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Moreover, if α is a unite speed curve. Then,

$$\tau = N' \bullet B = -(N \bullet B').$$

Example(1):

Let $\alpha : I \mapsto \mathbb{R}^3$ be given by $\alpha(t) = (2 + \frac{t}{\sqrt{5}}, \sqrt{\frac{3}{5}}t, \frac{t}{\sqrt{5}} + 5)$.

(i) Check if the curve α is a untie speed curve.

(ii) Compute the curvature κ of α.

(iii) Find the Serret-Frenet frame of α if exist.

Example(2):

Let $\alpha : I \mapsto \mathbb{R}^3$ be given by $\alpha(t) = (\cos t, \sin t, 2)$.

(i) Check if the curve α is a untie speed curve.

(ii) Compute the curvature κ of α.

(iii) Find the Serret-Frenet frame of α if exist.

(iv) Compute the torsion τ of α.

Example(3):

Let $\alpha : I \mapsto \mathbb{R}^3$ be given by $\alpha(t) = (\cos t, \sin t, t)$.

1. Show that the curve α is not a untie speed curve.
2. Reparametrisation the curve $\alpha(t)$ using the arc-length? (Find the normal reparametrisation of α)
3. Compute the curvature κ of α.
4. Find the Serret-Frenet frame of α if exist.
5. Compute the torsion τ of α.

Note:
In general, if we have a helix curve $\alpha(t) = (a\cos\frac{t}{c}, a\sin\frac{t}{c}, \frac{bt}{c})$, where a, b are constant not equal to 0, $c = \sqrt{a^2 + b^2}$, then

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$$\kappa(t) = \frac{a}{a^2+b^2},$$

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$$\kappa(t) = \frac{a}{a^2 + b^2},$$

the torsion is

$$\tau(t) = \frac{b}{a^2 + b^2}$$

Theorem (1):
Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit speed curve with $\kappa = 0$ on the interval I. Then the curve segment α is a straight line.

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Proof:

Thanks for listening.