Serret-Frenet Basis for Space Curves
Math 473
Introduction to Differential Geometry
Lecture 6

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Principal Normal and Binormal

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We know that the vector $T'(t)$ is either orthogonal to $T(t)$ or $T'(t) = 0$. But $T'(t) \neq 0$ since $|T'(t)| = \kappa(t) \cdot |\alpha'(t)| \neq 0$. We define the **principal normal** of α as

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$$B = T \times N.$$

Remark:

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The principal normal N is a unit vector orthogonal to T. The binormal B is orthogonal to both T and N and is unit length: $|B| = |T \times N| = |T| \cdot |N| = 1$ since $T \perp N$.

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They satisfy the equations

$$T \bullet T = N \bullet N = B \bullet B = 1,$$
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$$T \times N = B, \quad N \times B = T, \quad B \times T = N,$$
$$N \times T = -B, \quad B \times N = -T, \quad T \times B = -N.$$

Proof of Theorem 1

Defnation (2): Serret-Frenet Frame

The **Serret-Frenet frame** or the **Serret-Frenet basis** of the curve α is the orthonormal basis

$$\{T(t), N(t), B(t)\}$$

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Thanks for listening.