Serret-Frenet Basis for Space Curves and Torsion of Space Curves
Math 473
Introduction to Differential Geometry
Lecture 7

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Defnation (1):
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Torsion of Space Curves

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Example(1):

Let $\alpha : I \mapsto \mathbb{R}^3$ be given by $\alpha(t) = (2 + \frac{t}{\sqrt{5}}, \sqrt{\frac{3}{5}}t, \frac{t}{\sqrt{5}} + 5)$.

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2. (ii) Compute the curvature κ of α.
3. (iii) Find the Serret-Frenet frame of α if exist.

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(v) Compute the torsion τ of α.

Note:

In general, if we have a helix curve $\alpha(t) = (a\cos\frac{t}{c}, a\sin\frac{t}{c}, \frac{bt}{c})$, where a, b are constant not equal to 0, $c = \sqrt{a^2 + b^2}$, then

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Thanks for listening.