Engineering Mechanics
AGE 2330

Lect 10: Kinematics

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Engineering Mechanics: Dynamics

- Dynamics
  - Branch of mechanics that deals with the motion of bodies under the action of forces ***(Accelerated Motion)***
- Two distinct parts:
  - Kinematics
    - study of motion without reference to the forces that cause motion or are generated as a result of motion
  - Kinetics
    - relates the action of forces on bodies to their resulting motions

Engineering Mechanics: Dynamics

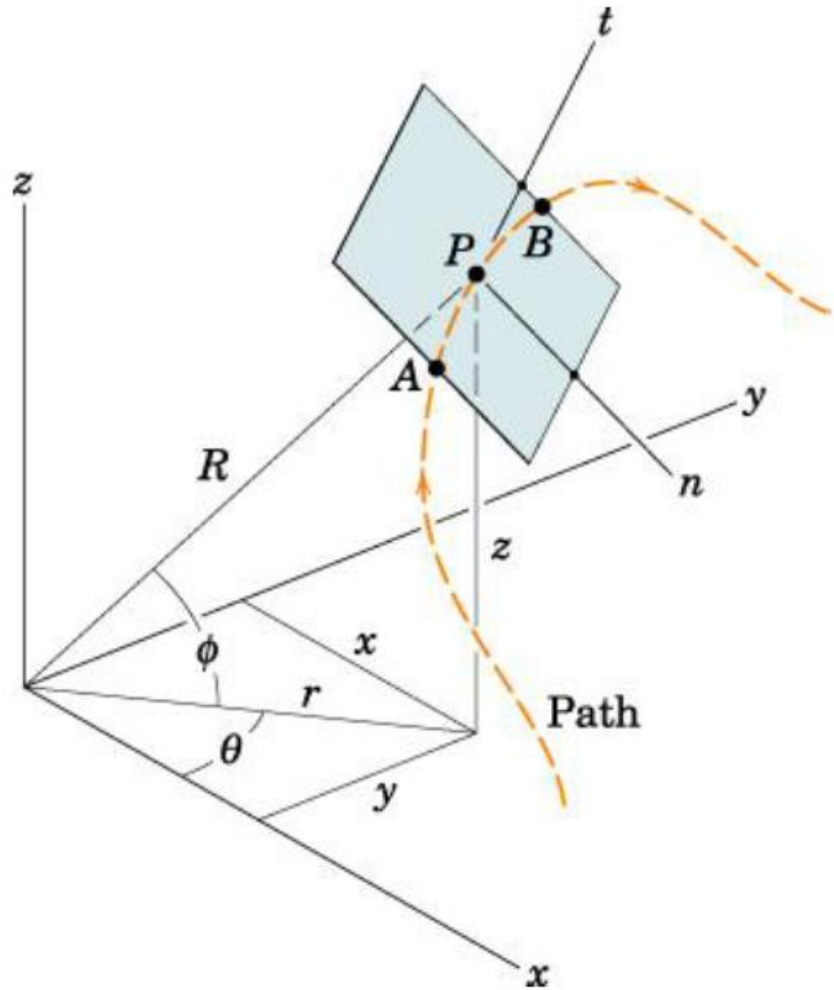
- Basis of rigid body dynamics
  - Newton's 2nd law of motion
    - A **particle** of **mass "m" acted upon by an unbalanced force "F"** experiences an **acceleration "a"** that has the **same direction as the force** and a **magnitude** that is **directly proportional to the force**
    - **a** is the resulting **acceleration** measured in a **non-accelerating frame of reference**

Kinematics of Particles

- **Motion**
  - Constrained :: confined to a specific path
  - Unconstrained :: not confined to a specific path
- **Choice of coordinates**
  - Position of P at any time t
    - rectangular (i.e., Cartesian) coordinates x, y, z
    - cylindrical coordinates r, θ, z
    - spherical coordinates R, θ, Φ
  - Path variables
    - Measurements along the tangent t and normal n to the curve

Kinematics of Particles

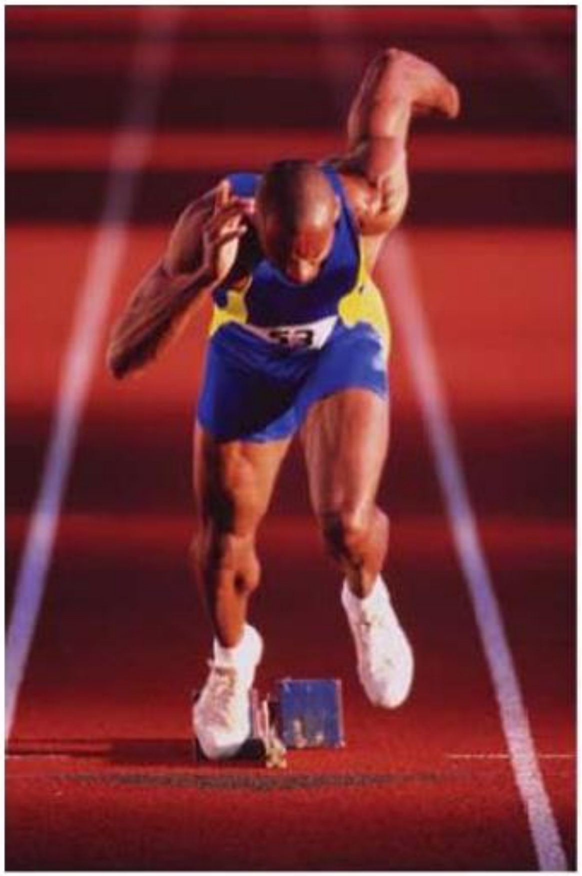
- Choice of coordinates



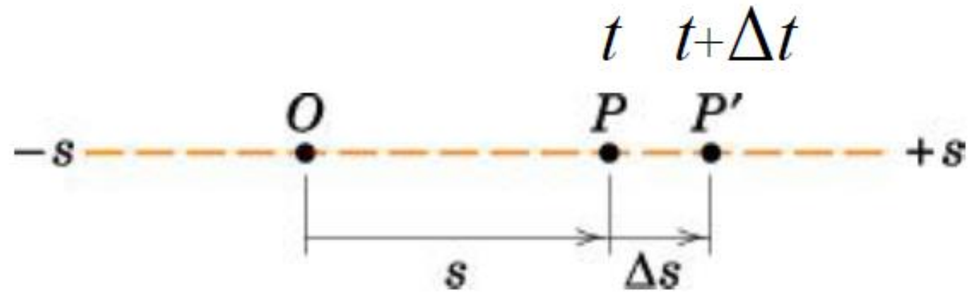
Kinematics of Particles

Rectilinear Motion

- Motion along a straight line

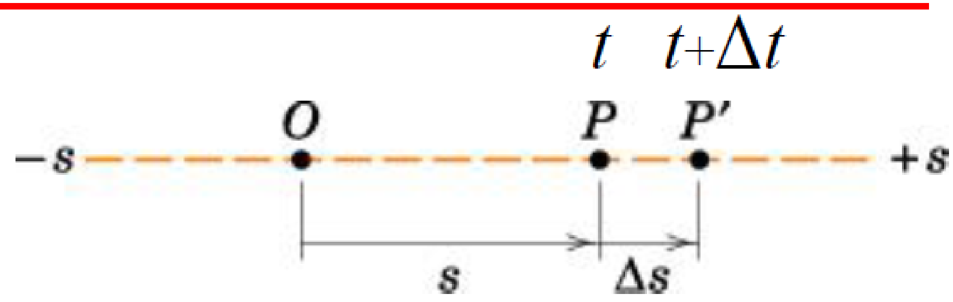


This sprinter will undergo rectilinear acceleration until he reaches his terminal speed.



Kinematics of Particles :: Rectilinear Motion

•Motion along a straight line



Position at any instance of time *t*

:: specified by its distance *s* measured from some convenient reference point *O* fixed on the line

:: (disp. is negative if the particle moves in the negative *s*-direction).

Velocity of the particle:

$$v = \frac{ds}{dt} = \dot{s}$$

Both are vector quantities

+ve or –ve depending on +ve or –ve displacement

Acceleration of the particle:

$$a = \frac{dv}{dt} = \dot{v} \quad \text{or} \quad a = \frac{d^2 s}{dt^2} = \ddot{s}$$

+ve or –ve depending on whether velocity is increasing or decreasing

$$v\,dv = a\,ds \qquad or \qquad \dot{s}\,d\dot{s} = \ddot{s}\,ds$$

Kinematics of Particles

Rectilinear Motion:

Graphical Interpretations

Using ***s-t*** curve, ***v-t*** & ***a-t*** curves can be plotted.

Area under *v-t* curve during time *dt* = *vdt* == *ds*

- Net disp from t_1 to t_2 = corresponding area under *v-t* curve →

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v dt$$

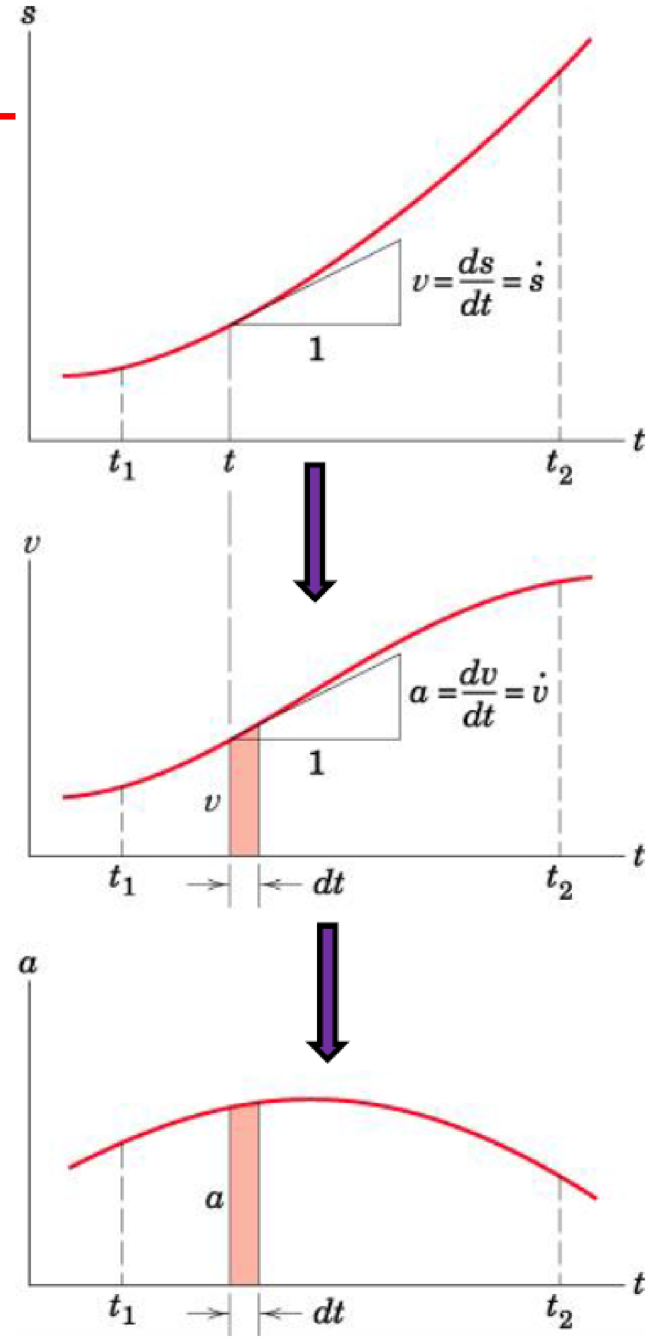
or **$s_2 - s_1$ = (area under *v-t* curve)**

Area under *a-t* curve during time *dt* = *adt* == *dv*

- Net change in vel from t_1 to t_2 = corresponding area under *a-t* curve →

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt$$

or **$v_2 - v_1$ = (area under *a-t* curve)**



Kinematics of Particles

Rectilinear Motion:

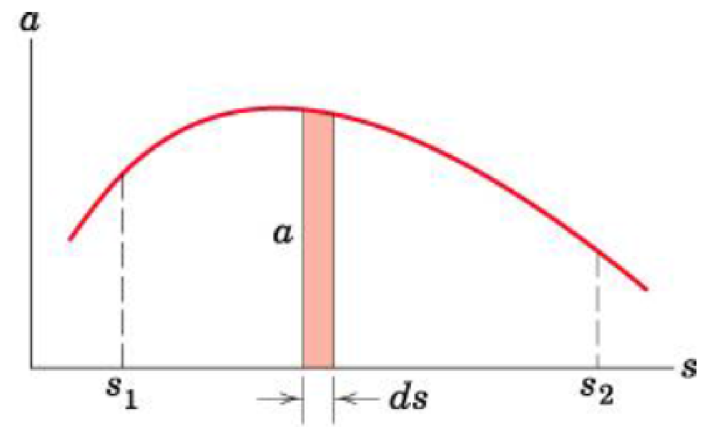
Graphical Interpretations

Two additional graphical relations:

Area under *a-s* curve during disp *ds*= *ads* == *vdv*

- Net area under *a-s* curve betn position coordinates s_1 and s_2 → $$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds$$

or **½ $(v_2^2 - v_1^2)$ = (area under *a-s* curve)**

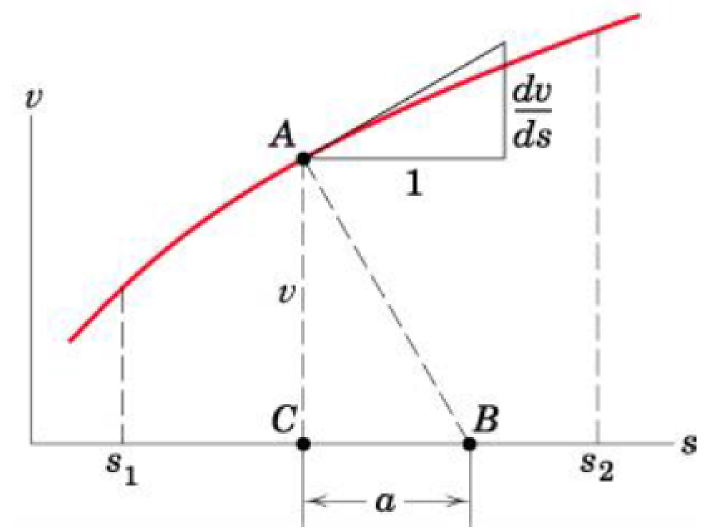


Slope of *v-s* curve at any point *A* = *dv/ds*

- Construct a normal *AB* to the curve at *A*. From similar triangles:

$$\frac{\overline{CB}}{v} = \frac{dv}{ds} \quad \rightarrow \quad \overline{CB} = v\frac{dv}{ds} = a \ \ (acceleration)$$

- Vel and posn coordinate axes should have the same numerical scales so that the accln read on the x-axis in meters will represent the actual accln in m/s^2



Kinematics of Particles :: Rectilinear Motion

Analytical Integration to find the position coordinate

Acceleration may be specified as a function of time, velocity, or position coordinate, or as a combined function of these.

(a) Constant Acceleration

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$

For a time interval t: integrating the following two equations

$$a = \frac{dv}{dt}$$

$$v\,dv = a\,ds$$

$$\int_{v_0}^{v} dv = a \int_0^t dt \qquad \text{or} \qquad v = v_0 + at$$

$$\int_{v_0}^{v} v\,dv = a \int_{s_0}^{s} ds \qquad \text{or} \qquad v^2 = {v_0}^2 + 2a(s - s_0)$$

Substituting in the following equation and integrating will give the position coordinate:

$$v = \frac{ds}{dt}$$

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\,dt \qquad \text{or} \qquad s = s_0 + v_0 t + \frac{1}{2}at^2$$

Equations applicable for Constant Acceleration and for time interval 0 to t

Kinematics of Particles :: Rectilinear Motion

Analytical Integration to find the position coordinate

(b) Acceleration given as a function of time, $a = f(t)$

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$
For a time interval t: integrating the following equation

$$a = \frac{dv}{dt} \rightarrow f(t) = \frac{dv}{dt} \qquad \int_{v_0}^{v} dv = \int_0^t f(t)\,dt \quad \text{or} \quad v = v_0 + \int_0^t f(t)\,dt$$

Substituting in the following equation and integrating will give the position coordinate:

$$v = \frac{ds}{dt} \qquad \int_{s_0}^{s} ds = \int_0^t v\,dt \quad \text{or} \quad s = s_0 + \int_0^t v\,dt$$

Alternatively, following second order differential equation may be solved to get the position coordinate:

$$a = \frac{d^2 s}{dt^2} = \ddot{s} \rightarrow \ddot{s} = f(t)$$

Kinematics of Particles :: Rectilinear Motion

Analytical Integration to find the position coordinate

(c) Acceleration given as a function of velocity, $a = f(v)$

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$

For a time interval t: Substituting a and integrating the following equation

$$a = \frac{dv}{dt} \rightarrow f(v) = \frac{dv}{dt} \qquad t = \int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)}$$

Solve for v as a function of t and integrate the following equation to get the position coordinate:

$$v = \frac{ds}{dt}$$

Alternatively, substitute $a = f(v)$ in the following equation and integrate to get the position coordinate :

$$v\,dv = a\,ds \qquad \int_{v_0}^{v} \frac{v\,dv}{f(v)} = \int_{s_0}^{s} ds \quad \text{or} \quad s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

Kinematics of Particles: Rectilinear Motion

Analytical Integration to find the position coordinate

(d) Acceleration given as a function of displacement, $a = f(s)$

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$

For a time interval t: substituting a and integrating the following equation

$$v\,dv = a\,ds$$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} f(s)\,ds \qquad \text{or} \qquad v^2 = v_0^2 + 2\int_{s_0}^{s} f(s)\,ds$$

Solve for v as a function of s : $v = g(s)$, substitute in the following equation and integrate to get the position coordinate:

$$v = \frac{ds}{dt}$$

$$\int_{s_0}^{s} \frac{ds}{g(s)} = \int_{0}^{t} dt \qquad \text{or} \qquad t = \int_{s_0}^{s} \frac{ds}{g(s)}$$

It gives t as a function of s. Rearrange to obtain s as a function of t to get the position coordinate.

In all these cases, if integration is difficult, graphical, analytical, or computer methods can be utilized.

Kinematics of Particles: Rectilinear Motion

Example

Position coordinate of a particle confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine: (a) time reqd for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$, (b) acceleration of the particle when $v = 30$ m/s, and (c) net disp of the particle during the interval from $t = 1$ s to $t = 4$ s.

Solution

Differentiating $s = 2t^3 - 24t + 6$ → $v = 6t^2 - 24$ m/s
→ $a = 12t$ m/s^2

(a) $v = 72$ m/s → $t = \pm 4$ s
(- 4 s happened before initiation of motion → no physical interest.)
→ $t = 4$ s

(b) $v = 30$ m/s → $t = 3$ sec → $a = 36$ m/s^2

(c) t = 1 s to 4 s. Using $s = 2t^3 - 24t + 6$
$\Delta s = s_4 - s_1 = [2(4^3) - 24(4) + 6] - [2(1^3) - 24(1) + 6]$
$\Delta s = 54$ m

Kinematics of Particles

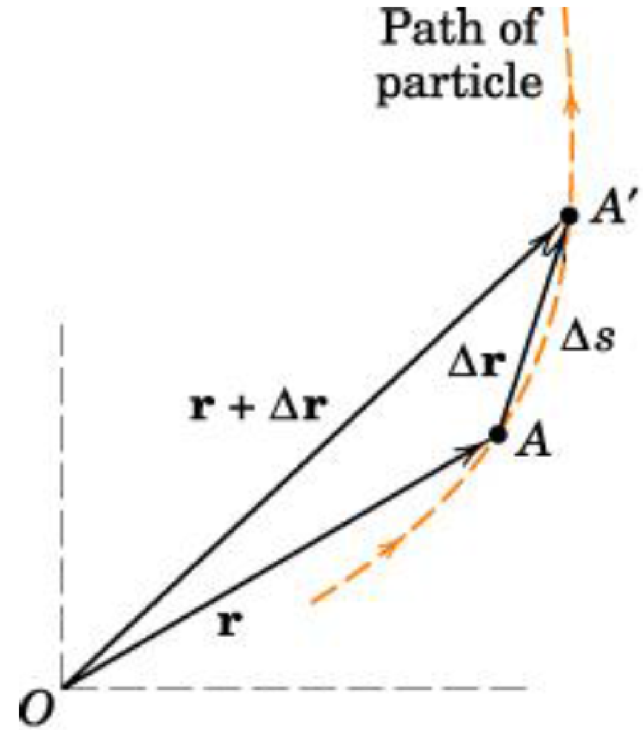
Plane Curvilinear Motion

Motion of a particle along a curved path which lies in a single plane.

For a short time during take-off and landing,
planes generally follow plane curvilinear motion

Kinematics of Particles

Plane Curvilinear Motion:



Between *A* and *A'*:

Average velocity of the particle : $\mathbf{v}_{av} = \Delta\mathbf{r} / \Delta t$

→ A vector whose direction is that of $\Delta\mathbf{r}$ and whose magnitude is magnitude of $\Delta\mathbf{r} / \Delta t$

Average speed of the particle = $\Delta s / \Delta t$

Instantaneous velocity of the particle is defined as the limiting value of the average velocity as the time interval approaches zero → $\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$

→ **v** is always a vector tangent to the path

Extending the definition of derivative of a scalar to include vector quantity:

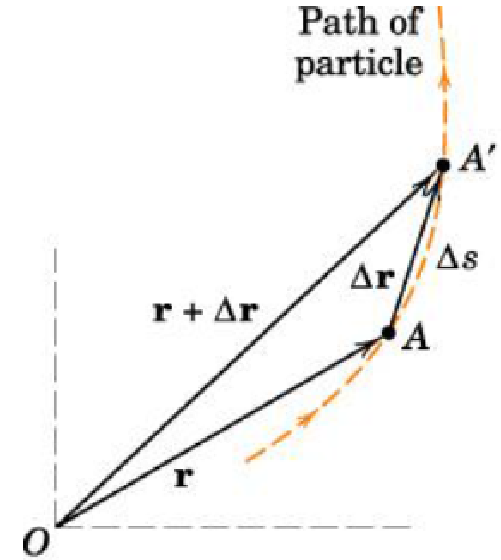
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

Derivative of a vector is a vector having a magnitude and a direction.

Magnitude of v is equal to speed (scalar) $v = |\mathbf{v}| = \frac{ds}{dt} = \dot{s}$

Kinematics of Particles

Plane Curvilinear Motion



Magnitude of the derivative:

$$\left|d\mathbf{r}/dt\right| = \left|\dot{\mathbf{r}}\right| = \dot{s} = \left|\mathbf{v}\right| = v$$

→ Magnitude of the velocity or the speed

Derivative of the magnitude:

$$d\left|\mathbf{r}\right|/dt = dr/dt = \dot{r}$$

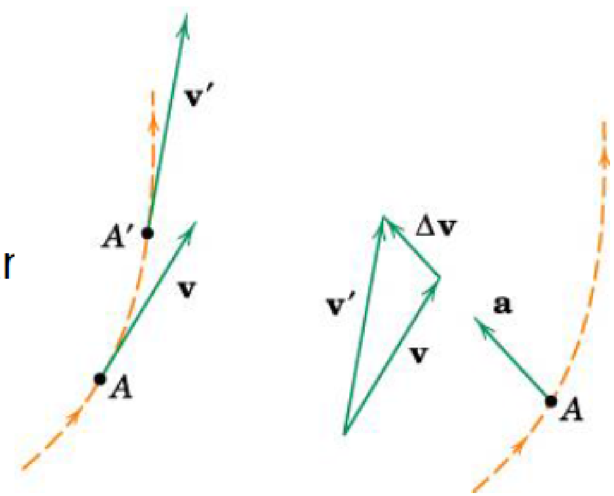
→ Rate at which the length of the position vector is changing

Velocity of the particle at *A* → tangent vector **v**

Velocity of the particle at *A'* → tangent vector **v'**

→ **v' – v = Δv**

→ Δ**v** Depends on both the change in magnitude of **v** ar
on the change in direction of **v**.



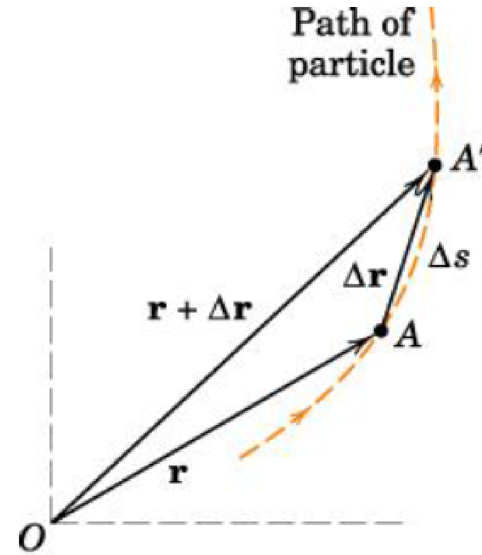
Kinematics of Particles

Plane Curvilinear Motion

Between *A* and *A'*:

Average acceleration of the particle : $\mathbf{a}_{av} = \Delta\mathbf{v}/\Delta t$

→ A vector whose direction is that of $\Delta\mathbf{v}$ and whose magnitude is the magnitude of $\Delta\mathbf{v}/\Delta t$

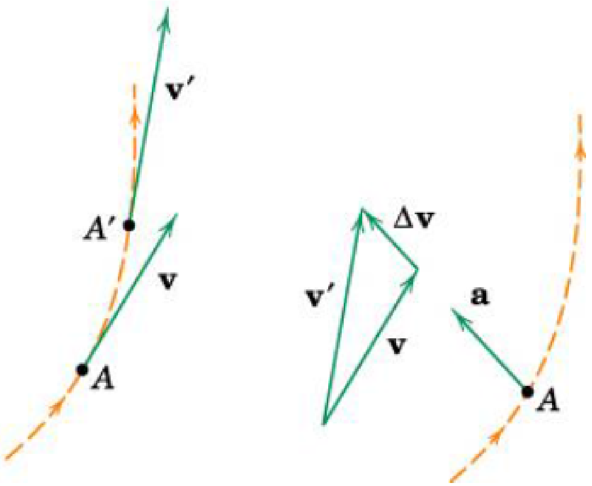


Instantaneous accln of the particle is defined as the limiting value of the average accln as the time interval approaches zero →

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

By definition of the derivative:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$



→ In general, direction of the acceleration of a particle in curvilinear motion neither tangent to the path nor normal to the path.

→ Acceleration component normal to the path points toward the center of curvature of the path.

Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

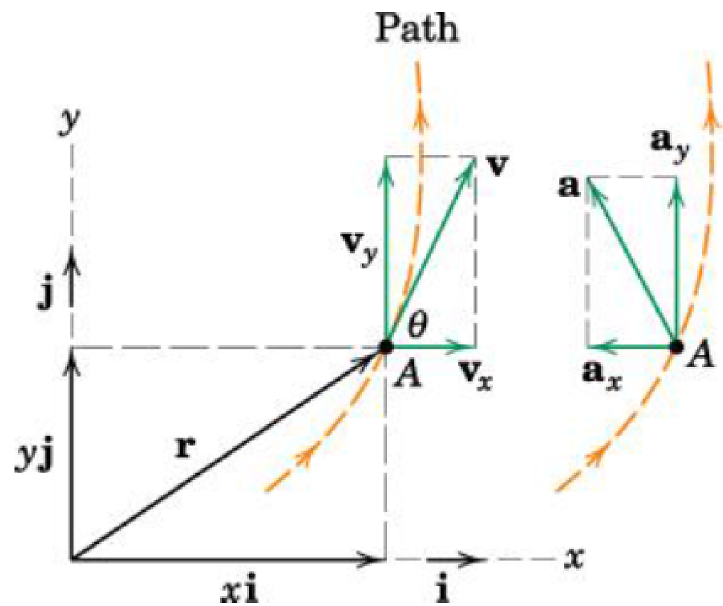
If all motion components are directly expressible in terms of horizontal and vertical coordinates

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

$v_x = \dot{x}$, $v_y = \dot{y}$ and $a_x = \dot{v}_x = \ddot{x}$, $a_y = \dot{v}_y = \ddot{y}$



$$v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}$$

Time derivatives of the unit vectors are zero because their magnitude and direction remains constant.

Also, $dy/dx = \tan\theta = v_y / v_x$

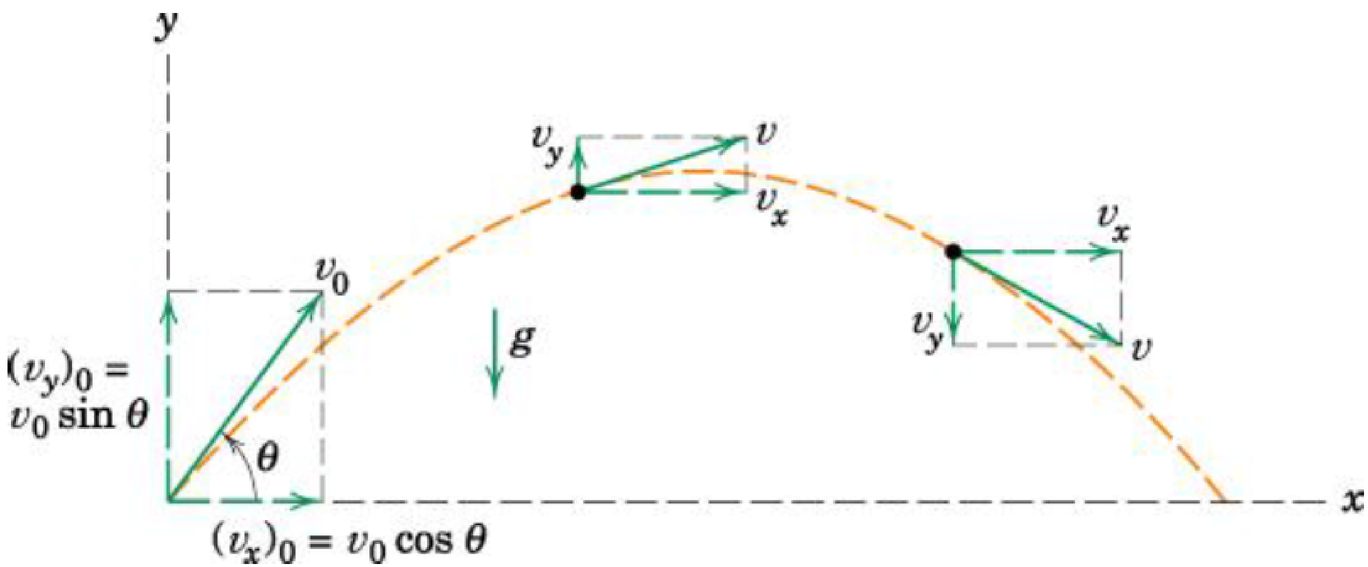
Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Projectile Motion → An important application

Assumptions: neglecting aerodynamic drag, Neglecting curvature and rotation of the earth, and altitude change is small enough such that *g* can be considered to be constant → Rectangular coordinates are useful for the trajectory analysis

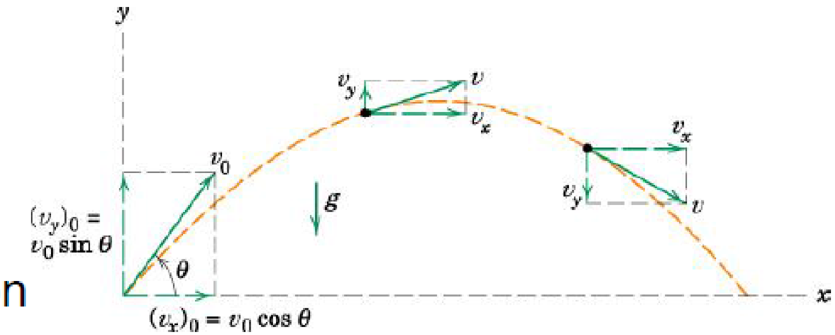
For the axes shown in the figure, the acceleration components are: $a_x = 0$, $a_y = -g$
Integrating these eqns for the condition of constant accln (slide 11) will give us equations necessary to solve the problem.



Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Projectile Motion



Horizontal Motion: $a_x = 0$

Integrating this eqn for constant accln condition

$$v = v_0 + at \quad \Rightarrow \quad v_x = (v_0)_x$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad x = x_0 + (v_0)_x t$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \Rightarrow \quad v_x = (v_0)_x$$

+ →

Vertical Motion: $a_y = -g$

Integrating this eqn for constant accln condition

$$v = v_0 + at \quad \Rightarrow \quad v_y = (v_0)_y - gt$$

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y = y_0 + (v_0)_y t - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 + 2a(y - y_0) \quad \Rightarrow \quad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

+↑

Subscript zero denotes initial conditions: $x_0 = y_0 = 0$

For the conditions under discussion:

→ *x*- and *y*- motions are independent

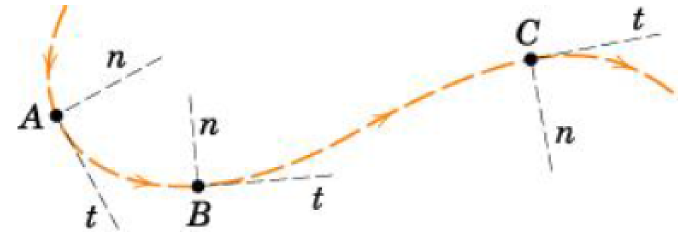
→ Path is parabolic

Kinematics of Particles: Plane Curvilinear Motion

Normal and Tangential Coordinates (*n*-*t*)

Common descriptions of curvilinear motion uses Path Variables: measurements made along the tangent and normal to the path of the particle.

- Positive *n* direction: towards the center of curvature of the path



Velocity and Acceleration

$\mathbf{e}_n$ = unit vector in the *n*-direction at point *A*

$\mathbf{e}_t$ = unit vector in the *t*-direction at point *A*

$$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

During differential increment of time *dt*, the particle moves a differential distance *ds* from *A* to *A'*.

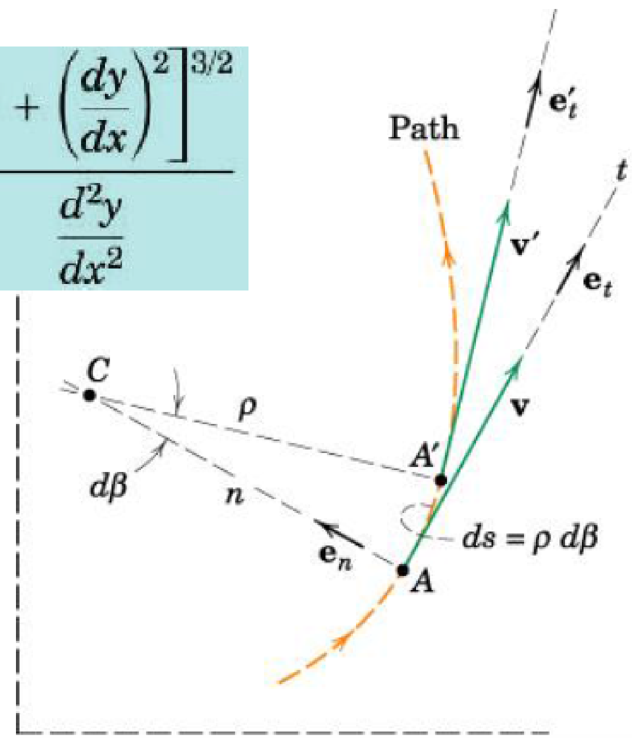
ρ = radius of curvature of the path at *A'*

→ $ds = \rho\, d\beta$

Magnitude of the velocity: $v = ds/dt = \rho\, d\beta/dt$

→ In vector form

$$\mathbf{v} = v\mathbf{e}_t = \rho\dot{\beta}\mathbf{e}_t$$

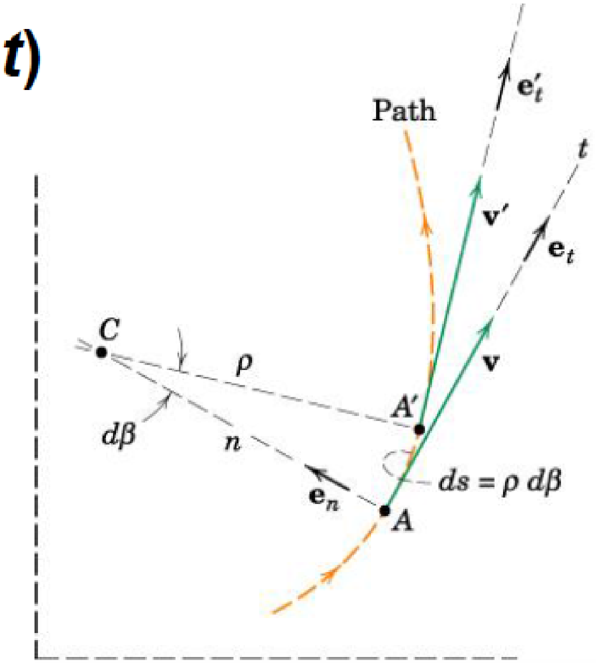


Differentiating: $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{e}_t)}{dt} = v\dot{\mathbf{e}}_t + \dot{v}\mathbf{e}_t$

Unit vector $\mathbf{e}_t$ has non-zero derivative because its direction changes.

Kinematics of Particles: Plane Curvilinear Motion

Normal and Tangential Coordinates (*n*-*t*)



Determination of $\dot{\mathbf{e}}_t$:

→ change in $\mathbf{e}_t$ during motion from *A* to *A'*

→ The unit vector changes to $\mathbf{e}'_t$

The vector difference $d\mathbf{e}_t$ is shown in the bottom figure.

- In the limit $d\mathbf{e}_t$ has magnitude equal to length of the arc $|\mathbf{e}_t|\ d\beta = d\beta$
- Direction of $d\mathbf{e}_t$ is given by $\mathbf{e}_n$

→ We can write: $d\mathbf{e}_t = \mathbf{e}_n\, d\beta$ → $\dfrac{d\mathbf{e}_t}{d\beta} = \mathbf{e}_n$

Dividing by *dt*: $d\mathbf{e}_t / dt = \mathbf{e}_n (d\beta/dt)\ \mathbf{e}_n$ → $\dot{\mathbf{e}}_t = \dot{\beta}\mathbf{e}_n$

Substituting this and $v = \rho\, d\beta/dt$ = $v = \rho\dot{\beta}$ in equation for acceleration:

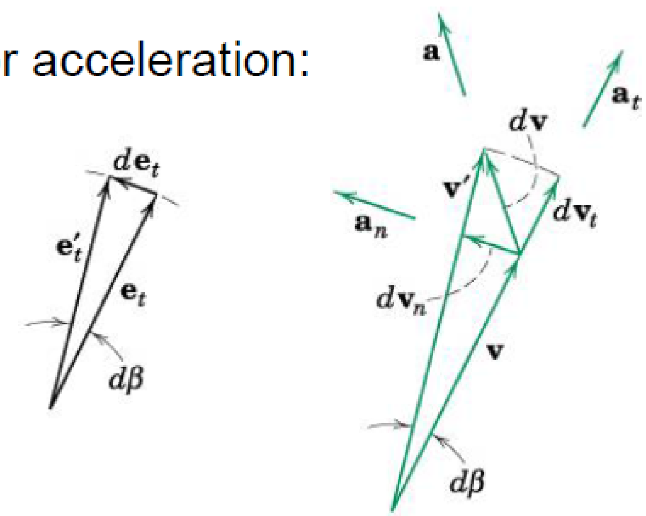
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{e}_t)}{dt} = v\dot{\mathbf{e}}_t + \dot{v}\mathbf{e}_t \rightarrow \mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t$$

Here:

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s}$$

$$a = \sqrt{a_n^2 + a_t^2}$$



Kinematics of Particles: Plane Curvilinear Motion

Normal and Tangential Coordinates (*n-t*)

Important Equations

$$v = \rho\dot{\beta}$$

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s}$$

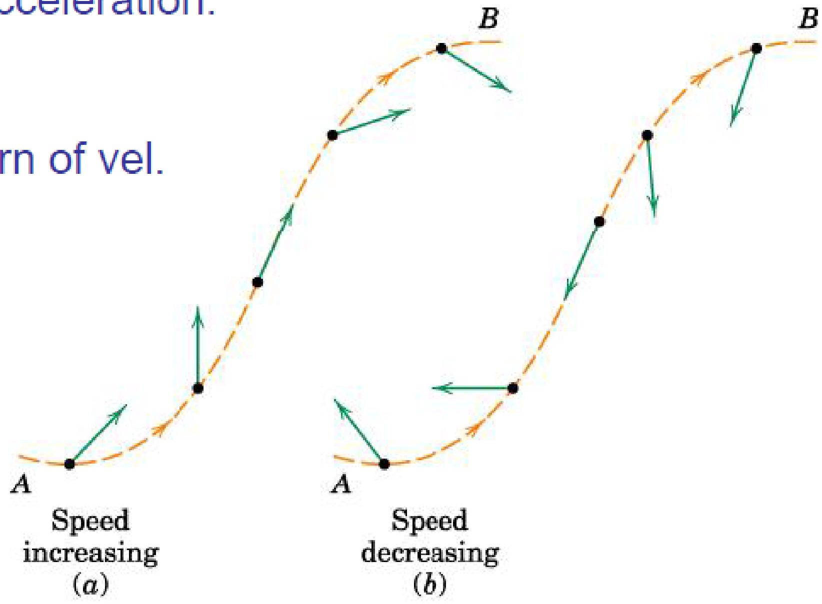
$$a = \sqrt{a_n^2 + a_t^2}$$

- In *n-t* coordinate system, there is no component of velocity in the normal direction because of constant ρ for any section of curve (normal velocity would be rate of change of ρ).
- Normal component of the acceleration a_n is always directed towards the center of the curvature → sometimes referred as centripetal acceleration.
  - If the particle moves with constant speed, $a_t = 0$, and $a = a_n = v^2/\rho$

  → a_n represents the time rate of change in the dirn of vel.
- Tangential component a_t will be in the +ve *t*-dirn of motion if the speed *v* is increasing, and in the - ve *t*-direction if the speed is decreasing.
  - If the particle moves in a straight line, $\rho = \infty$ $a_n = 0$, and $a = a_t = \dot{v} = \ddot{s}$

  → a_t represents the time rate of change in the magnitude of velocity.

Directions of tangential components of acceleration are shown in the figure.



Acceleration vectors for particle moving from *A* to *B*

Kinematics of Particles: Plane Curvilinear Motion

Normal and Tangential Coordinates (*n*-*t*)

Circular Motion: Important special case of plane curvilinear motion

- Radius of curvature becomes constant (radius *r* of the circle).
- Angle β is replaced by the angle θ measured from any radial reference to *OP*

Velocity and acceleration components for the circular motion of the particle:

$$v = \rho\dot{\beta}$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

general motion

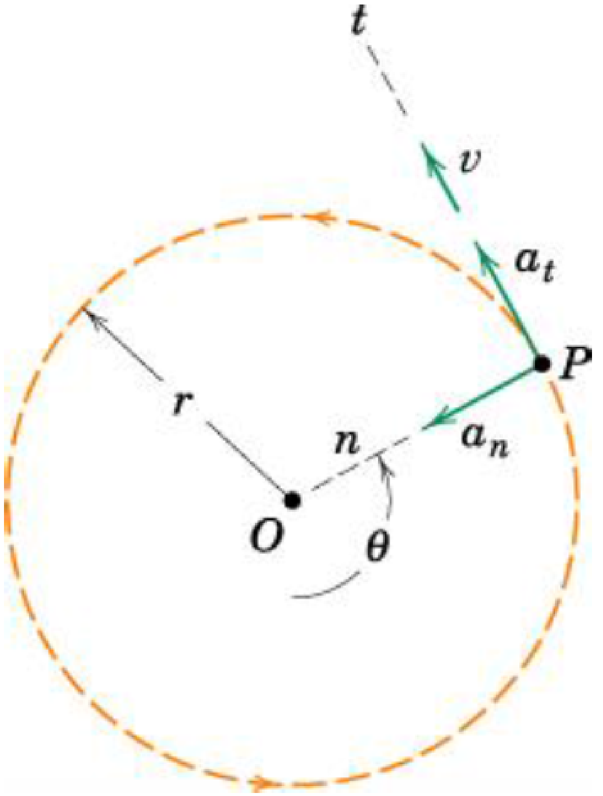
→

$$v = r\dot{\theta}$$

$$a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}$$

$$a_t = \dot{v} = r\ddot{\theta}$$

circular motion



Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Example

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$. At $t = 0$, $x = 0$. v_x is in m/s², x and y are in m, and t is in s. Plot the path of the particle and determine its velocity and acceleration at $y = 0$.

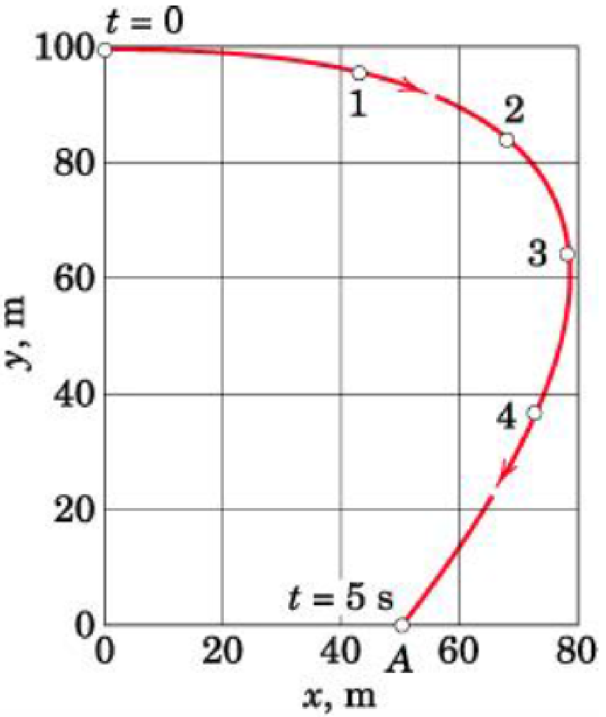
Solution:

$$\left[\int dx = \int v_x\,dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\,dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

$$[v_y = \dot{y}] \qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$$

$$[a_y = \dot{v}_y] \qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$$



Calculate *x* and *y* for various *t* values and plot

Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Example

Solution:

When $y = 0 \rightarrow 0 = 100 - 4t^2 \rightarrow t = 5$ s

$$v_x = 50 - 16(5) = -30 \text{ m/s}$$

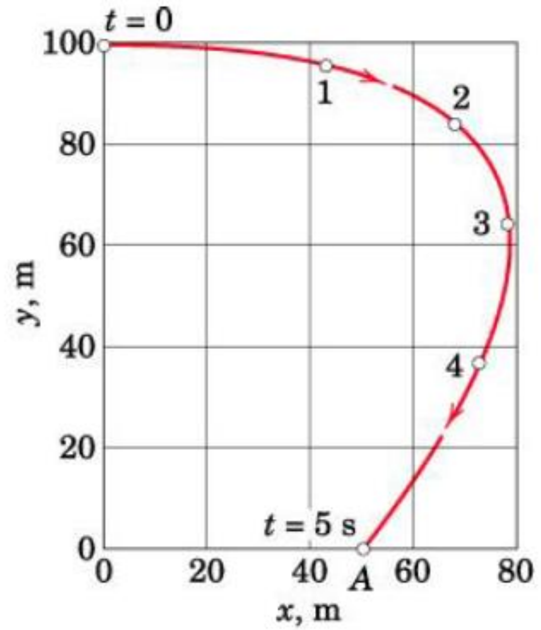
$$v_y = -8(5) = -40 \text{ m/s}$$

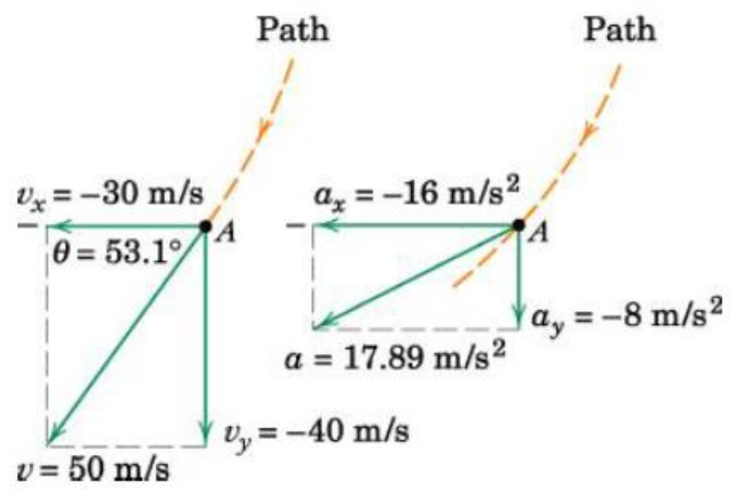
$$v = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

$$a = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$

$$\mathbf{v} = -30\mathbf{i} - 40\mathbf{j} \text{ m/s}$$

$$\mathbf{a} = -16\mathbf{i} - 8\mathbf{j} \text{ m/s}^2$$





Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Example: The rider jumps off the slope at 30^0 from a height of 1 m, and remained in air for 1.5 s. Neglect the size of the bike and of the rider. Determine:

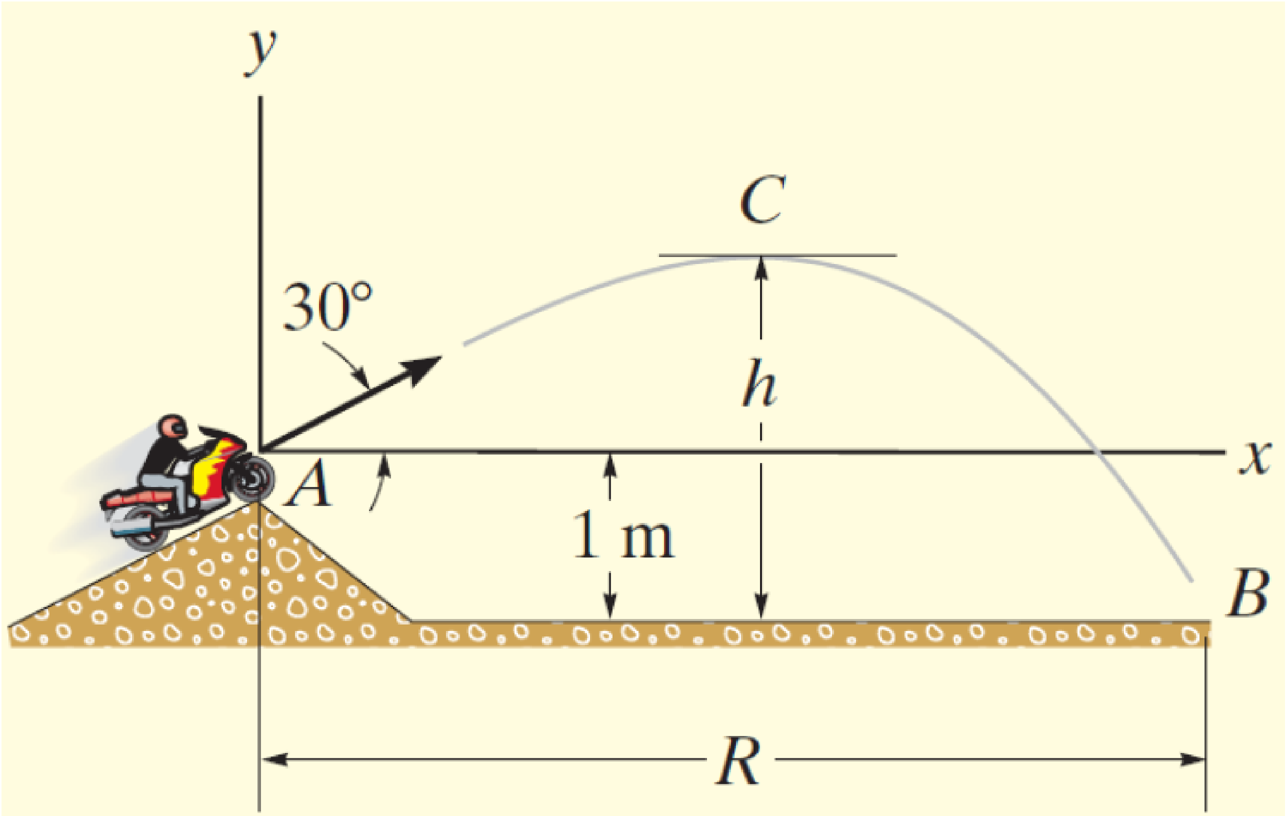
(a) the speed at which he was travelling off the slope,

(b) the horizontal distance he travelled before striking the ground, and

(c) the maximum height he attains.

Solution:

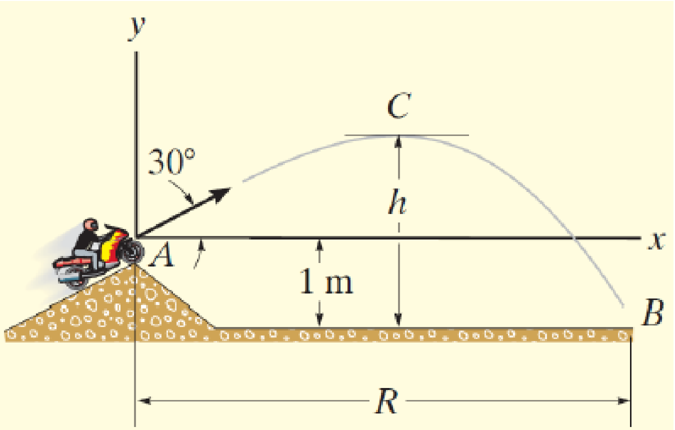
Let the origin of the coordinates be at A.



Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Example:



Solution: For projectile motion:

$a_x = 0$, $a_y = -g = -9.81$ m/s^2 → Constant Acceleration

(a) speed at which he was travelling off the slope?

Let v_0 be the initial velocity of the bike at *A*.

For vertical Motion: $a_y = -g$; *subsequent integrations will give following equations*

$$v = v_0 + at \quad \Rightarrow \quad v_y = (v_0)_y - gt$$

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y = y_0 + (v_0)_y t - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 + 2a(y - y_0) \quad \Rightarrow \quad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

+↑

Using second eqn: $-1 = 0 + (v_0)_y(1.5) - 0.5(9.81)(1.5)^2$

Initial velocity along *y*-direction $(v_0)_y = v_0 \sin 30 = 0.5v_0$

→ $-1 = 0 + 0.5v_0(1.5) - 0.5(9.81)(1.5)^2$

→ Initial Velocity of the bike: $v_0 = 13.38$ m/s (velocity at *A*)

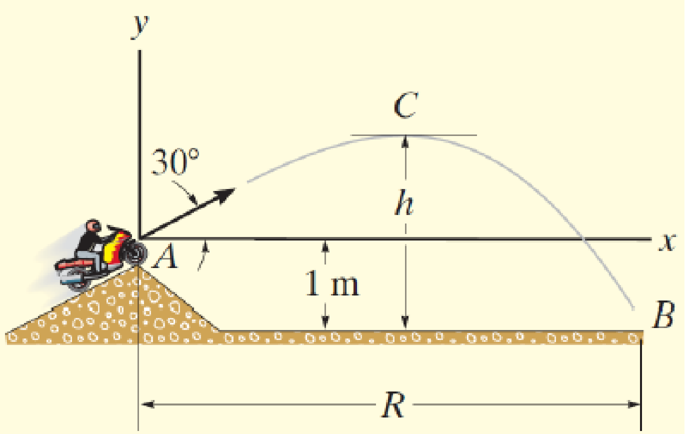
Kinematics of Particles: Plane Curvilinear Motion

Rectangular Coordinates (*x*-*y*)

Example:

Solution: For projectile motion:

$a_x = 0$, $a_y = -g = -9.81\ \text{m/s}^2$ → Constant Acceleration



(b) horizontal distance he travelled before striking the ground?

Let *R* be the horizontal distance between *A* and *B*.

For horizontal Motion: $a_x = 0$*; subsequent integrations will give following equations*

$$v = v_0 + at \quad \Rightarrow \quad v_x = (v_0)_x$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad x = x_0 + (v_0)_x t$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \Rightarrow \quad v_x = (v_0)_x$$

$\overset{+}{\rightarrow}$

Using second eqn: $R = 0 + (v_0)_x(1.5) = 13.38\cos 30(1.5)$

→ Horz distance: $R = 17.4$ m

Kinematics of Particles: Plane Curvilinear Motion

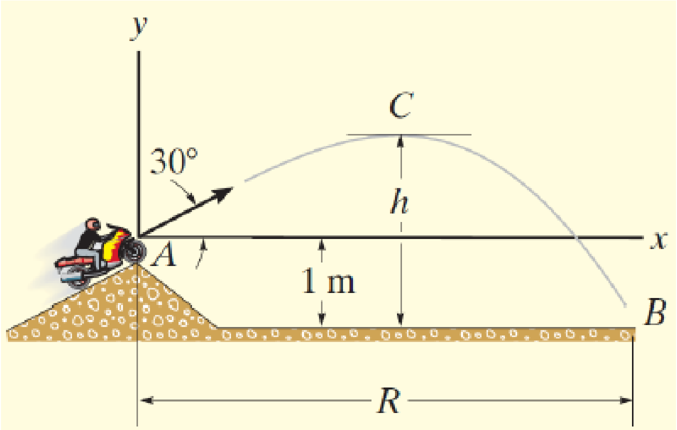
Rectangular Coordinates (*x*-*y*)

Example:

Solution: For projectile motion:

$a_x = 0$, $a_y = -g = -9.81\ \text{m/s}^2$ → Constant Acceleration

(c) Maximum height attained by the bike?



Let (h - 1) m be the maximum height attained from x-axis at point C.

For Vertical Motion: $a_y = -g$,

$$v = v_0 + at \quad \Rightarrow \quad v_y = (v_0)_y - gt$$

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y = y_0 + (v_0)_y t - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 + 2a(y - y_0) \quad \Rightarrow \quad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

+↑

Using the third eqn between A and C: All the quantities are known except the height of point C ($y = h$-1) and the velocity at point C → $v_y = 0$ at C

→ $0 = (0.5\text{x}13.38)^2 - 2(9.81)(h - 1 - 0)$

→ $h = 3.28$ m (total height attained above ground level)

Kinematics of Particles: Plane Curvilinear Motion

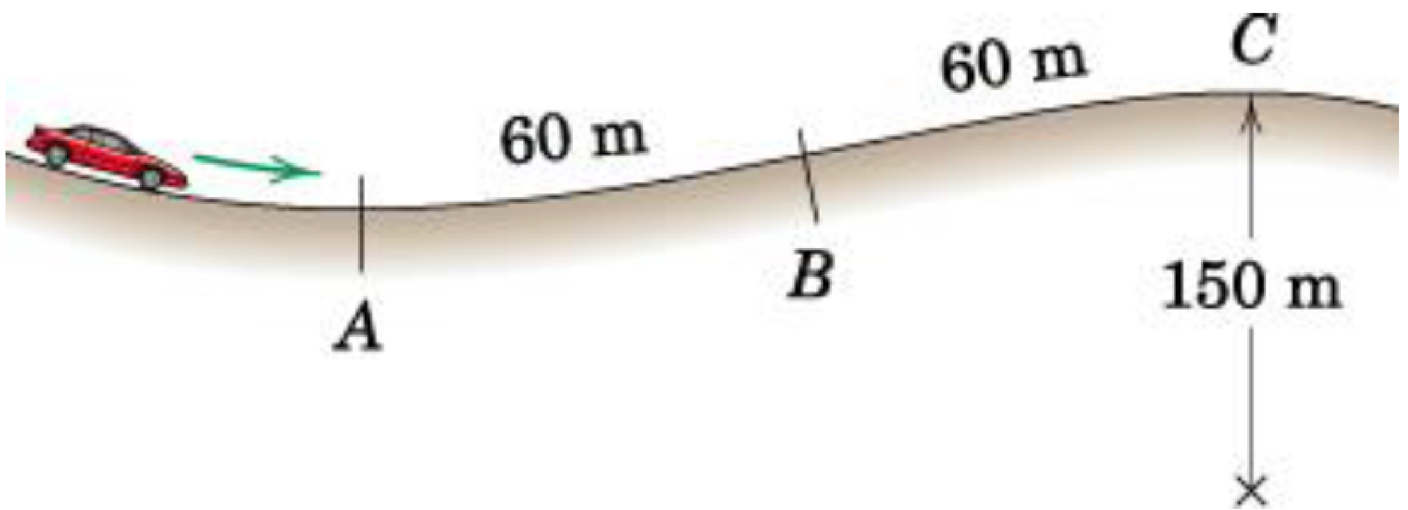
Normal and Tangential Coordinates (*n*-*t*)

Example: At the position shown, the driver applies brakes to produce a uniform deceleration. Speed of the car is 100 km/h at *A* (bottom of the dip), and 50 km/h at *C* (top of the hump). Distance between *A* and *C* is 120 m along the road. Passengers experience a total acceleration of 3 m/s^2 at *A*. Radius of curvature of the hump at *C* is 150 m. Calculate:

(a) radius of curvature at *A*

(b) total acceleration at inflection point *B*, and

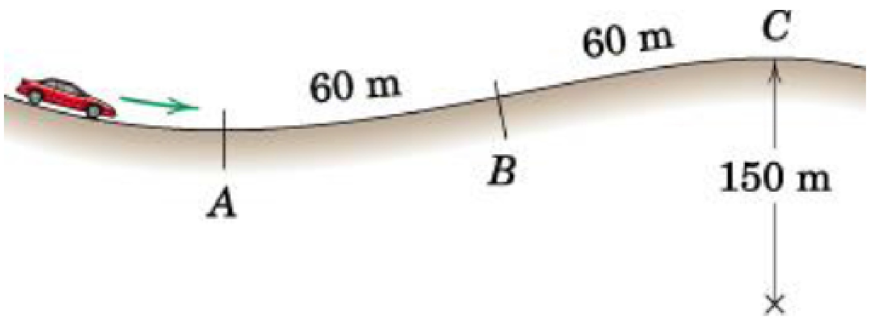
(c) total acceleration at *C*.



Normal and Tangential Coordinates (*n-t*)

Example

Solution:



Converting the units of Velocity:

v_A = 100 km/h [1000/(60x60)] = 27.8 m/s

v_C = 50 km/h [1000/(60x60)] = 13.89 m/s

For Constant Deceleration, we can use the following formulae:

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Using the third equation between *A* and *C* to find the constant deceleration of the car:

$(13.89)^2 = (27.8)^2 + 2a(120 - 0)$

→ $a = -2.41\ \text{m/s}^2$

This acceleration is the tangential component of the total acceleration → $\mathbf{a_t = -2.41\ m/s^2}$

(a) radius of curvature at *A*?

Total accln at *A* is given as: $a = 3\ \text{m/s}^2$

Using the third eqn: $(3)^2 = (a_n)^2 + (-2.41)^2$

→ $a_n = 1.785\ \text{m/s}^2$

Using the first eqn: $\rho_A = (27.8)^2/1.785$ → $\rho_A = 432$ m

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

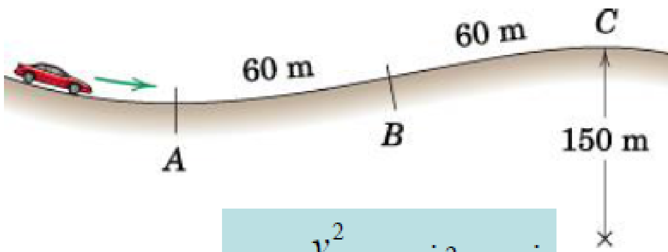
$$a_t = \dot{v} = \ddot{s}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

Kinematics of Particles: Plane Curvilinear Motion

Normal and Tangential Coordinates (*n-t*)

Example



Solution:

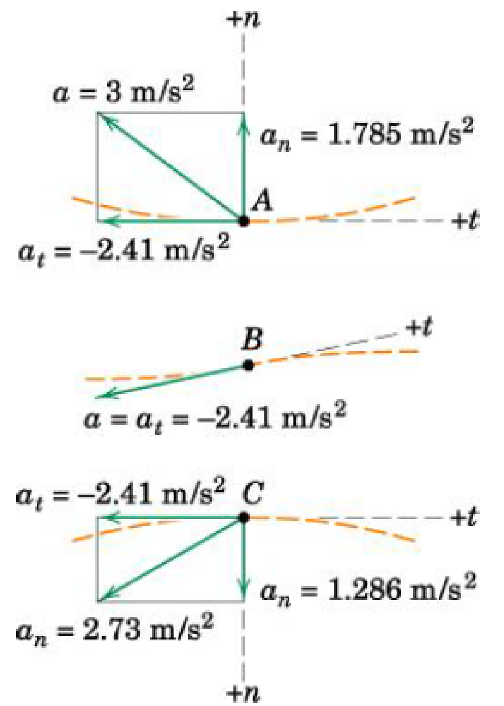
v_A = 100 km/h [1000/(60x60)] = 27.8 m/s
v_C = 50 km/h [1000/(60x60)] = 13.89 m/s

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$
$$a_t = \dot{v} = \ddot{s}$$
$$a = \sqrt{a_n^2 + a_t^2}$$

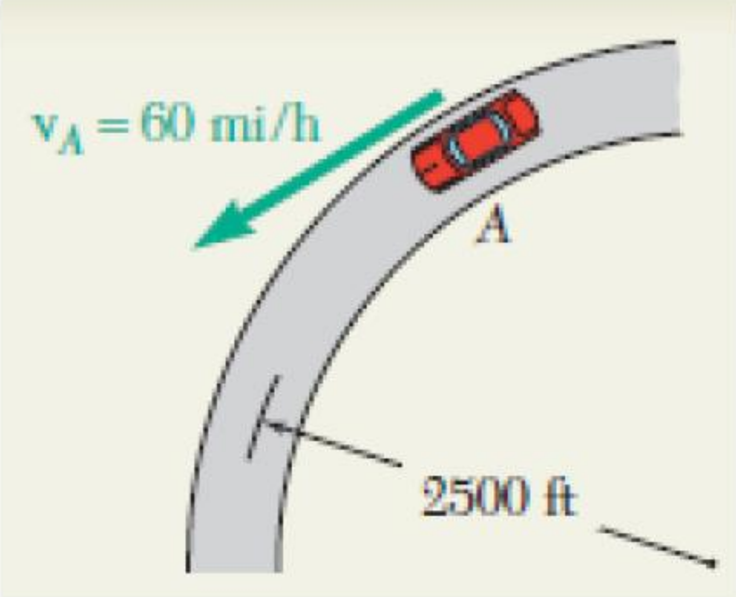
(b) total acceleration at inflection point *B*?
Tangential component of acceleration at *B*, a_t = -2.41 m/s^2
At inflection point radius of curvature is infinity,
Therefore, normal component of acceleration, a_n = 0
→ Total acceleration at *B*: $a = a_t$ = -2.41 m/s^2

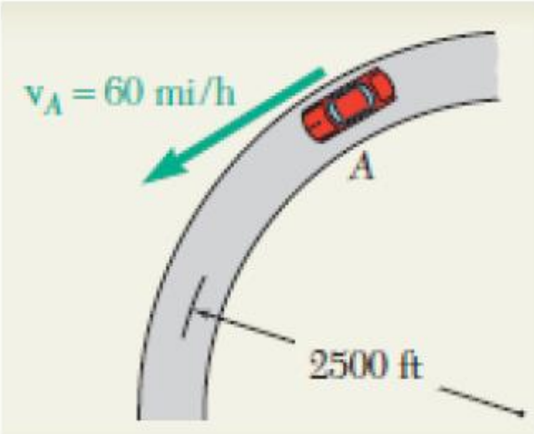
(c) total acceleration at *C*?
Tangential component of acceleration at *C*, a_t = -2.41 m/s^2
Normal component can be found from first eqn:
a_n = (13.89)2/150 = 1.286 m/s^2

Total acceleration at C: $a^2 = (1.286)^2 + (-2.41)^2$
→ Total acceleration at *C*: a = 2.73 m/s^2



A motorist is traveling on a curved section of highway of radius 2500 ft at the speed of 60 mi/h. The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. Knowing that after 8 s the speed has been reduced to 45 mi/h, determine the acceleration of the automobile immediately after the brakes have been applied.





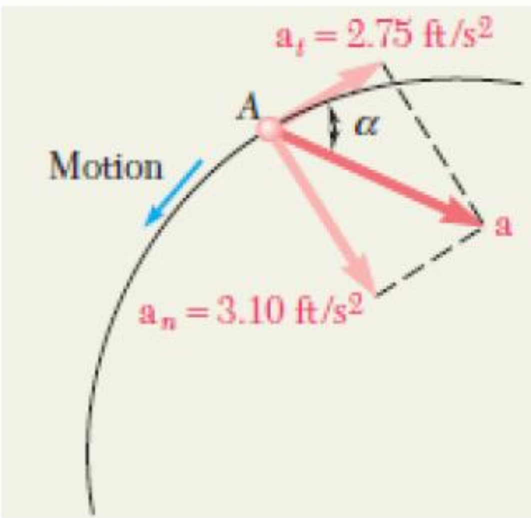
Tangential Component of Acceleration. First the speeds are expressed in ft/s.

$$60 \text{ mi/h} = \left(60 \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 88 \text{ ft/s}$$

$$45 \text{ mi/h} = 66 \text{ ft/s}$$

Since the automobile slows down at a constant rate, we have

$$a_t = \text{average } a_t = \frac{\Delta v}{\Delta t} = \frac{66 \text{ ft/s} - 88 \text{ ft/s}}{8 \text{ s}} = -2.75 \text{ ft/s}^2$$



Normal Component of Acceleration. Immediately after the brakes have been applied, the speed is still 88 ft/s, and we have

$$a_n = \frac{v^2}{\rho} = \frac{(88 \text{ ft/s})^2}{2500 \text{ ft}} = 3.10 \text{ ft/s}^2$$

Magnitude and Direction of Acceleration. The magnitude and direction of the resultant **a** of the components $\mathbf{a}_n$ and $\mathbf{a}_t$ are

$$\tan \alpha = \frac{a_n}{a_t} = \frac{3.10 \text{ ft/s}^2}{2.75 \text{ ft/s}^2} \qquad \alpha = 48.4° \quad ◀$$

$$a = \frac{a_n}{\sin \alpha} = \frac{3.10 \text{ ft/s}^2}{\sin 48.4°} \qquad a = 4.14 \text{ ft/s}^2 \quad ◀$$