Engineering Mechanics
AGE 2330

Lect 12: Lect 10 Review

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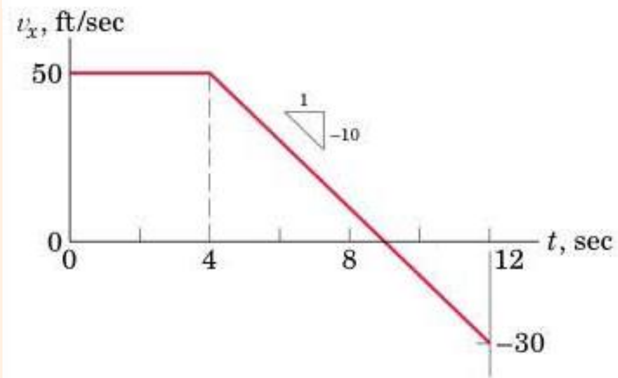
A particle moves along the x-axis with an initial velocity $v_x = 50$ ft/sec at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10$ ft/sec^2. Calculate the velocity and the x-coordinate of the particle for the conditions of $t = 8$ sec and $t = 12$ sec and find the maximum positive x-coordinate reached by the particle.



Solution. The velocity of the particle after $t = 4$ sec is computed from

$$\left[\int dv = \int a\,dt\right] \qquad \int_{50}^{v_x} dv_x = -10\int_4^t dt \qquad v_x = 90 - 10t \text{ ft/sec}$$

and is plotted as shown. At the specified times, the velocities are

$$t = 8 \text{ sec}, \qquad v_x = 90 - 10(8) = 10 \text{ ft/sec}$$

$$t = 12 \text{ sec}, \qquad v_x = 90 - 10(12) = -30 \text{ ft/sec} \qquad \textit{Ans.}$$

The x-coordinate of the particle at any time greater than 4 seconds is the distance traveled during the first 4 seconds plus the distance traveled after the discontinuity in acceleration occurred. Thus,

$$\left[\int ds = \int v\,dt\right] \qquad x = 50(4) + \int_4^t (90 - 10t)\,dt = -5t^2 + 90t - 80 \text{ ft}$$

For the two specified times,

$$t = 8 \text{ sec}, \qquad x = -5(8^2) + 90(8) - 80 = 320 \text{ ft}$$

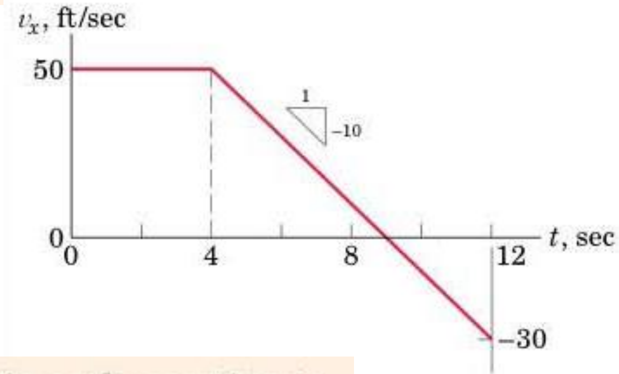
$$t = 12 \text{ sec}, \qquad x = -5(12^2) + 90(12) - 80 = 280 \text{ ft}$$

The x-coordinate for $t = 12$ sec is less than that for $t = 8$ sec s
in the negative x-direction after $t = 9$ sec. The maximum positi
then, the value of x for $t = 9$ sec which is

$$x_{max} = -5(9^2) + 90(9) - 80 = 325 \text{ ft}$$

These displacements are seen to be the net positive areas unde
to the values of t in question.

A particle moves along the x-axis with an initial velocity $v_x = 50$ ft/sec at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10$ ft/sec^2. Calculate the velocity and the x-coordinate of the particle for the conditions of $t = 8$ sec and $t = 12$ sec and find the maximum positive x-coordinate reached by the particle.



The x-coordinate for $t = 12$ sec is less than that for $t = 8$ sec since the motion is in the negative x-direction after $t = 9$ sec. The maximum positive x-coordinate is, then, the value of x for $t = 9$ sec which is

$$x_{max} = -5(9^2) + 90(9) - 80 = 325 \text{ ft} \qquad \textit{Ans.}$$

These displacements are seen to be the net positive areas under the v-t graph up to the values of t in question.

The velocity of a particle which moves along the s-axis is given by $v = 2 - 4t + 5t^{3/2}$, where t is in seconds and v is in meters per second. Evaluate the position s, velocity v, and acceleration a when $t = 3$ s. The particle is at the position $s_0 = 3$ m when $t = 0$.

m/s^2

$\boxed{2/3}$ $v = 2 - 4t + 5t^{3/2}$

$$a = \frac{dv}{dt} = -4 + \frac{15}{2} t^{1/2}$$

$$\frac{ds}{dt} = 2 - 4t + 5t^{3/2}$$

$$\int_{s_0 = 3}^{s} ds = \int_0^t (2 - 4t + 5t^{3/2})\, dt$$

$$s = 3 + 2t - 2t^2 + 2t^{5/2}$$

At $t = 3s$: $\begin{cases} s = 22.2 \text{ m} \\ v = 15.98 \text{ m/s} \\ a = 8.99 \text{ m/s}^2 \end{cases}$

2/5 The acceleration of a particle is given by $a = 2t - 10$, where a is in meters per second squared and t is in seconds. Determine the velocity and displacement as functions of time. The initial displacement at $t = 0$ is $s_0 = -4$ m, and the initial velocity is $v_0 = 3$ m/s.

Ans. $v = 3 - 10t + t^2$ (m/s)
$s = -4 + 3t - 5t^2 + \frac{1}{3}t^3$ (m)

2/5

$$a = \frac{dv}{dt} = 2t - 10$$

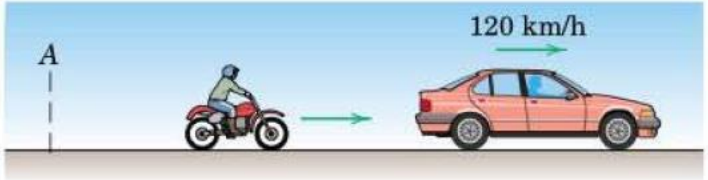
$$\int_{v_0 = 3}^{v} dv = \int_0^t (2t - 10)\, dt, \quad \underline{v = 3 - 10t + t^2} \text{ (m/s)}$$

$$\frac{ds}{dt} = 3 - 10t + t^2$$

$$\int_{s_0 = -4}^{s} ds = \int_0^t (3 - 10t + t^2)\, dt$$

$$\underline{s = -4 + 3t - 5t^2 + \tfrac{1}{3}t^3} \text{ (m)}$$

2/32 A motorcycle patrolman starts from rest at A two seconds after a car, speeding at the constant rate of 120 km/h, passes point A. If the patrolman accelerates at the rate of 6 m/s^2 until he reaches his maximum permissible speed of 150 km/h, which he maintains, calculate the distance s from point A to the point at which he overtakes the car.



2/32 $s_{car} = vt = \frac{120}{3.6} t$

$s_{cycle} = v_{av} t_1 + v_{max} t_2 = \frac{1}{2} \frac{150}{3.6} t_1 + \frac{150}{3.6} t_2$

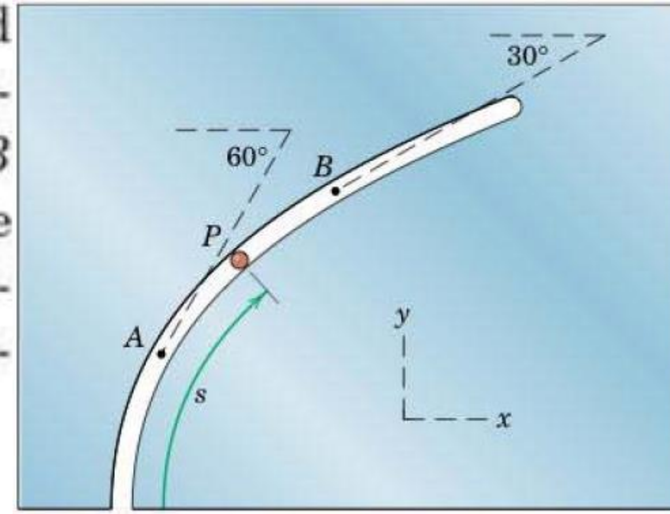
where $t_1 = \frac{v_{max}}{a} = \frac{150}{3.6 \times 6} = 6.94$ s & $t_2 = t - 6.94 - 2$

$s_{car} = s_{cycle}$; $\frac{120}{3.6} t = \frac{75}{3.6} 6.94 + \frac{150}{3.6}(t - 8.94)$

$30t = 820.8$, $t = 27.36$ s

$s = \frac{120}{3.6}(27.36) = \underline{912 \text{ m}}$

The particle P moves along the curved slot, a portion of which is shown. Its distance in meters measured along the slot is given by $s = t^2/4$, where t is in seconds. The particle is at A when $t = 2.00$ s and at B when $t = 2.20$ s. Determine the magnitude a_{av} of the average acceleration of P between A and B. Also express the acceleration as a vector $\mathbf{a}_{av}$ using unit vectors $\mathbf{i}$ and $\mathbf{j}$.



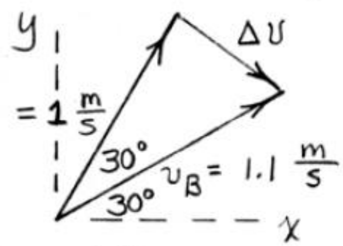
$\boxed{2/65}$ $v = \dot{s} = \frac{t}{2}$, $v_A = \frac{2}{2} = 1$ m/s, $v_B = \frac{2.2}{2} = 1.1 \frac{m}{s}$

$\Delta v_x = v_{Bx} - v_{Ax} = 1.1 \cos 30° - 1.0 \cos 60° = 0.453 \frac{m}{s}$

$\Delta v_y = v_{By} - v_{Ay} = 1.1 \sin 30° - 1.0 \sin 60° = -0.316 \frac{m}{s}$

$\Delta v = \sqrt{0.453^2 + 0.316^2}$

$= 0.552$ m/s



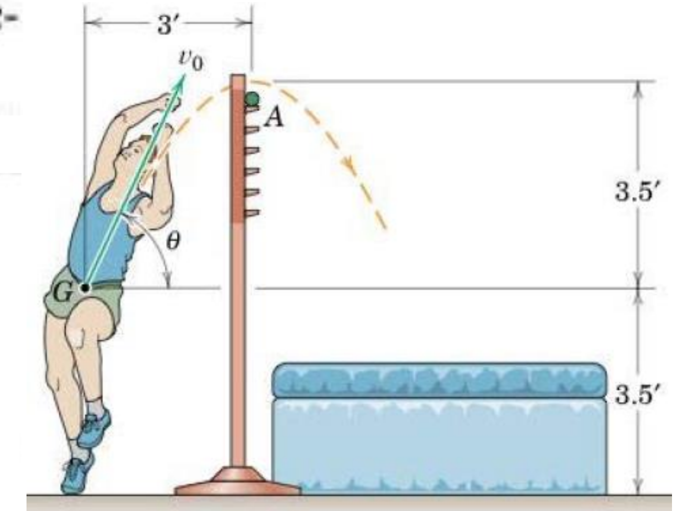
The average acceleration is

$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0.552}{0.20} = \underline{2.76 \text{ m/s}^2}$

$\underline{a}_{av} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.453\underline{i} - 0.316\underline{j}}{0.20}$

$= \underline{2.26\underline{i} - 1.580\underline{j} \text{ m/s}^2}$

The center of mass G of a high jumper follows the trajectory shown. Determine the component v_0, measured in the vertical plane of the figure, of his take-off velocity and angle θ if the apex of the trajectory just clears the bar at A. (In general, must the mass center G of the jumper clear the bar during a successful jump?)



2/71 Set up x-y axes at the initial location of G.

$$x = x_0 + v_{x_0} t \quad : \quad 3 = (v_0 \cos\theta) t$$

$$y = y_0 + v_{y_0} t - \tfrac{1}{2} g t^2 \; : \; 3.5 = (v_0 \sin\theta) t - 16.1 t^2$$

$$v_y = v_{y_0} - g t \quad : \quad 0 = v_0 \sin\theta - 32.2 t$$

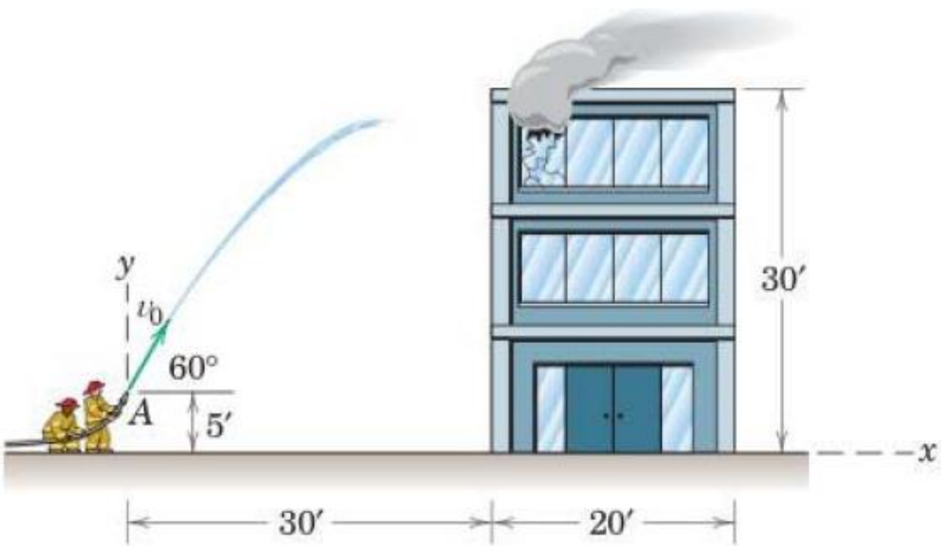
Solve simultaneously:

$t = 0.466$ sec

$\underline{v_0 = 16.33 \text{ ft/sec}}$

$\underline{\theta = 66.8^\circ}$

2/74 Water issues from the nozzle at A, which is 5 ft above the ground. Determine the coordinates of the point of impact of the stream if the initial water speed is (*a*) $v_0 = 45$ ft/sec and (*b*) $v_0 = 60$ ft/sec.



2/74 Use x-y coordinates of the figure.

(a) $v_0 = 45$ ft/sec

$x = x_0 + v_{x_0} t$ @ left wall : $30 = 0 + 45\cos 60°\, t$

$t = 1.333$ sec

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $y = 5 + 45\sin 60° (1.333) - 16.1 (1.333)^2$

$= 28.3$ ft (hits wall)

Ans. : $\underline{(x, y) = (30', 28.3')}$

(b) $v_0 = 60$ ft/sec

Repeat above procedure to find $y = 40.9'$ when $x = 30'$, so water clears left wall.

$x = x_0 + v_{x_0} t$ @ right wall : $50 = 0 + 60 \cos 60° t$

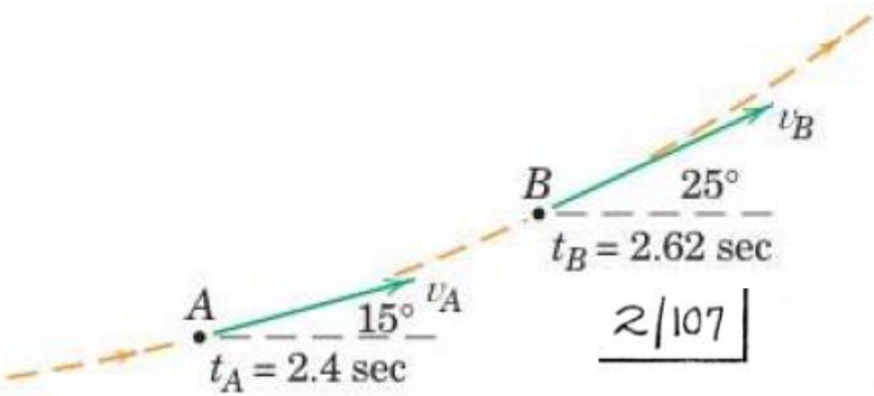
$t = 1.667$ sec

y eq. yields $y = 46.9$ ft @ $t = 1.667$ sec, so water clears building! For horizontal range: From $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ $\underline{y = 0}$, $y_0 = 5$ ft, we find $t = -0.0935$ s & $t = 3.32$ s. From $x = x_0 + v_{x_0} t$: $x = 0 + 60 \cos 60°(3.32) = \underline{99.6 \text{ ft}}$

A particle moves along the curved path shown. The particle has a speed $v_A = 12$ ft/sec at time t_A and a speed $v_B = 14$ ft/sec at time t_B. Determine the average values of the normal and tangential accelerations of the particle between points A and B.

Ans. $a_n = 10.31\ \text{ft/sec}^2$

$a_t = 9.09\ \text{ft/sec}^2$



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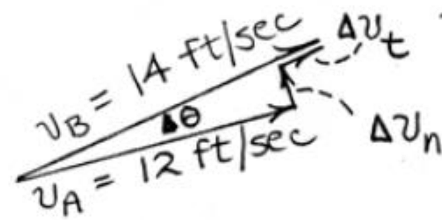
$$\Delta\theta = (25-15)\frac{\pi}{180} = 0.1745 \text{ rad}$$

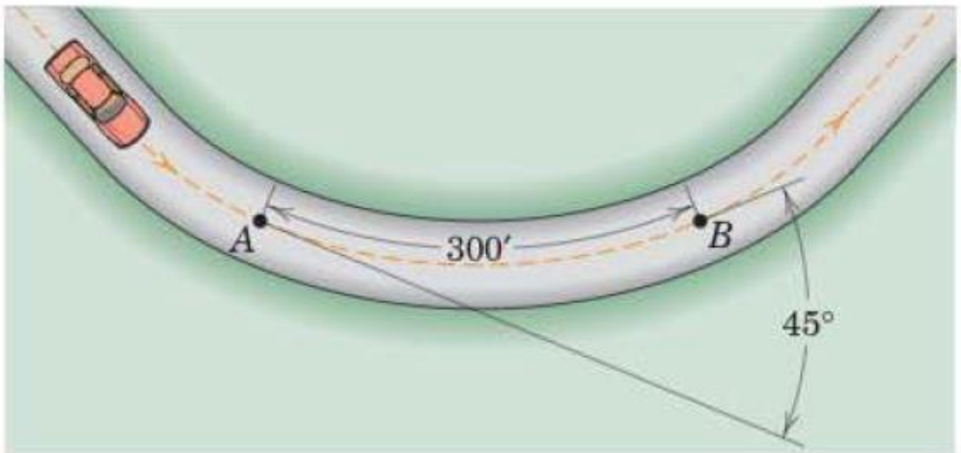
$$v_{av} = \frac{12+14}{2} = 13 \text{ ft/sec}$$



$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v_{av}(\Delta\theta)}{\Delta t} = \frac{13(0.1745)}{2.62-2.4} = \underline{10.31 \frac{\text{ft}}{\text{sec}^2}}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{14-12}{0.22} = \underline{9.09 \frac{\text{ft}}{\text{sec}^2}}$$

2/112 A car rounds a turn of constant curvature between A and B with a steady speed of 45 mi/hr. If an accelerometer were mounted in the car, what magnitude of acceleration would it record between A and B?

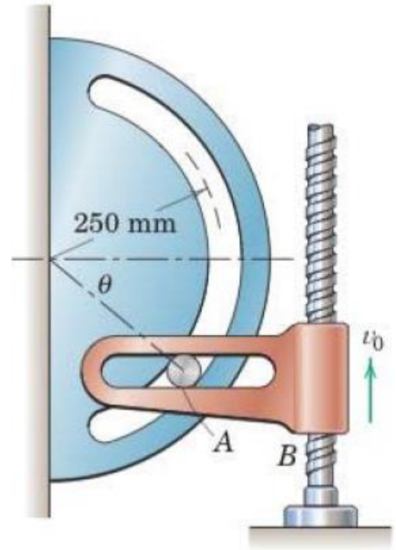


2/112 $a_n = v^2/\rho$; $\rho = s/\theta = \dfrac{300}{\pi/4} = 382$ ft

$v = 45 \dfrac{44}{30} = 66$ ft/sec

$a = a_n = 66^2/382 = \underline{11.40 \text{ ft/sec}^2}$

In the design of a timing mechanism, the motion of the pin A in the fixed circular slot is controlled by the guide B, which is being elevated by its lead screw with a constant upward velocity $v_0 = 2$ m/s for an interval of its motion. Calculate both the normal and tangential components of acceleration of pin A as it passes the position for which $\theta = 30°$.



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$$v = v_0 / \cos 30° = 2/\cos 30° = 2.31 \text{ m/s}$$

$$a_n = v^2/r = 2.31^2/0.250$$

$$= \underline{21.3 \text{ m/s}^2}$$

$$a_t = -a_n \tan 30° = \underline{-12.32 \ \frac{\text{m}}{\text{s}^2}}$$