Engineering Mechanics
AGE 2330

Lect 14: Kinematics of Rigid bodies

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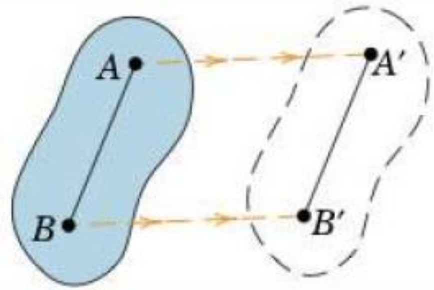
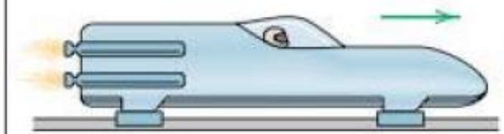
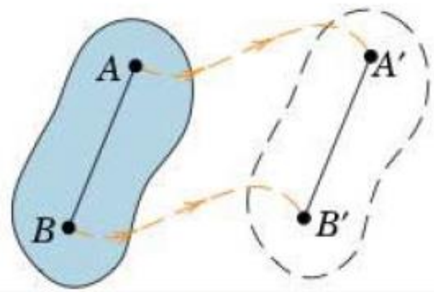
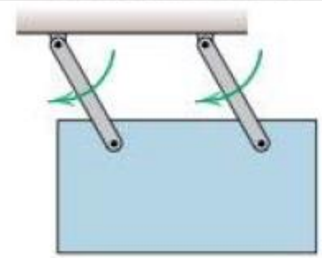
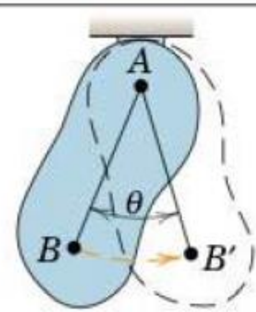
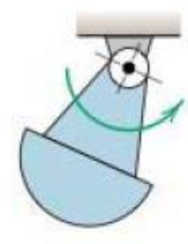
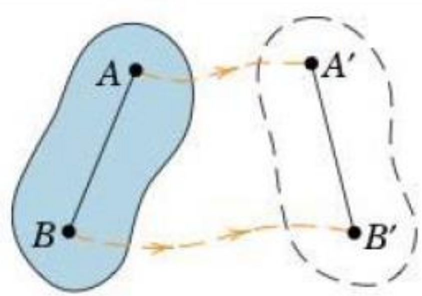
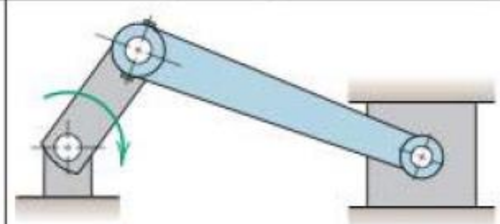
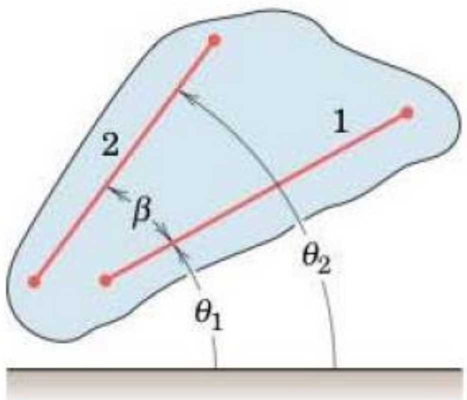
	Type of Rigid-Body Plane Motion	Example
(*a*) Rectilinear translation	 	 Rocket test sled
(*b*) Curvilinear translation	 	 Parallel-link swinging plate
(*c*) Fixed-axis rotation	 	 Compound pendulum
(*d*) General plane motion	 	 Connecting rod in a reciprocating engine

Rotation



$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \qquad \text{or} \qquad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega\, d\omega = \alpha\, d\theta \qquad \text{or} \qquad \dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta$$

constant angular acceleration,

$$\omega = \omega_0 + \alpha t$$

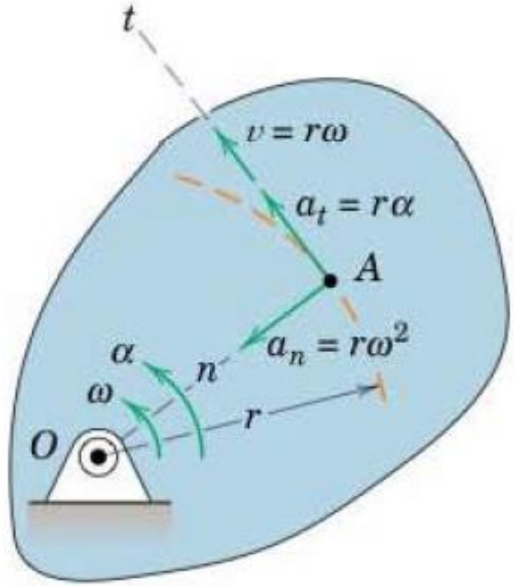
$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$v = r\omega$$

$$a_n = r\omega^2 = v^2/r = v\omega$$

$$a_t = r\alpha$$



A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4t$ rad/s^2, where t is the time in seconds during which the torque is applied. Determine (a) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (b) the time required for the flywheel to reverse its direction of rotation, and (c) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

(a) Since α is a known function of the time, we may integrate it to obtain angular velocity. With the initial angular velocity of $-1800(2\pi)/60 = -60\pi$ rad/s, we have

$$[d\omega = \alpha\, dt] \qquad \int_{-60\pi}^{\omega} d\omega = \int_0^t 4t\, dt \qquad \omega = -60\pi + 2t^2$$

Substituting the clockwise angular speed of 900 rev/min or $\omega = -900(2\pi)/60 = -30\pi$ rad/s gives

$$-30\pi = -60\pi + 2t^2 \qquad t^2 = 15\pi \qquad t = 6.86 \text{ s} \qquad \textit{Ans.}$$

(b) The flywheel changes direction when its angular velocity is momentarily zero. Thus,

$$0 = -60\pi + 2t^2 \qquad t^2 = 30\pi \qquad t = 9.71 \text{ s} \qquad \textit{Ans.}$$

(c) The total number of revolutions through which the flywheel turns during 14 seconds is the number of clockwise turns N_1 during the first 9.71 seconds, plus the number of counterclockwise turns N_2 during the remainder of the interval. Integrating the expression for ω in terms of t gives us the angular displacement in radians. Thus, for the first interval

$$[d\theta = \omega\, dt] \qquad \int_0^{\theta_1} d\theta = \int_0^{9.71} (-60\pi + 2t^2)\, dt$$

$$\theta_1 = \left[-60\pi t + \frac{2}{3}t^3\right]_0^{9.71} = -1220 \text{ rad}$$

or $N_1 = 1220/2\pi = 194.2$ revolutions clockwise.

For the second interval

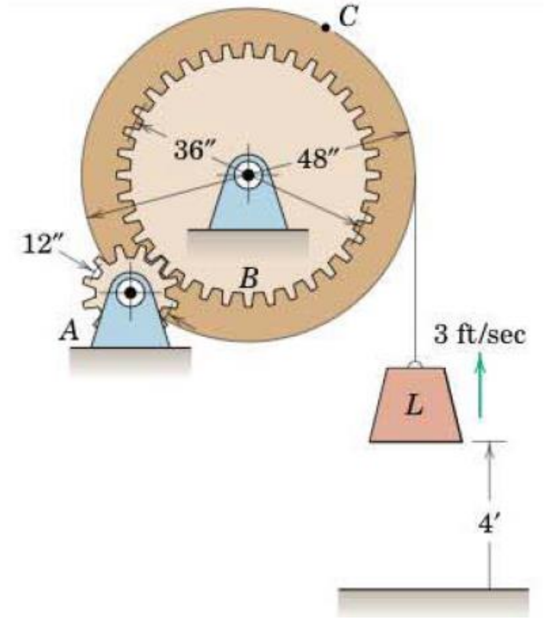
$$\int_0^{\theta_2} d\theta = \int_{9.71}^{14} (-60\pi + 2t^2)\, dt$$

$$\theta_2 = \left[-60\pi t + \frac{2}{3}t^3\right]_{9.71}^{14} = 410 \text{ rad}$$

or $N_2 = 410/2\pi = 65.3$ revolutions counterclockwise. Thus, the total number of revolutions turned during the 14 seconds is

$$N = N_1 + N_2 = 194.2 + 65.3 = 259 \text{ rev} \qquad \textit{Ans.}$$

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 3 ft/sec in a vertical rise of 4 ft with constant acceleration. As the load passes this position, compute (a) the acceleration of point C on the cable in contact with the drum and (b) the angular velocity and angular acceleration of the pinion A.



Solution. (a) If the cable does not slip on the drum, the vertical velocity and acceleration of the load L are, of necessity, the same as the tangential velocity v and tangential acceleration a_t of point C. For the rectilinear motion of L with constant acceleration, the n- and t-components of the acceleration of C become

$[v^2 = 2as]$ $\qquad a = a_t = v^2/2s = 3^2/[2(4)] = 1.125 \text{ ft/sec}^2$

$[a_n = v^2/r]$ $\qquad a_n = 3^2/(24/12) = 4.5 \text{ ft/sec}^2$

$[a = \sqrt{a_n{}^2 + a_t{}^2}]$ $\qquad a_C = \sqrt{(4.5)^2 + (1.125)^2} = 4.64 \text{ ft/sec}^2$ *Ans.*

(b) The angular motion of gear A is determined from the angular motion of gear B by the velocity v_1 and tangential acceleration a_1 of their common point of contact. First, the angular motion of gear B is determined from the motion of point C on the attached drum. Thus,

$[v = r\omega]$ $\qquad \omega_B = v/r = 3/(24/12) = 1.5 \text{ rad/sec}$

$[a_t = r\alpha]$ $\qquad \alpha_B = a_t/r = 1.125/(24/12) = 0.562 \text{ rad/sec}^2$

Then from $v_1 = r_A\omega_A = r_B\omega_B$ and $a_1 = r_A\alpha_A = r_B\alpha_B$, we have

$$\omega_A = \frac{r_B}{r_A}\omega_B = \frac{18/12}{6/12}1.5 = 4.5 \text{ rad/sec CW}$$ *Ans.*

$$\alpha_A = \frac{r_B}{r_A}\alpha_B = \frac{18/12}{6/12}0.562 = 1.688 \text{ rad/sec}^2 \text{ CW}$$ *Ans.*