Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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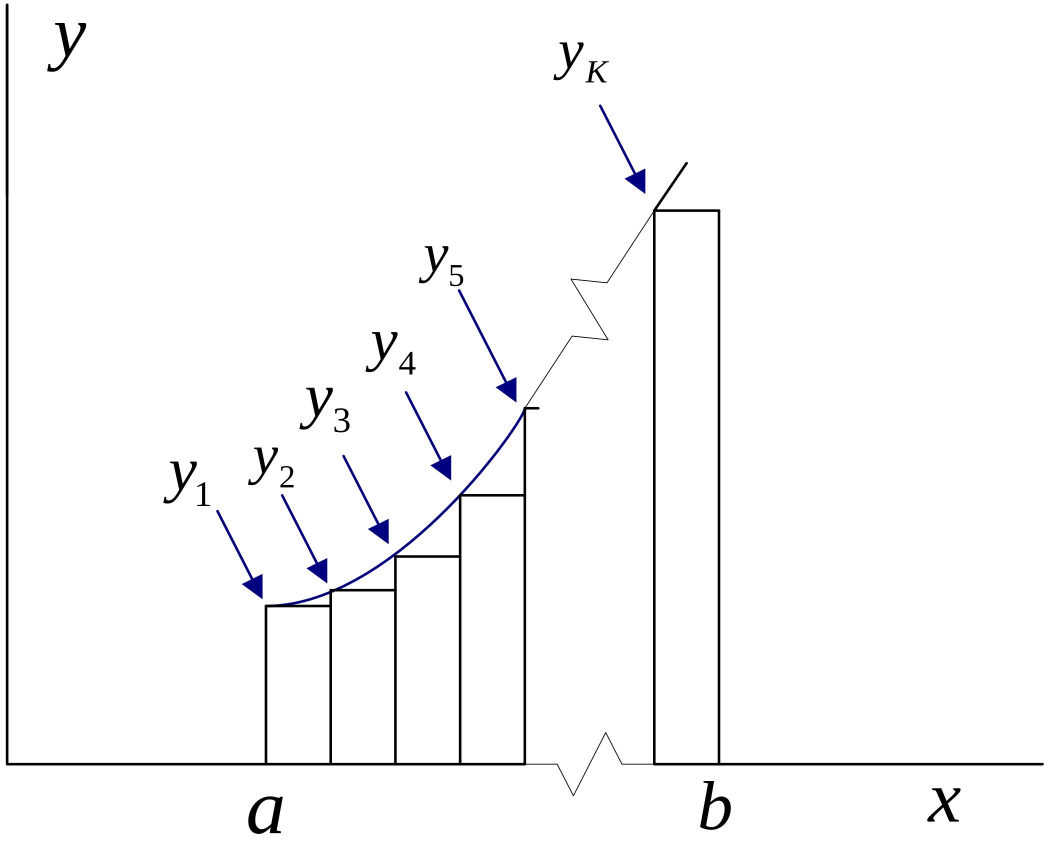
Integral Calculus

Indefinite and Definite Integrals

Indefinite $$\int f(x)dx$$

Definite $$\int_{x_1}^{x_2} f(x)dx$$

Definite Integral as Area Under the Curve



$$\text{Approximate Area} = \sum_k y_k \Delta x$$

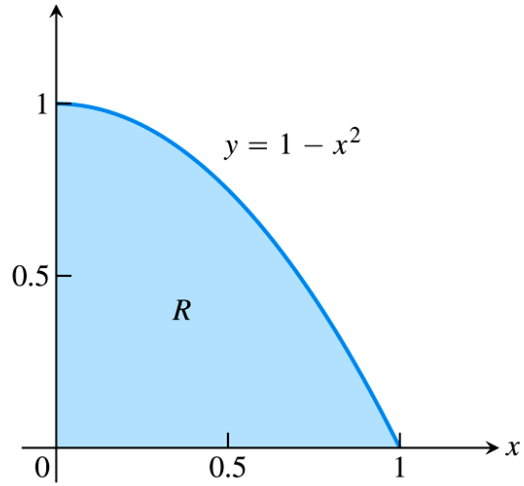
Exact Area as Definite Integral

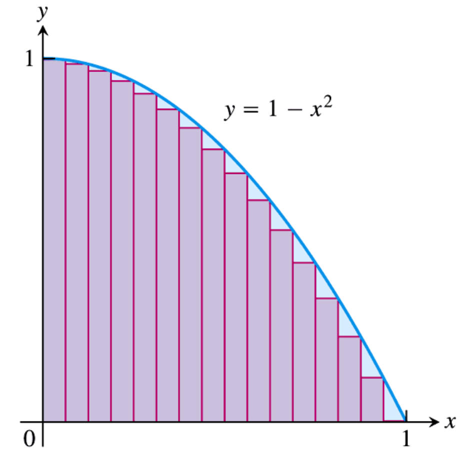
$$\int_a^b y\,dx = \lim_{\Delta x \to dx} \sum_k y_k \Delta x$$

Definite Integral with Variable Upper Limit

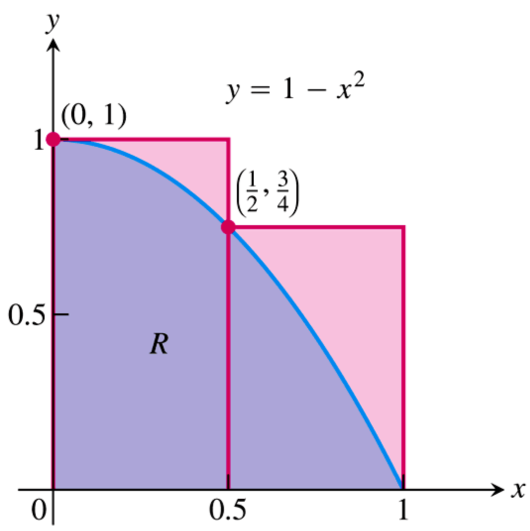
$$\int_a^x y dx$$

The Definite Integral

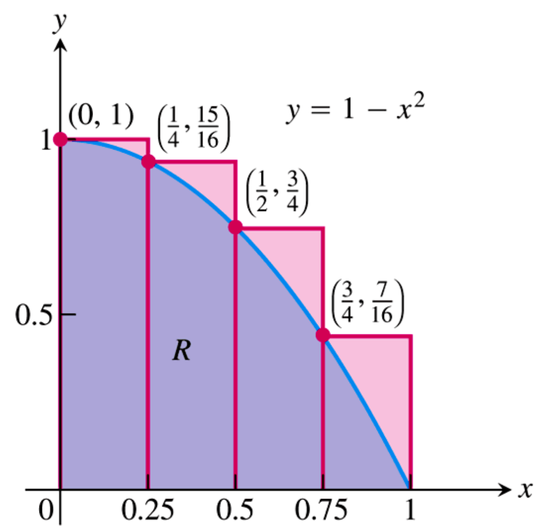




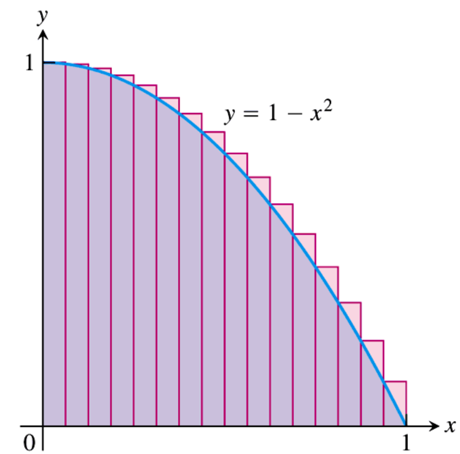
(a)



(a)



(b)



(b)

As the number of rectangles increased, the approximation of the area under the curve approaches a value.

The Definite Integral

Definition

The *definite integral* from *a* to *b*, $\int_a^b f(x)\,dx$ is the number to which all Riemann sums tend as the number of rectangles approaches infinity and as the width of all rectangles tend to zero:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

where Δx_i is the width of the *i*th rectangle and c_i is the *x*-coordinate of the point where the *i*th rectangle touches $f(x)$.

Note: The function f(x) must be continuous on the interval [a, b].

The Definite Integral

Parts of the Definite Integral

The function is the integrand.

Upper limit of integration

x is the variable of integration.

Integral sign

$$\underbrace{\int_a^b f(x)\,dx}_{\text{Integral of } f \text{ from } a \text{ to } b}$$

Lower limit of integration

When you find the value of the integral, you have evaluated the integral.

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Properties of the Definite Integral

Rules satisfied by definite integrals

1.	*Order of Integration:*	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	A Definition
2.	*Zero Width Interval:*	$\int_a^a f(x)\,dx = 0$	A Definition when $f(a)$ exists
3.	*Constant Multiple:*	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$	Any constant k
4.	*Sum and Difference:*	$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$	
5.	*Additivity:*	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$	

Using the Properties of the Definite Integral

Given: $\int_1^3 f(x)dx = 6 \qquad \int_3^7 f(x)dx = 9 \qquad \int_1^3 g(x)dx = -4$

$$\int_1^3 3f(x)dx = 3\int_1^3 f(x)dx = 3(6) = 18$$

$$\int_1^3 (2f(x) - 4g(x))dx = 2\int_1^3 f(x)dx - 4\int_1^3 g(x)dx = 2(6) - 4(-4) = 28$$

$$\int_1^7 f(x)dx = \int_1^3 f(x)dx + \int_3^7 f(x)dx = 6 + 9 = 15$$

$$\int_3^1 f(x)dx = -\int_1^3 f(x)dx = -6$$

Rules of the Definite Integral

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b x\,dx = \frac{b^2}{2} - \frac{a^2}{2}$$

$$\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$$

Examples

$$\int_2^6 4\,dx = \quad 4(6-2) = \quad 16$$

$$\int_4^8 x\,dx = \quad \frac{8^2}{2} - \frac{4^2}{2} = \ 32 - 8 = \ 24$$

$$\int_3^5 x^2\,dx = \frac{5^3}{3} - \frac{3^3}{3} = \quad \frac{125}{3} - \frac{27}{3} = \ \frac{98}{3} = 32.67$$

$$\int_3^4 x^2 + 3x - 2\,dx = \int_3^4 x^2\,dx + 3\int_3^4 x\,dx - \int_3^4 2\,dx = \frac{4^3}{3} - \frac{3^3}{3} + 3\left(\frac{4^2}{2} - \frac{3^2}{2}\right) - \ 2(4-3) =$$

$$\frac{64}{3} - \frac{27}{3} + 3\left(\frac{16}{2} - \frac{9}{2}\right) - \ 2(1) = \ 20.83$$

The Fundamental Theorem of Calculus

If f is continuous at every point in [a, b] and F is any antiderivative of f on [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a).$$

Examples

$$\int_1^5 5x\,dx = \frac{5x^2}{2}\Bigg|_1^5 = \frac{5(5)^2}{2} - \frac{5(1)^2}{2} = \frac{125}{2} - \frac{5}{2} = \frac{120}{2} = 60$$

$$\int_{\pi/6}^{2\pi/3} sinx\,dx = -cosx\Bigg|_a^b = -cos\left(\frac{2\pi}{3}\right) - \left(-cos\left(\frac{\pi}{6}\right)\right) = -\left(-\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right) = 0.866$$

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a).$$

Examples

$$\int_3^4 x^2 + 3x - 2\,dx = \left.\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right|_3^4 = \frac{4^3}{3} + \frac{3(4)^2}{2} - 2(4) - \left(\frac{3^3}{3} + \frac{3(3)^2}{2} - 2(3)\right)$$

$$37.33 - 16.5 = 20.83$$

$$\int_1^{32} \frac{1}{x^{6/5}}\,dx = \int_1^{32} x^{-6/5}\,dx = \left.\frac{x^{-1/5}}{-\frac{1}{5}}\right|_1^{32} = \left.\frac{-5}{x^{1/5}}\right|_1^{32} = -\frac{5}{2} - \left(-\frac{5}{1}\right) = 2.5$$

Differentiating a Definite Integral

$If\ f\ is\ continuous\ on\ [a,b]\ and\ x\ is\ any\ point\ on\ [a,b], then$

$$F'(x) = \frac{d}{dx}\int_{a}^{x} f(t)dt = f(x)$$

$$\frac{d}{dx}\int_{1}^{x} t^2\,dt = \frac{d}{dx}\left(\frac{t^3}{3}\right)\Bigg|_{1}^{x} = \frac{d}{dx}\left(\frac{x^3}{3} - \frac{1^3}{3}\right) = \frac{d}{dx}\left(\frac{x^3}{3} - \frac{1}{3}\right) = x^2$$

$$\frac{d}{dx}\int_{1}^{4x} t^2\,dt = \frac{d}{dx}\left(\frac{t^3}{3}\right)\Bigg|_{1}^{4x} = \frac{d}{dx}\left(\frac{(4x)^3}{3} - \frac{1^3}{3}\right) = \frac{d}{dx}\left(\frac{64x^3}{3} - \frac{1}{3}\right) = 64x^2$$

$$\frac{d}{dx}\int_{1}^{x^2} t^2\,dt = \frac{d}{dx}\left(\frac{t^3}{3}\right)\Bigg|_{1}^{x^2} = \frac{d}{dx}\left(\frac{(x^2)^3}{3} - \frac{1^3}{3}\right) = \frac{d}{dx}\left(\frac{x^6}{3} - \frac{1}{3}\right) = \frac{6x^5}{3} = 2x^5$$

The Fundamental Theorem of Calculus

Differentiating a Definite Integral

$If\ f\ is\ continuous\ on\ [a,b]\ and\ x\ is\ any\ point\ on\ [a,b], then$

$$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

$$\frac{d}{dx}\int_1^x t^2\,dt = (x)^2(1) = x^2$$

$$\frac{d}{dx}\int_1^{4x} t^2\,dt = (4x)^2(4) = 64x^2$$

$$\frac{d}{dx}\int_1^{x^2} t^2\,dt = (x^2)^2(2x) = 2x^5$$

Common Integrals.

$f(x)$	$F(x) = \int f(x)dx$	Integral Number
$af(x)$	$aF(x)$	I-1
$u(x)+v(x)$	$\int u(x)dx + \int v(x)dx$	I-2
a	ax	I-3
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	I-4
e^{ax}	$\frac{e^{ax}}{a}$	I-5
$\frac{1}{x}$	$\ln x$	I-6
$\sin ax$	$-\frac{1}{a}\cos ax$	I-7
$\cos ax$	$\frac{1}{a}\sin ax$	I-8
$\sin^2 ax$	$\frac{1}{2}x - \frac{1}{4a}\sin 2ax$	I-9

Continuation.

$\cos^2 ax$	$\frac{1}{2}x + \frac{1}{4a}\sin 2ax$	I-10
$x\sin ax$	$\frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$	I-11
$x\cos ax$	$\frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$	I-12
$\sin ax\cos ax$	$\frac{1}{2a}\sin^2 ax$	I-13
$\sin ax\cos bx$ for $a^2 \neq b^2$	$-\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$	I-14
xe^{ax}	$\frac{e^{ax}}{a^2}\left(ax-1\right)$	I-15
$\ln x$	$x\left(\ln x - 1\right)$	I-16
$\frac{1}{ax^2+b}$	$\frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right)$	I-17