



Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS

MATH 1110

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Reciprocal Identities

- This relationship can be summarized:

$$\sin \theta = \frac{1}{\csc \theta} \qquad \cos \theta = \frac{1}{\sec \theta} \qquad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

- Each identity is true for angles except those that make a denominator equal to zero
- These reciprocal identities must be memorized

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1,$$

$$\tan^2 \theta + 1 = \sec^2 \theta,$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

MUST MEMORIZE!!!

Double Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

$$\tan 2\theta = 2 \frac{\tan \theta}{1 - \tan^2 \theta}$$

Half Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

Angle Addition Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

Sum, Difference and Product of Trigonometric Functions

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A-B) + \sin(A+B)]$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Trigonometric Integrals

Recall Basic Identities

- Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

These will be used to
integrate powers of
sin and cos

- Half-Angle Formulas

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

Integral of $\sin^n x$, n Odd

- Split into product of an even and $\sin x$

$$\int \sin^5 x \, dx = \int \sin^4 x \cdot \sin x \, dx$$

- Make the even power a power of $\sin^2 x$

$$\int \sin^4 x \cdot \sin x \, dx = \int (\sin^2 x)^2 \sin x \, dx$$

- Use the Pythagorean identity

$$\int (\sin^2 x)^2 \sin x \, dx = \int (1 - \cos^2 x)^2 \sin x \, dx$$

- Let $u = \cos x$, $du = -\sin x \, dx$

$$-\int (1 - u^2)^2 \, du = -\int 1 - 2u^2 + u^4 \, du = \dots$$

Integral of $\sin^n x$, n Odd

- Integrate and un-substitute

$$\begin{aligned} -\int 1 - 2u^2 + u^4 \, du &= -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C \\ &= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C \end{aligned}$$

- Similar strategy with $\cos^n x$, n odd