Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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EXAMPLE 6:

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = 2\frac{\tan\theta}{1-\tan^2\theta}$$

Find $\int \cos^4 x\, dx.$

Use $\cos^2 x = \dfrac{1+\cos 2x}{2}$.

$$\int \cos^4 x\, dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx =$$

$$\int \frac{1 + 2\cos 2x + \cos^2 2x}{4} dx =$$

$$\int \left(\frac{1}{4} + \frac{\cos 2x}{2} + \frac{\cos^2 2x}{4}\right) dx =$$

$$\frac{x}{4} + \frac{\sin 2x}{4} + \frac{1}{4}\int \frac{1+\cos 4x}{2} dx =$$

$$\frac{x}{4} + \frac{\sin 2x}{4} + \frac{1}{8}x + \frac{1}{32}\sin 4x =$$

$$\frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C.$$

EXAMPLE 7:

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Find $\int sec^4x\, tan^{10}x\, dx$

We will let $u = tan\ x$, $du = sec^2x\, dx$, and use the identity

$$1 + tan^2x = sec^2x\ .$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Peel off a factor of sec^2x .

$$\int sec^4x\, tan^{10}x\, dx = \int sec^2x\, tan^{10}x\, sec^2x\, dx =$$

$$\int (1 + tan^2x)tan^{10}x\, sec^2x\, dx =$$

$$\int \left(tan^{10}x + tan^{12}x\right)sec^2x\, dx =$$

$$\int (u^{10} + u^{12})du =$$

$$\frac{tan^{11}x}{11} + \frac{tan^{13}x}{13} + C\ .$$

EXAMPLE 8:

Find $\int sec^3 x\, tan^3 x\, dx.$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Technique: Peel off a $\boldsymbol{sec\, x\, tan\, x}$.

Then let

$u = sec\, x,\ du\ = sec\, x\, tan\, x\, dx,$

and use the identity $1 + tan^2 x = sec^2 x$.

$$\int sec^3 x\, tan^3 x\, dx =$$

$$\int sec^2 x\, tan^2 x\, (\boldsymbol{sec\, x\, tan\, x})dx$$

$u = sec\, x,\ du\ = sec\, x\, tan\, x\, dx$

$tan^2 x = sec^2 x - 1 = u^2 - 1$

$$\int sec^2 x \, sec\, x \, tan^2 x \, tan\, x \, dx =$$

$$\int sec^2 x \left(sec^2 x - 1\right) sec\, x \, tan\, x \, dx$$

$$= \int \left(sec^4 x - sec^2 x\right)(sec\, x \, tan\, x)dx =$$

$$\int \left(u^4 - u^2\right) du$$

(where $u = sec\, x$, $du = sec\, x \, tan\, x \, dx$)

$$= \frac{u^5}{5} - \frac{u^3}{3} = \frac{sec^5 x}{5} - \frac{sec^3 x}{3} + C\,.$$

EXAMPLE 9:

Find $\int sec^6 x\, tan^5 x\, dx.$

Now we have a choice.
We can keep one $sec^2 x$, use the identity $sec^2 x = 1 + tan^2 x$, and substitute $u = tan\, x$.

$$\int sec^2 x\, sec^4 x\, tan^5 x\, dx =$$

$$\int (1 + tan^2 x)^2 tan^5 x\, sec^2 x\, dx =$$

$$\int (1 + u^2)^2 u^5 du = \int \left(1 + 2u^2 + u^4\right) u^5 du =$$

$$\int \left(u^5 + 2u^7 + u^9\right) du = \frac{u^6}{6} + \frac{2u^8}{8} + \frac{u^{10}}{10} =$$

$$\frac{tan^6 x}{6} + \frac{tan^8 x}{4} + \frac{tan^{10} x}{10} + C\,.$$

EXAMPLE 10:

Find $\int sec^6 x\, tan^5 x\, dx$ a 2nd way.

We can peel off a $sec\, x\, tan\, x$ and make the substitution $u = sec\, x,\ du = sec\, x\, tan\, x\, dx.$

$$\int sec^6 x\, tan^5 x\, dx =$$

$$\int sec^5 x\, tan^4 x\, (sec\, x\, tan\, x\, dx) =$$

$$\int sec^5 x (sec^2 x - 1)^2 (sec\, x\, tan\, x\, dx) =$$

$$\int u^5 (u^2 - 1)^2 du = \int u^5 \left(u^4 - 2u^2 + 1\right) du =$$

$$\int \left(u^9 - 2u^7 + u^5\right) du =$$

$$\frac{sec^{10} x}{10} - \frac{sec^8 x}{4} + \frac{sec^6 x}{6} + C\,.$$

EXAMPLE 11:

Find $\int csc^4 x\, cot^6 x\, dx$.

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\mathbf{1 + \cot^2 \theta = \csc^2 \theta}$$

Let $u = cot\, x$, $du = -\,csc^2 x\, dx$,

$$1 + cot^2 x = csc^2 x\,.$$

$$\int cot^6 x\, csc^2 x\, csc^2 x\, dx =$$

$$-\int u^6 (1 + u^2) du =$$

$$-\int (u^6 + u^8) du = -\frac{u^7}{7} - \frac{u^9}{9} =$$

$$-\frac{cot^7 x}{7} - \frac{cot^9 x}{9} + C.$$

EXAMPLE 12:

Find $\int tan^2 x \, dx$.

Use $tan^2 x = sec^2 x - 1$.

$$\int tan^2 x \, dx = \int (sec^2 x - 1) dx =$$

$$\int sec^2 x \, dx - \int 1 \, dx =$$

$$tan \, x - x + C \, .$$