Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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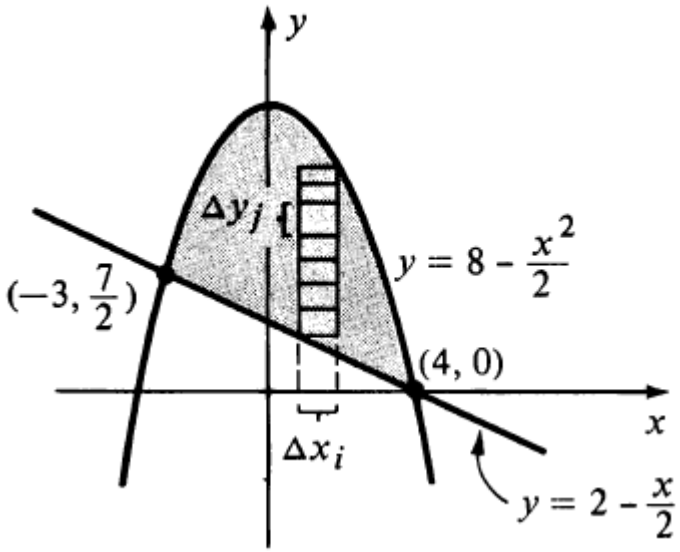
Areas and Volumes

Area Equals to:

$$A = \iint_R dA = \int_a^b \int_{g_1(x)}^{g_2(x)} dy\, dx$$

$$A = \iint_R dA = \int_c^d \int_{h_1(y)}^{h_2(y)} dx\, dy$$

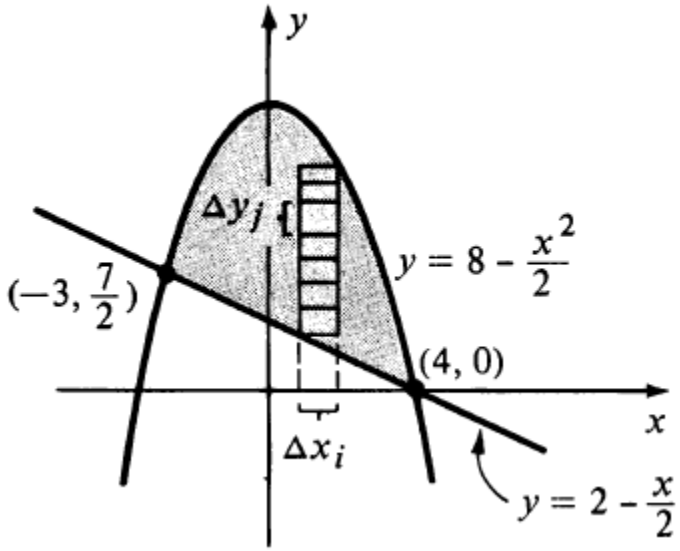
Example 1 Find the area A of the region bounded by the graphs of $2y = 16 - x^2$ and $x + 2y - 4 = 0$.



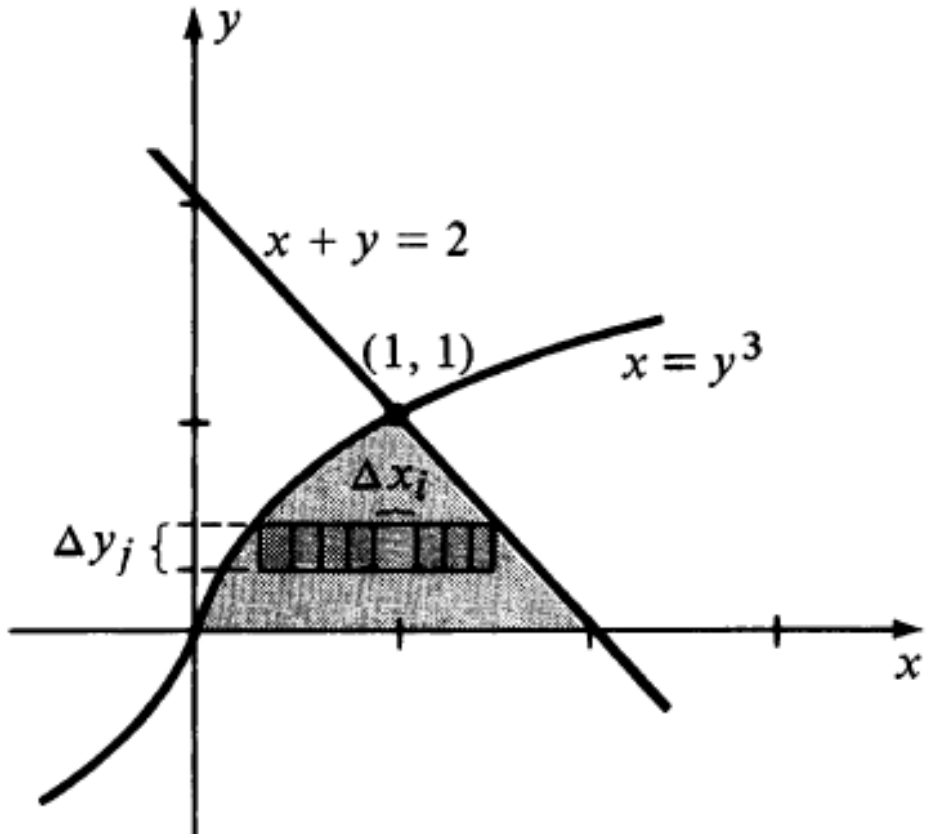
Solution-1

$$A = \int_{-3}^{4} \int_{2-(x/2)}^{8-(x^2/2)} dy\, dx = \int_{-3}^{4} \left[\left(8 - \frac{x^2}{2}\right) - \left(2 - \frac{x}{2}\right) \right] dx$$

$$= 6x - \frac{x^3}{6} + \frac{x^2}{4} \Bigg]_{-3}^{4} = \frac{343}{12}.$$



Example 2 Find the area A of the region in the xy-plane bounded by the graphs of $x = y^3$, $x + y = 2$, and $y = 0$.



Solution-2

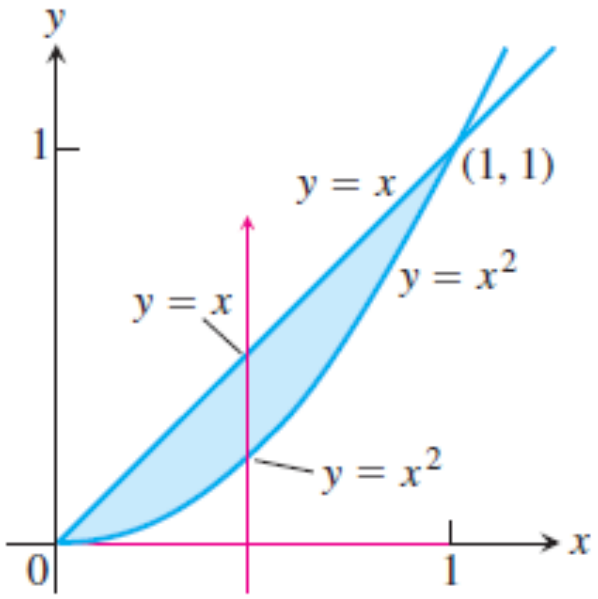
Using an iterated integral gives us

$$A = \iint_R dA = \int_0^1 \int_{y^3}^{2-y} dx\, dy = \int_0^1 x \Big]_{y^3}^{2-y} dy$$

$$= \int_0^1 (2 - y - y^3)\, dy = 2y - \frac{y^2}{2} - \frac{y^4}{4}\Big]_0^1 = \frac{5}{4}.$$

Example 3

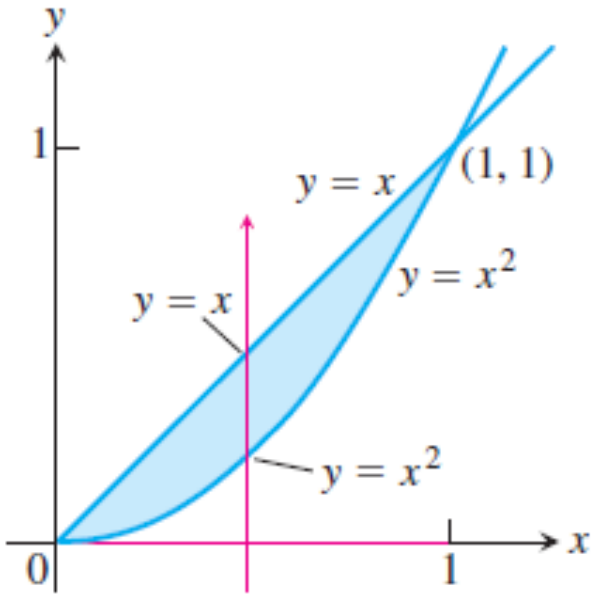
Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.



Solution-3

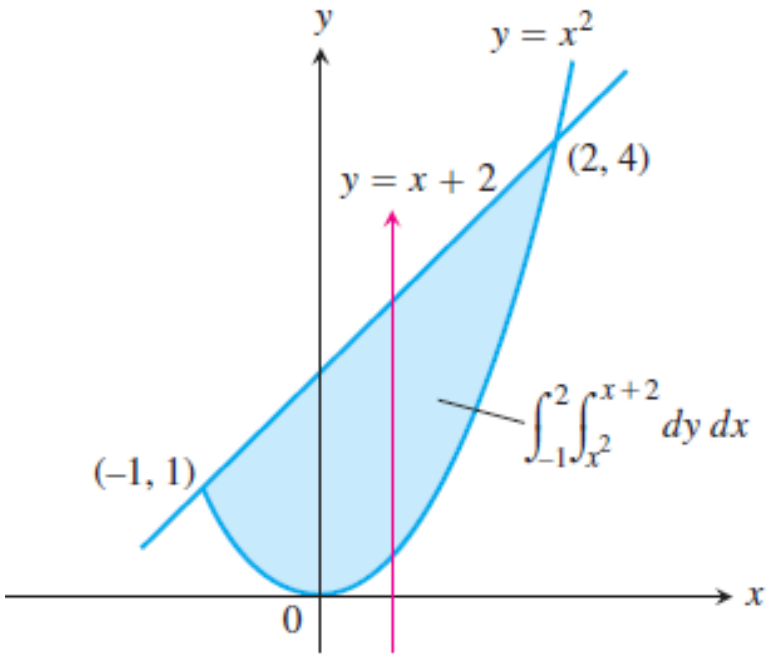
$$A = \int_0^1 \int_{x^2}^{x} dy\, dx = \int_0^1 \Big[y \Big]_{x^2}^{x} dx$$

$$= \int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{6}.$$

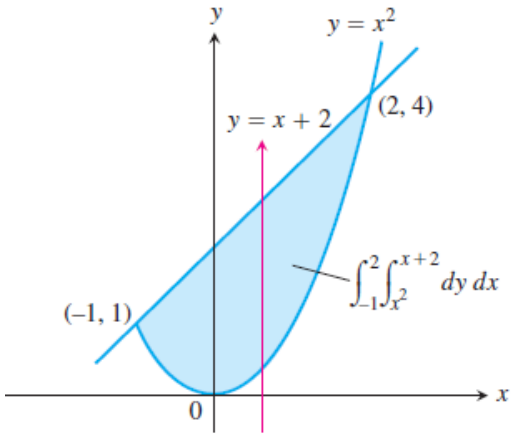


Example 4

Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.



Solution-4



$$A = \int_{-1}^{2} \int_{x^2}^{x+2} dy\, dx.$$

$$A = \int_{-1}^{2} \Big[y \Big]_{x^2}^{x+2} dx$$

$$= \int_{-1}^{2} (x + 2 - x^2)\, dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2} = \frac{9}{2}.$$