Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS
MATH 1110

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Solved Examples

Example 1:

For the following problems
(a) Find the series' radius and interval of convergence

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

$$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1$$

$$\lim_{n \to \infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| < 1$$

$$|x| \lim_{n \to \infty} \left(\frac{1}{n+1} \right) < 1 \text{ for all x}$$

the radius is ∞; the series converges for all x

the series converges absolutely for all x

Example 2:

For the following problems
(a) Find the series' radius and interval of convergence

$$\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$$

$$\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$$

$$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1$$

$$\lim_{n \to \infty} \left| \frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} \right| < 1$$

$$3\,|x| \lim_{n \to \infty} \left(\frac{1}{n+1} \right) < 1 \text{ for all x}$$

the radius is ∞; the series converges for all x

the series converges absolutely for all x

Example 3:

For the following problems
(a) Find the series' radius and interval of convergence

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$$

$$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1$$

$$\lim_{n \to \infty} \left| \frac{x^{2n+3}}{(n+1)!} \cdot \frac{n!}{x^{2n+1}} \right| < 1$$

$$x^2 \lim_{n \to \infty} \left(\frac{1}{n+1} \right) < 1 \text{ for all x}$$

the radius is ∞; the series converges for all x

the series converges absolutely for all x

Example 4:

For the following problems
(a) Find the series' radius and interval of convergence

$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2 + 3}}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2 + 3}}$$

$$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1 \Rightarrow$$

$$\lim_{n \to \infty} \left| \frac{x^{n+1}}{\sqrt{(n+1)^2 + 3}} \cdot \frac{\sqrt{n^2 + 3}}{x^n} \right| < 1$$

$$|x| \sqrt{\lim_{n \to \infty} \frac{n^2 + 3}{n^2 + 2n + 4}} < 1 \Rightarrow |x| < 1$$

$-1 < x < 1$; when $x = -1$ we have

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$, a conditionally convergent series; when $x = 1$ we have

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3}}, \text{ a divergent series}$$

the radius is 1; the interval of convergence is $-1 \leq x < 1$

the interval of absolute convergence is $-1 < x < 1$

the series converges conditionally at $x = -1$

Example 5:

For the following problems
(a) Find the series' radius and interval of convergence

$$\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$$

$$\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$$

$$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1$$

$$\lim_{n \to \infty} \left| \frac{(n+1)(x+3)^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n(x+3)^n} \right| < 1$$

$$\frac{|x+3|}{5} \lim_{n \to \infty} \left(\frac{n+1}{n}\right) < 1 \Rightarrow \frac{|x+3|}{5} < 1$$

$$|x+3| < 5$$

$$-5 < x+3 < 5$$

$-8 < x < 2$; when $x = -8$ we have

$$\sum_{n=1}^{\infty} \frac{n(-5)^n}{5^n} = \sum_{n=1}^{\infty} (-1)^n n, \text{ a divergent}$$

series; when $x = 2$ we have $\sum_{n=1}^{\infty} \frac{n5^n}{5^n} = \sum_{n=1}^{\infty} n$, a divergent series

(a) the radius is 5; the interval of convergence is $-8 < x < 2$

(b) the interval of absolute convergence is $-8 < x < 2$