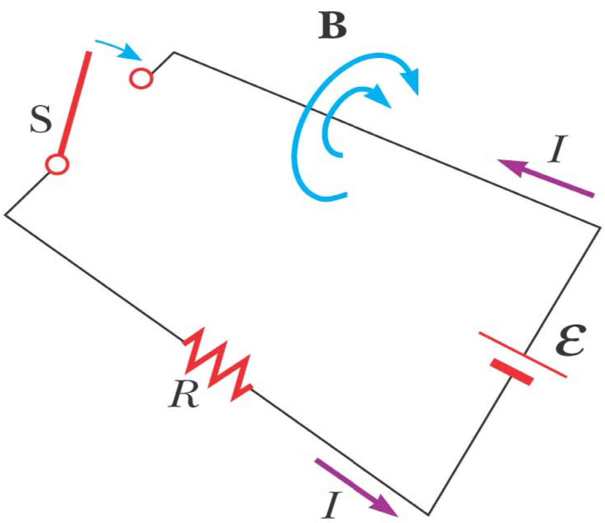
Chapter 32

Inductance

32.1 Self-Induction and Inductance

As the source current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit.

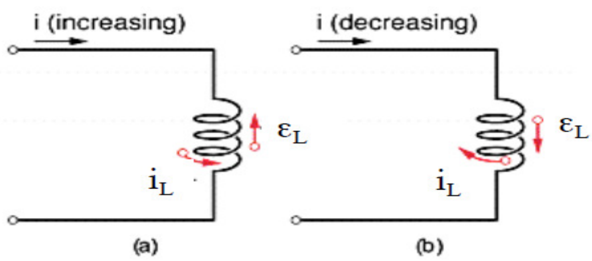
The direction of the induced emf is such that it would cause an induced current in the loop (if a current were not already flowing in the loop), which would establish a magnetic field that would oppose the change in the source magnetic field.



©2004 Thomson - Brooks/Cole

After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop.

As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.



$$\Phi_B \; \alpha I$$

$$\Phi_B = K\, I$$

Using Faraday's law, the self-induced emf can be written as,

$$\varepsilon_L = -N\frac{d\Phi}{dt} = -N\frac{d\Phi}{dI}\frac{dI}{dt}$$

$$= -N\,K\frac{dI}{dt} \qquad 32.1$$

$$\varepsilon_L = -L\frac{dI}{dt}$$

Where L is the proportionality constant-called the inductance of the loop that depends on the geometry of the loop and other physical characteristics.

We know that: $\varepsilon_L = -N\frac{d\Phi_B}{dt}$ so we substitute this in eq. (32.1) as following,

$$-N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \qquad 32.2$$
$$\Rightarrow L = \frac{N\Phi_B}{I}$$

The unit of L is henry (H).

Example 32.1

EXAMPLE 32.2 Calculating Inductance and emf

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

Solution Using Equation 32.4, we obtain

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})\ \frac{(300)^2(4.00 \times 10^{-4}\ \mathrm{m}^2)}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T\cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

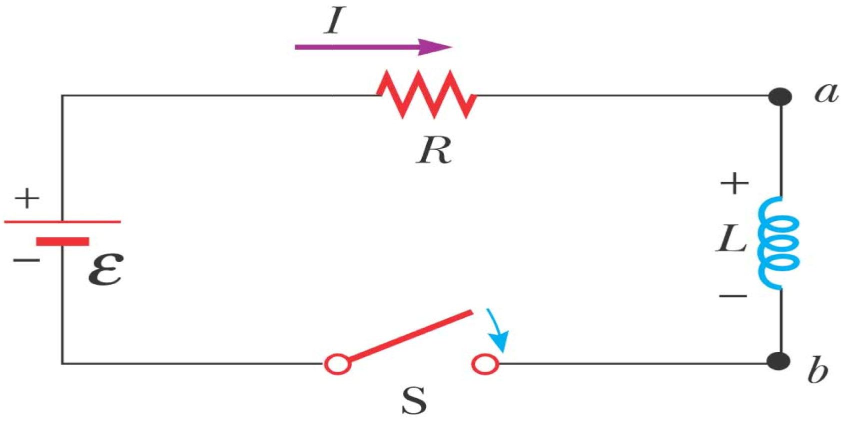
(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Solution Using Equation 32.1 and given that $dI/dt = -50.0$ A/s, we obtain

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$

32.2 Energy in Magnetic field:



In the figure, the battery in the circuit must provide more energy than in a circuit without the inductor. The energy supplied by the battery appears as internal energy in the resistance and the remaining energy is stored in the magnetic field of the inductor. This can be formulated as following,

$$\varepsilon = IR + L\frac{dI}{dt} \qquad 32.3$$

Multiplying this equation by I,

$$I\varepsilon = I^2R + LI\frac{dI}{dt} \qquad 32.4$$

Ok, what this equation does represent?

The last term $\left(LI\frac{dI}{dt}\right)$ represents the rate at which energy U is being stored in the indictor.

$$\frac{dU}{dt} = LI\frac{dI}{dt} \qquad 32.5$$

By integrating this equation one can find the total energy stored in the inductor at any instant,

$$U = \int dU = L\int_0^I IdI = \frac{1}{2}LI^2 \qquad 32.5$$

Also, we can find the energy density of a magnetic field created by a solenoid. For a solenoid, we have derived the magnetic field and the inductance:

$$L = \mu_0 n^2 V$$

$$B = \mu_0 n\ I \Rightarrow I = \frac{B}{\mu_0 n}$$

Substituting these in eq. (32.5),

$$U = \frac{1}{2}\left(\mu_0 n^2 V\right)\left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2\mu_0}B^2 V$$

$$u_B = \frac{U}{V} = \frac{1}{2\mu_0}B^2 \qquad 32.6$$

Where u_B indicates the energy density.

Examples:

31. Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of $3.70 \times 10^{-4}\ \mathrm{T \cdot m^2}$ in each turn.

$$L = \frac{N\Phi_B}{I} = \frac{200(3.70 \times 10^{-4})}{1.75} = 42.3\ \mathrm{mH}$$

so $\quad U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423\mathrm{H})(1.75\mathrm{A})^2 = \boxed{0.0648\ \mathrm{J}}$

$\boxed{33.}$ An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

$$L = \mu_0 \frac{N^2 A}{1} = \mu_0 \frac{(68.0)^2\ \pi(0.600 \times 10^{-2})^2}{0.0800} = 8.21\ \mu\mathrm{H}$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(8.21 \times 10^{-6}\ \mathrm{H})(0.770\ \mathrm{A})^2 = \boxed{2.44\ \mu\mathrm{J}}$$

38. A uniform electric field with a magnitude of 680 kV/m throughout a cylindrical volume results in a total energy of 3.40 μJ. What magnetic field over this same region stores the same total energy?

We have $u = \epsilon_0 \dfrac{E^2}{2}$ and $u = \dfrac{B^2}{2\mu_0}$

Therefore $\epsilon_0 \dfrac{E^2}{2} = \dfrac{B^2}{2\mu_0}$ so $B^2 = \epsilon_0 \mu_0 E^2$

$$B = E\sqrt{\epsilon_0 \mu_0} = \frac{6.80 \times 10^5 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.27 \times 10^{-3} \text{ T}}$$