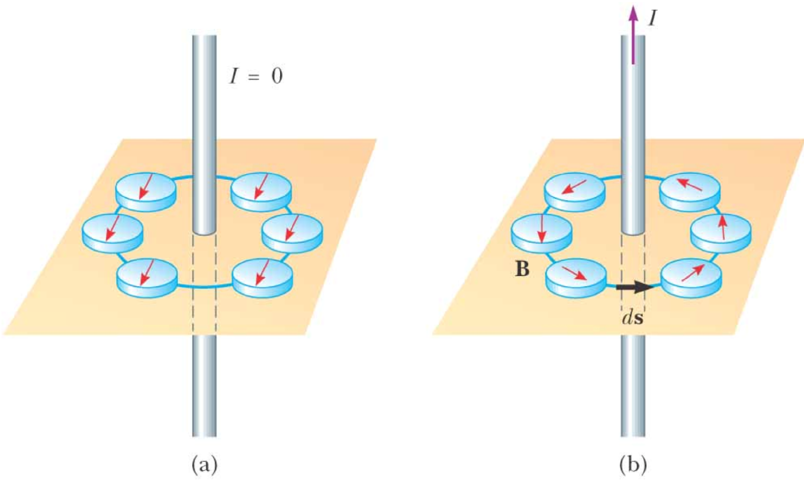
30.3 Ampere's Law

Because the compass needles point in the direction of B, we conclude that the lines of B form circles around the wire,

- The magnitude of B is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire.
- By varying the current and distance *a from the wire, we find that B is proportional to the current and inversely* proportional to the distance from the wire,



$$\oint \vec{B}.d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I \qquad 30.7$$

Ampere's law "the line integral of $\vec{B}.d\vec{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

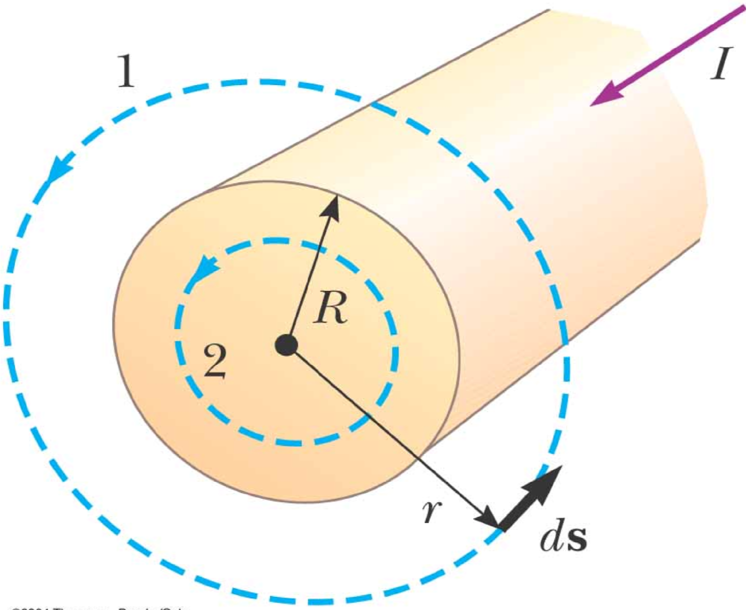
$$\oint \vec{B}.d\vec{s} = \mu_0 I$$

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a *high degree of symmetry*. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

Example:

A long, straight wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire (Fig. 30.11). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Let us choose for our path of integration circle 1 in Figure in front from symmetry, B must be constant in magnitude and parallel to *ds at every* point on this circle. Because the total current passing through the plane of the circle is I_0, *Ampère's law gives*



$$\oint \mathbf{B} \cdot d\mathbf{s} = B\oint ds = B(2\pi r) = \mu_0 I_0$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r}} \qquad (\text{for } r \geq R)$$

Now consider the interior of the wire, where $r < R$. *Here* the current I *passing through the plane of circle 2 is less than* the total current I_0. *Because the current is uniform over the* cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area r^2 *enclosed by* circle 2 to the cross-sectional area R^2 *of the wire:*

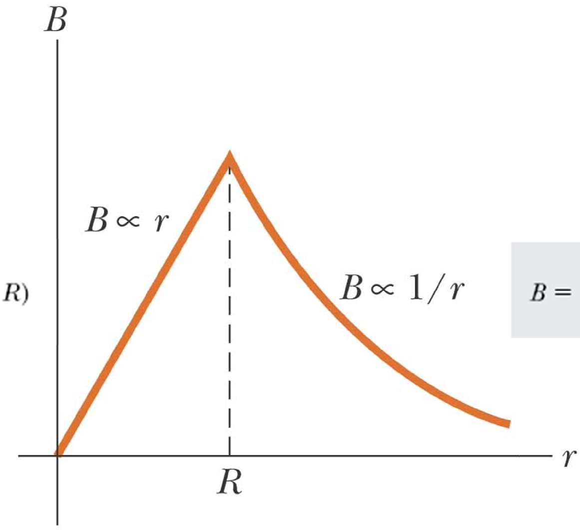
$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2}$$

$$I = \frac{r^2}{R^2} I_0$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I = \mu_0 \left(\frac{r^2}{R^2} I_0 \right)$$

$$\boxed{B = \left(\frac{\mu_0 I_0}{2\pi R^2} \right) r} \qquad \text{(for } r < R\text{)}$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r}} \qquad \text{(for } r \geq R\text{)}$$



$$\boxed{B = \left(\frac{\mu_0 I_0}{2\pi R^2} \right) r} \qquad \text{(for } r < R\text{)}$$