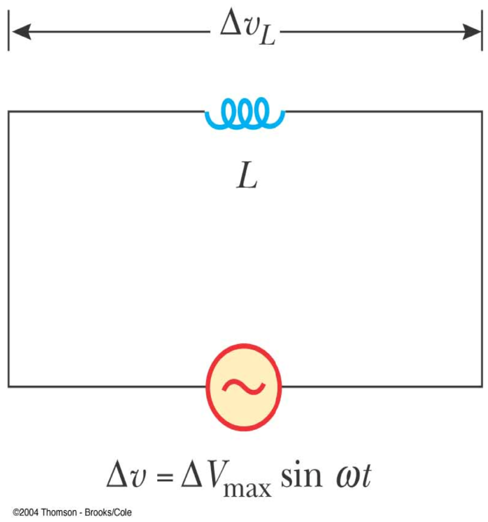
33.3 Inductors in AC circuit

In the figure in front, a simple AC circuit contains an inductor and an AC source.



Similar to our treatment in the previous section, we find that:

$$\Delta v - L\frac{di_L}{dt} = 0 \qquad 33.6$$

$$V_{max} \sin \omega t = L\frac{di_L}{dt}$$

$$\Rightarrow i_L = \frac{V_{max}}{L}\int \sin \omega t \ dt = -\frac{V_{max}}{\omega L}\cos \omega t$$

Using $\cos \omega t = -\sin\left(\omega t - \frac{\pi}{2}\right)$

$$i_L = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad 33.7$$

Where $I_{max} = \frac{V_{max}}{\omega L} = \frac{V_{max}}{X_L}$

Also, X_L is the inductive reactance, $X_L = \omega L$.

Plot of the instantaneous current i_L and instantaneous voltage Δv_L across the inductor as functions of time is shown in the figure below.

It shows that the instantaneous current and the instantaneous voltage across the circuit is out of phase by $\frac{\pi}{2}\text{rad} = 90^0$.

Remarks about the points a,b,c,d,e, and f are discussed

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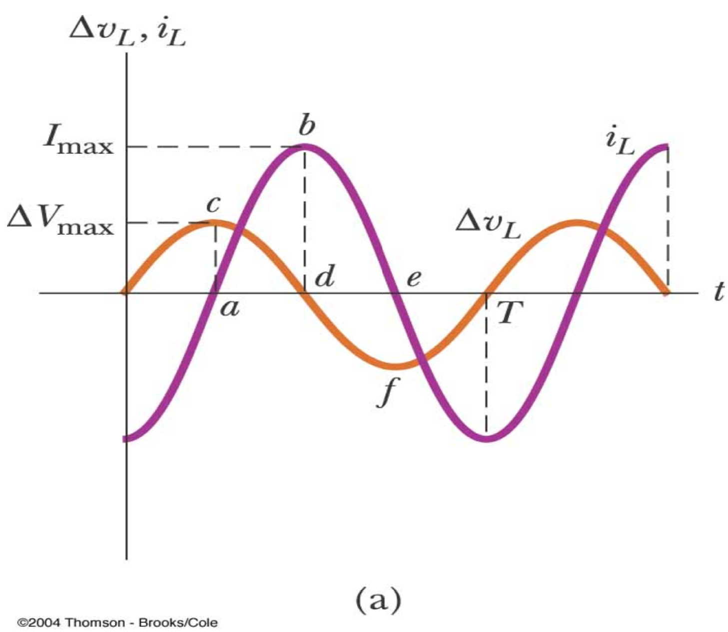

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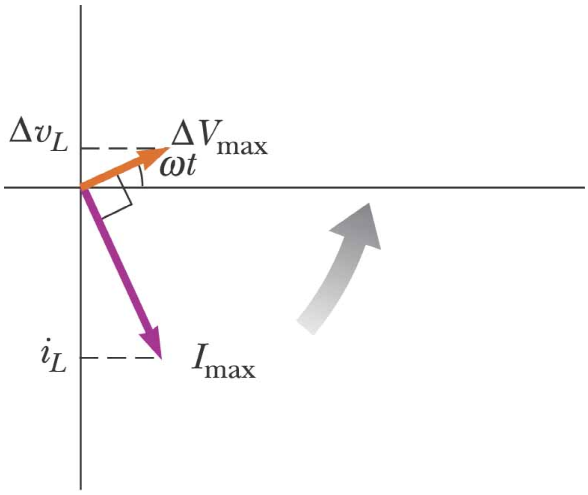
Phasor diagram for the inductive circuit shows that the current lags behind the voltage by 90°.

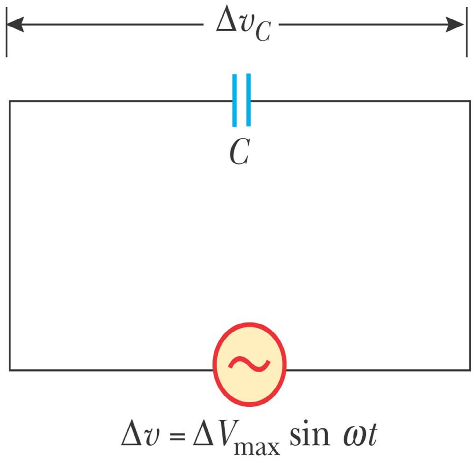


(b)

Example 33.2

33.4 Inductors in AC circuit

In the figure in front, an AC circuit connects to a capacitor. By using Kirchhoff's law, we find



$$\Delta v + \Delta v_C = 0$$

$$\Delta v - \frac{q}{C} = 0 \qquad 33.8$$

$$q = \Delta v C = CV_{max} \sin \omega t$$

We know that $I = \frac{dq}{dt}$. Then,

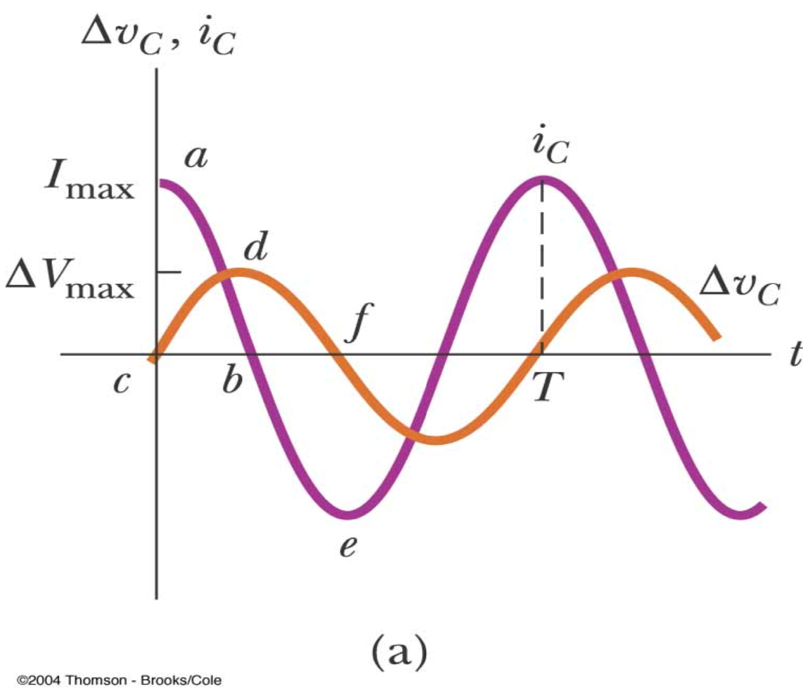
$$\frac{dq}{dt} = \frac{d}{dt} CV_{max} \sin \omega t = \omega CV_{max} \cos \omega t \qquad 33.9$$

$$\Rightarrow i_C = \omega CV_{max} \cos \omega t = I_{max} \sin(\omega t + \frac{\pi}{2})$$

Where $I_{max} = \omega C V_{max} = \frac{V_{max}}{X_C}$

$X_C = (\frac{1}{\omega C})$ and is called the capacitive reactance.

Plot of the instantaneous current i_C and instantaneous voltage Δv_C across the capacitor as functions of time is shown in the figure below.



The current leads the voltage by one-fourth of a cycle.

What can we tell about the current and the voltage at the points a,b,c,d,e, and f?

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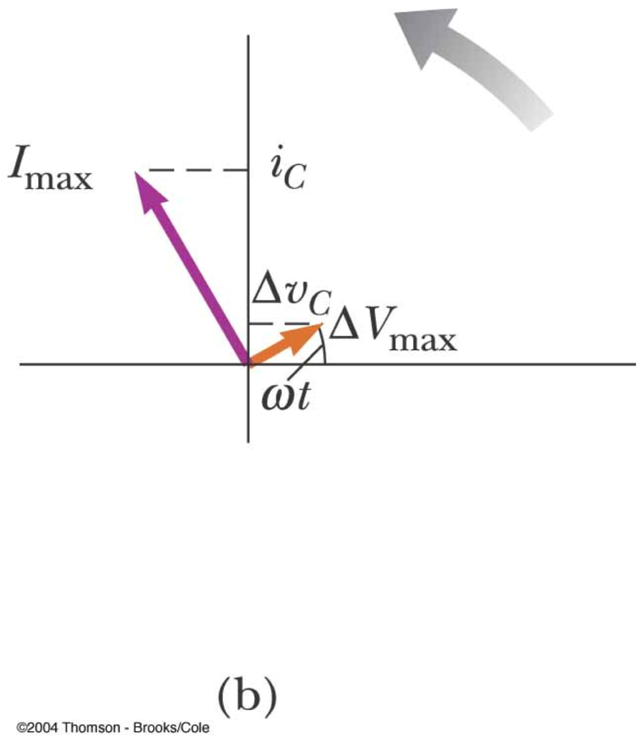
Phasor diagram for the capacitive circuit shows that the current and the voltage phasors are at 90° to each other.



Example 33.3