Chapter 8

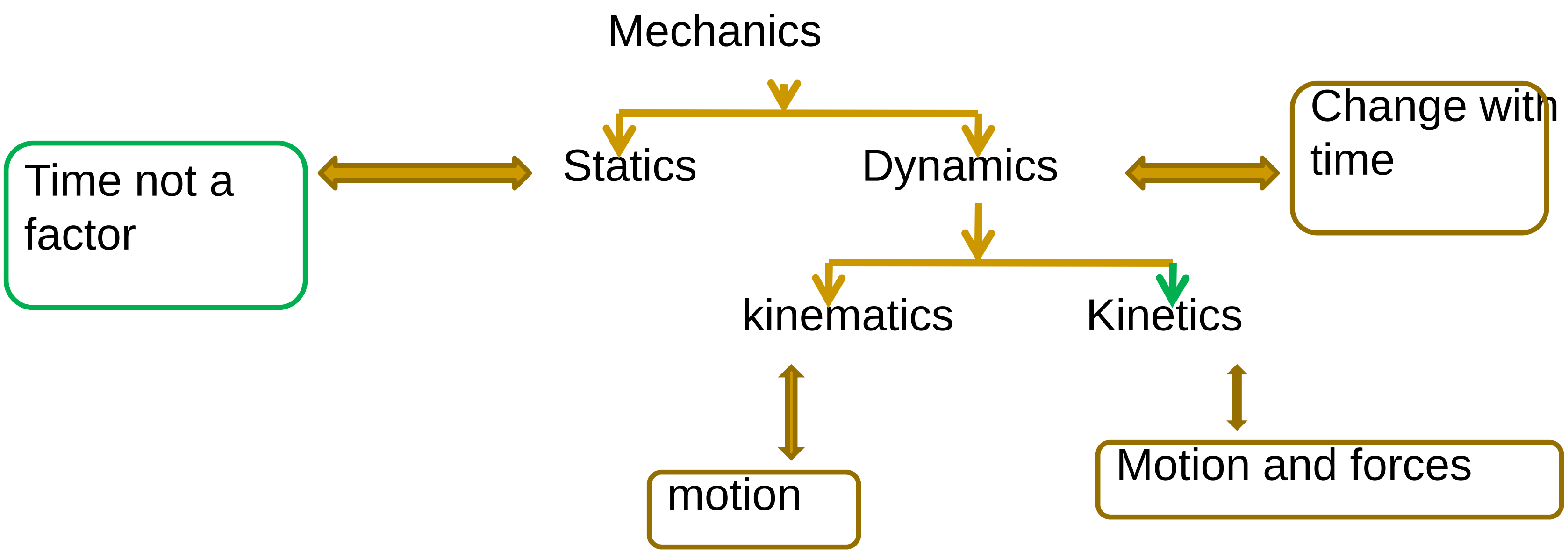
Screws, Fasteners, and the Design of Nonpermanent Joints

Machine design—what is it?

Subset of Mechanical design…which is
Subset of Engineering design…which is
Subset of Design….which is
Subset of the topic of Problem Solving

What is a machine? …a combination of resistant bodies arranged so that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions.

Big picture


Mechanics
Time not a factor
Statics
Dynamics
Change with time
kinematics
Kinetics
motion
Motion and forces

The Design Process

- Recognize need/define problem
- Create a solution/design
- Prepare model/prototype/solution
- Test and evaluate
- Communicate design

Important to review the fundamentals of….

- Statics
- Dynamics
- Materials/material properties
- Mechanics of Materials
- Design of Mechanical Systems 1
- Dynamics of Machinery

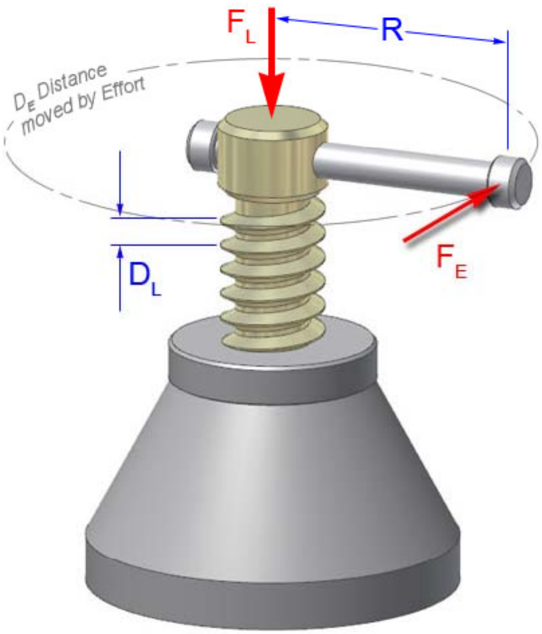
Power Screws

- Transmit angular motion to linear motion
- Transmit large or produce large axial force
- It is always desired to reduce number of screws

Uses of Power Screws

- Obtain high mechanical advantage in order to move large loads with a minimum effort. e.g screw jack.
- Generate large forces e.g tensile testing machine, compactor press.
- Obtain precise axial movements e.g. camera calibration rigs.

Applications



Screw Jacks

Toolmakers Clamp

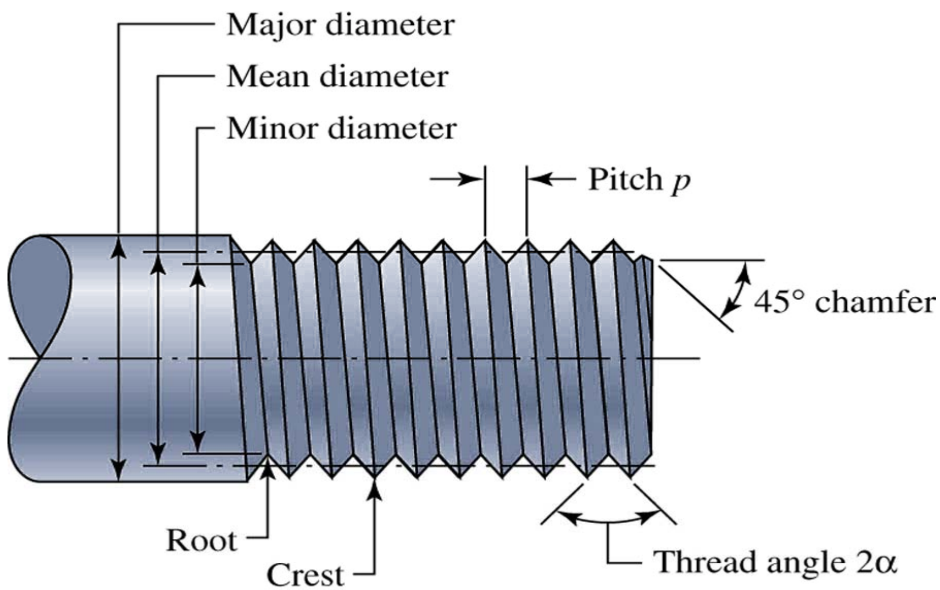
X-Y Precision Table

Advantages of power screws

- Compact design and takes less space
- Large load carrying capability
- Simple to design and easy to manufacture
- Can obtain a large mechanical advantage
- Precise and accurate linear motion
- Easy maintenance
- Self-locking feature

Thread Standards and Definitions

- The terminology of screw threads can be defined as the following



Major diameter d, Minor diameter d_r Mean dia or pitch diameter d_p

Lead l, distance the nut moves for one turn rotation

Thread Standards and Definitions

The terminology of screw threads, illustrated in Fig. 8–1, is explained as follows:

The *pitch* is the distance between adjacent thread forms measured parallel to the thread axis. The pitch in U.S. units is the reciprocal of the number of thread forms per inch N.

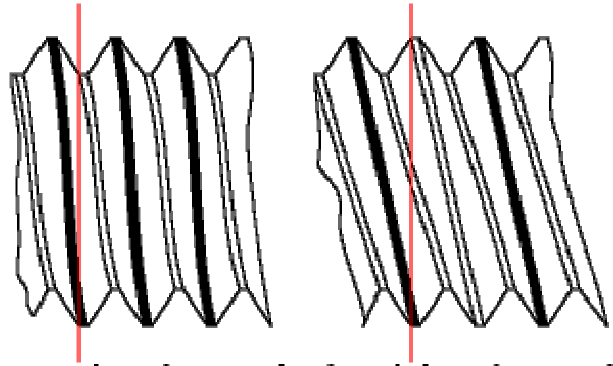
The *major diameter* d is the largest diameter of a screw thread.

The *minor* (or root) *diameter* d_r is the smallest diameter of a screw thread.

The pitch diameter d_p is a theoretical diameter between the major and minor diameters.

The *lead* l, not shown, is the distance the nut moves parallel to the screw axis when the nut is given one turn. For a single thread, as in Fig. 8–1, the lead is the same as the pitch.

Single and Double threaded screws



Double threaded screws are stronger and moves faster

- A single-threaded screw is made by cutting a single helical groove on the cylinder. For a single thread as in our figure, the lead is the same as the pitch.
- A multiple-threaded product is one having two or more threads cut beside each other. A multiple-threaded screw advances a nut more rapidly than a single-threaded screw of the same pitch.
- Double-threaded screw has a lead equal to twice the pitch, a triple-threaded screw has a lead equal to 3 times the pitch, and so on.
- The thread angle is 60^{o}
- The crests of the thread may be either flat or rounded.

Screw Designations

- United National Standard UNS
- International Standard Organization ISO

- UNC (Coarse thread): is the most common and recommended for ordinary applications, where the screw is threaded into a softer material. It is used for general assembly work.

- UNF (Fine thread): is more resistant to loosening, because of its smaller helix angle. Fine threads are widely employed in automotive, aircraft, and other applications where vibrations are likely to occur.

- Many tensile tests of threaded rods have shown that an unthreaded rod having a diameter equal to the mean of the pitch diameter and minor diameter will have the same tensile strength as the threaded rod. The area of this unthreaded rod is called the tensile-stress area A_t of the thread rod.

- Metric threads are specified by writing the diameter and pitch in millimeters, in that order, thus M12X1.75 is a thread having a nominal major diameter of 12mm and pitch of 1.75mm.

Screw Designations

- Class of screw, defines its fit, Class 1 fits have widest tolerances, Class 2 is the most commonly used
- Class three for very precision application
- Example:1in-12 UNRF-2A-LH, A for Ext. Thread and B for Internal, R root radius
- Metric M10x1.5 10 diameter mm major diameter,1.5 pitch

Screw Classifications

	Unified National Standard	ISO (Metric)
Thread Pitch	UNC –coarse UNF –fine UNEF –extra fine	coarse fine
Tolerance	Class 1 Class 2 Class 3	several levels

d=0.25”

fine

Class 2

¼-20 UNF –2A

20 threads/in.

external threads

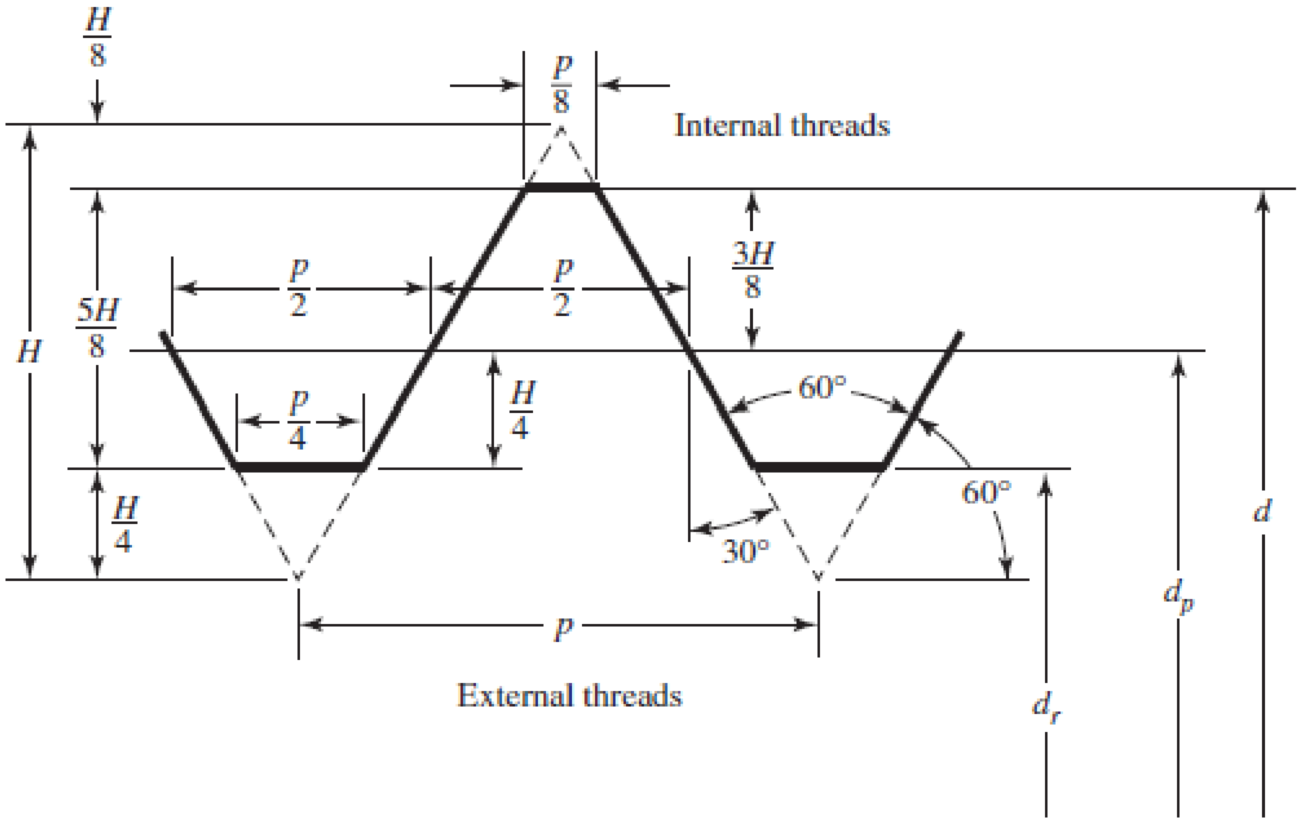
d=12mm

metric

M12 x 1.75

p=1.75 mm/thread

The thread geometry of the metric M and MJ profiles



Basic profile for metric M and MJ threads.

d = major diameter

d_r = minor diameter

d_p = pitch diameter

p = pitch

$H = \frac{\sqrt{3}}{2} p$

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series Pitch p mm	Coarse-Pitch Series Tensile-Stress Area A_t mm^2	Coarse-Pitch Series Minor-Diameter Area A_r mm^2	Fine-Pitch Series Pitch p mm	Fine-Pitch Series Tensile-Stress Area A_t mm^2	Fine-Pitch Series Minor-Diameter Area A_r mm^2
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

A great many tensile tests of threaded rods have shown that an unthreaded rod having a diameter equal to the mean of the pitch diameter and minor diameter will have the same tensile strength as the threaded rod. The area of this unthreaded rod is called the tensile-stress area A_t of the threaded rod; values of A_t are listed in both tables.

Table 8–2

Diameters and Area of Unified Screw Threads UNC and UNF*

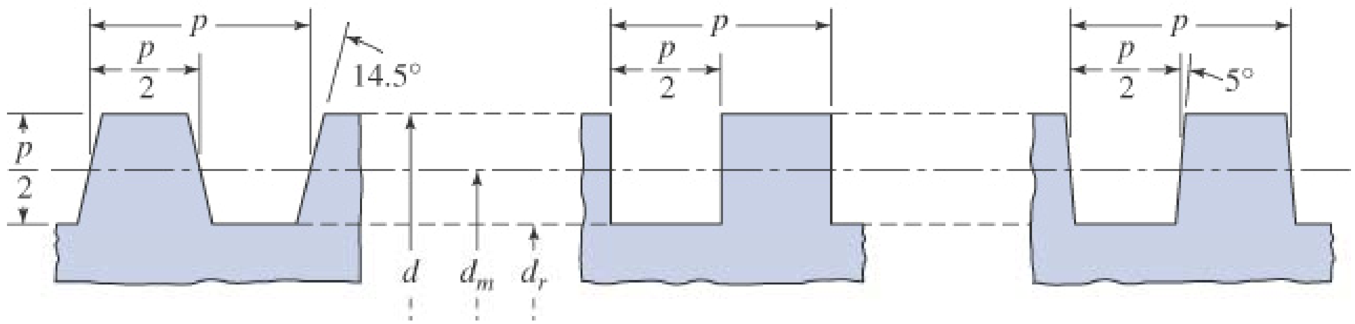
Size Designation	Nominal Major Diameter in	Coarse Series—UNC Threads per Inch N	Coarse Series—UNC Tensile-Stress Area A_t in^2	Coarse Series—UNC Minor-Diameter Area A_r in^2	Fine Series—UNF Threads per Inch N	Fine Series—UNF Tensile-Stress Area A_t in^2	Fine Series—UNF Minor-Diameter Area A_r in^2
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6
$\frac{1}{4}$	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6
$\frac{5}{16}$	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.052 4
$\frac{3}{8}$	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.080 9
$\frac{7}{16}$	0.4375	14	0.106 3	0.093 3	20	0.118 7	0.109 0
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6
$\frac{9}{16}$	0.5625	12	0.182	0.162	18	0.203	0.189
$\frac{5}{8}$	0.6250	11	0.226	0.202	18	0.256	0.240
$\frac{3}{4}$	0.7500	10	0.334	0.302	16	0.373	0.351
$\frac{7}{8}$	0.8750	9	0.462	0.419	14	0.509	0.480
1	1.0000	8	0.606	0.551	12	0.663	0.625
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521

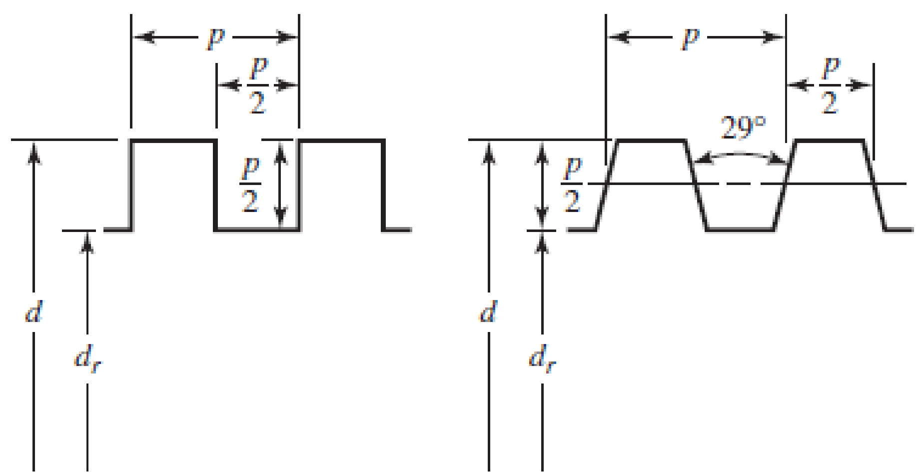
*This table was compiled from ANSI B1.1-1974. The minor diameter was found from the equation $d_r = d - 1.299\ 038p$, and the pitch diameter from $d_p = d - 0.649\ 519p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.

Square and Acme threads

- Are used for power screws
- Acme screw is in widespread usage. They are sometimes modified to a stub (end) form by making the thread shorter. This results in a large minor diameter and slightly stronger screw.
- A square thread provides somewhat greater strength and efficiency but is rarely used, due to difficulties in manufacturing the 0^{o} thread angle. The 5^{o} thread angle of the modified square thread partially overcomes this and some other objections.

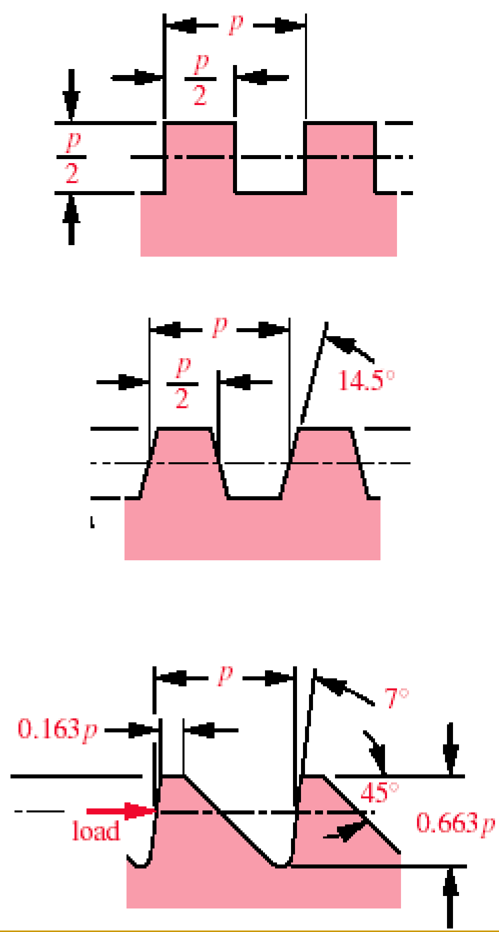
Square and Acme Threads are used for the power screw





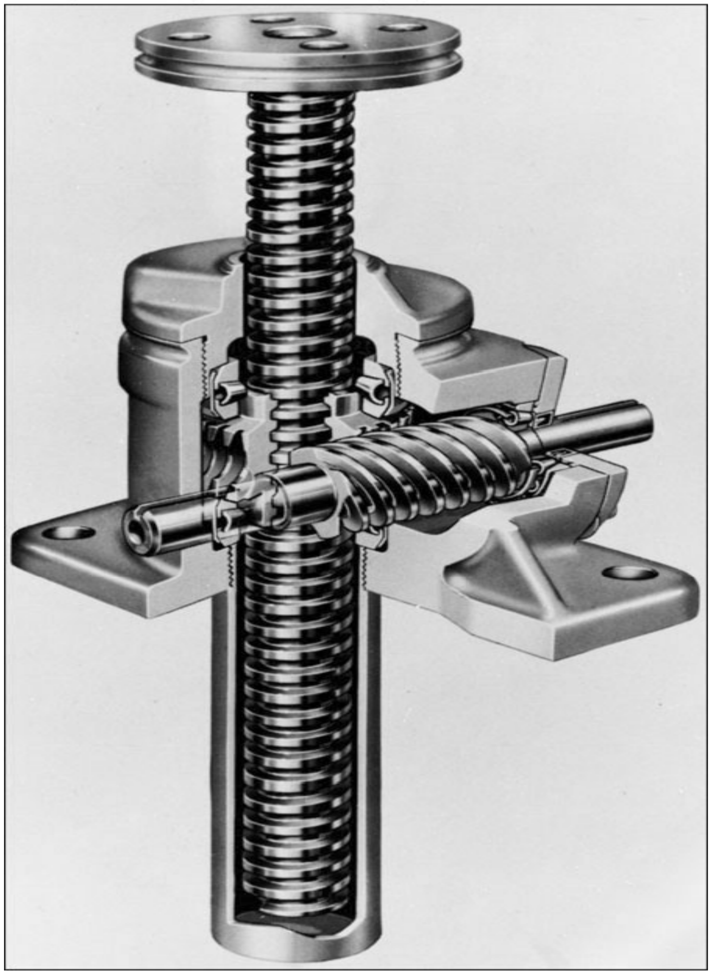
(a) (b)

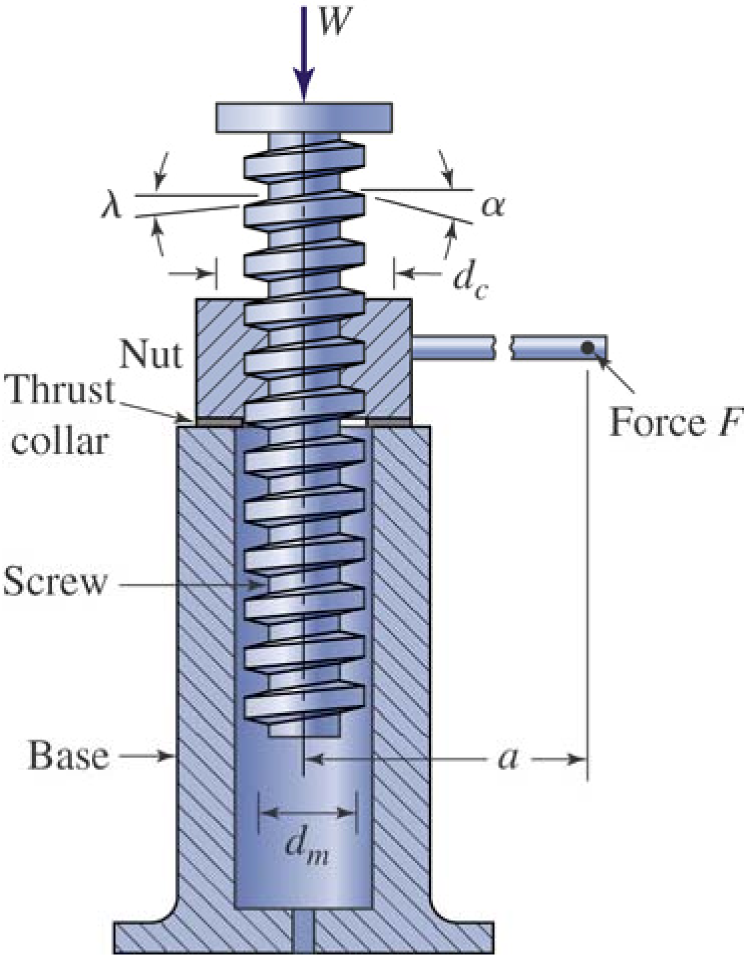
Power Screw Types



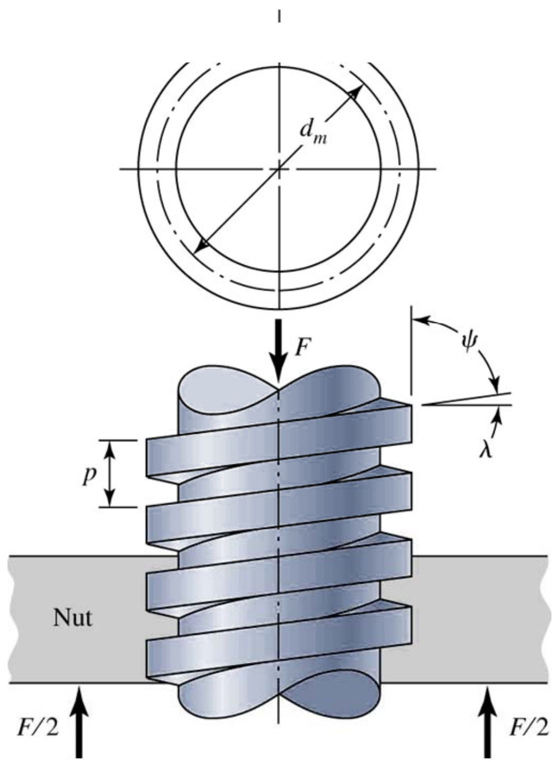
- Square
  - strongest
  - no radial load
  - hard to manufacture
- Acme
  - 29° included angle
  - easier to manufacture
  - common choice for loading in both directions
- Buttress (contrafuerte)
  - great strength
  - only unidirectional loading

Mechanics of Power Screws

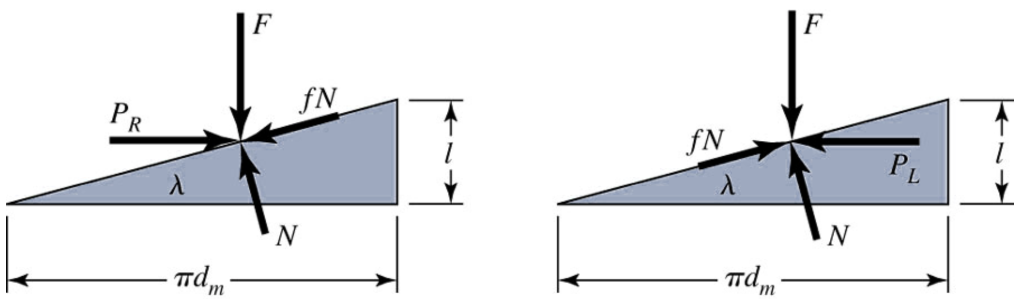




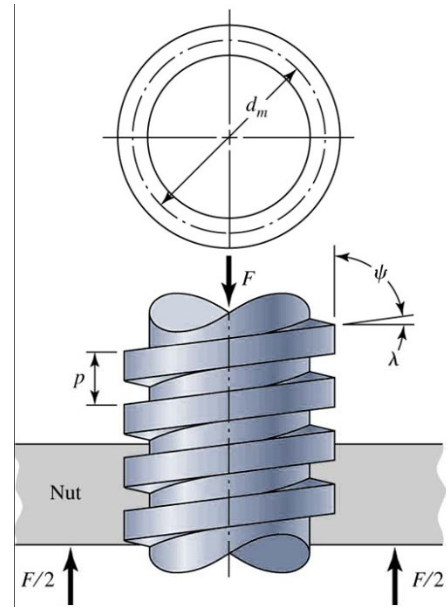
- A **square-threaded** power screw with a single thread having a mean diameter dm, a pitch p, a lead angle λ, and a helix angle ψ is loaded by the axial compressive force F.



The torque required to raise or to lower the load:



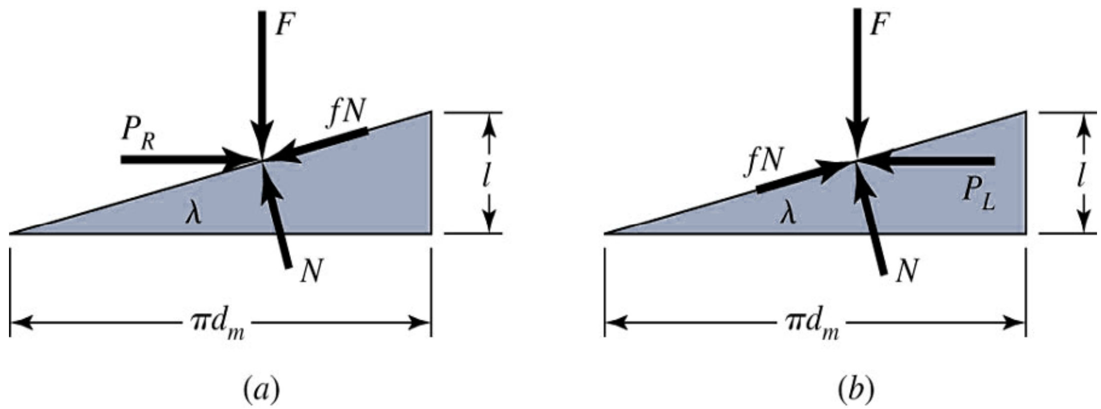
(a) (b)



Lifting the load

Lowering the load

- First, imagine that a single thread of the screw is unrolled for exactly a single turn
- The base is the circumference of the mean-thread-diameter circle and the height is the lead.



- Figure (*a*) represents lifting the load and figure (*b*) represent lowering the load.
- The summation of all the unit axial forces acting upon the normal thread area by F
- To raise the load, a force P_R acts to the right (Fig. *a*), and to lower the load, a force P_L acts to the left (Fig. *b*)
- The friction force is the product of the coefficient of friction f with the normal force N, and acts to oppose the motion

- The system is in equilibrium under the action of these forces:
- For raising the load

$$\sum F_H = P_R - N\sin\lambda - fN\cos\lambda = 0$$

$$\sum F_V = F + fN\sin\lambda - N\cos\lambda = 0$$

- For lowering the load

$$\sum F_H = -P_L - N\sin\lambda + fN\cos\lambda = 0$$

$$\sum F_V = F - fN\sin\lambda - N\cos\lambda = 0$$

- Solve each two equations for P_R and P_L, we have:

$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda}$$

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda}$$

- Divide the numerator and the denominator of these equations by $\cos\lambda$ and use the relation $\tan\lambda = l/\pi d_m$ we then have respectively:

$$P_R = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)}$$

$$P_L = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)}$$

- The torque then can be found by multiply the force by the mean radius $d_m/2$. Therefore,

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right)$$

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right)$$

The force and torque required to raise and lower the load :

$$P_R = \frac{F\left[\left(l/\pi d_m\right) + f\right]}{1 - \left(fl/\pi d_m\right)} \qquad T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right)$$

$$P_L = \frac{F\left[f - \left(l/\pi d_m\right)\right]}{1 + \left(fl/\pi d_m\right)} \qquad T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right)$$

- T_R is the torque required for two purposes: to overcome thread friction and to raise the load.
- T_L is the torque required to lower the load. This torque requires overcoming a part of the friction in lowering the load. If the lead is larger or the friction is low, the load will lower itself by causing the screw to spin without any external effort.
- In such cases T_L is negative or zero in equation. So, if

$$T_L > 0 \Rightarrow \text{the screw is said to be self-locking}$$

- Condition for self-locking:

$$\frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right) > 0 \qquad \pi f d_m > l$$

- This leads to

$$f = \frac{l}{\pi d_m}$$

- But

$$\tan\lambda = \frac{l}{\pi d_m}$$

- Thus

$$f = \tan\lambda \qquad f > \tan\lambda$$

  - This relation states that self-locking is obtained whenever the coefficient of thread friction is equal to or greater than the tangent of the thread lead angle.

self-locking – screw cannot turn from load F

back-driving – screw can be turned from load F

Efficiency:

- If we let the friction equal to zero then T_R reduced to:

$$T_o = \frac{Fl}{2\pi} \qquad\qquad T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right)$$

- Thus, the efficiency can be written as

$$e = \frac{T_R\big|_{f=0}}{T_R} = \frac{T_o}{T_R} = \frac{Fl}{2\pi T_R}$$

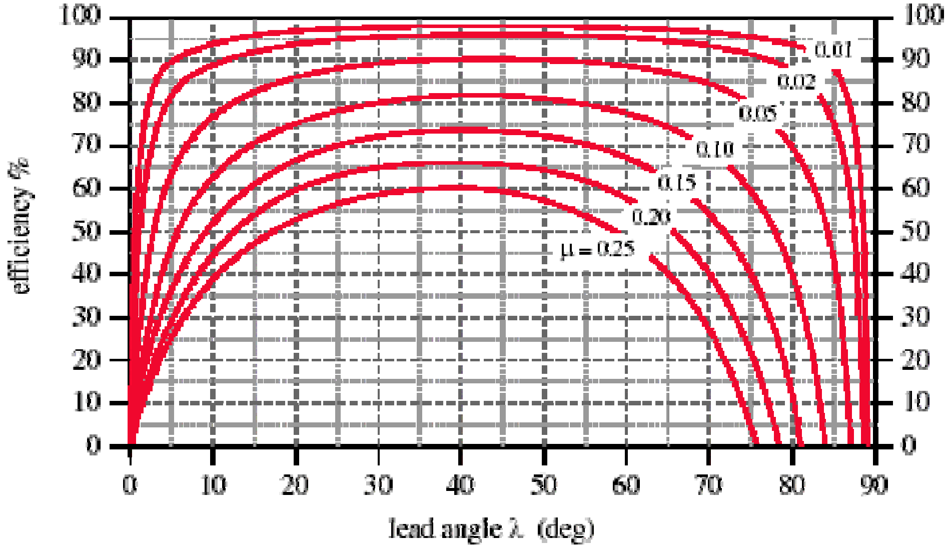
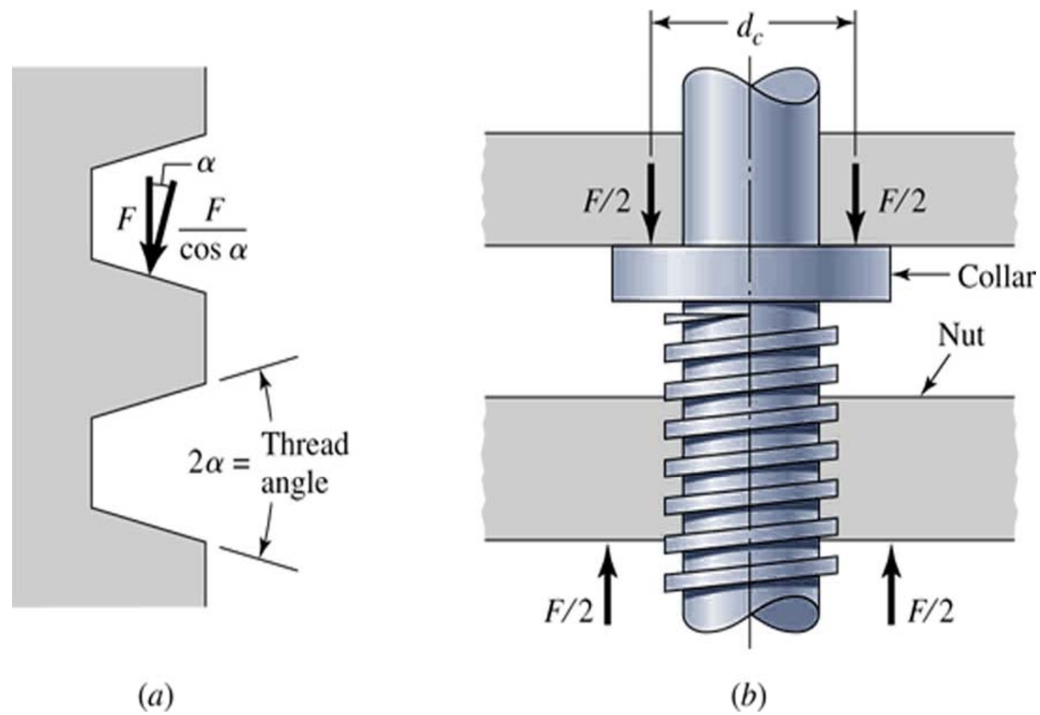


FIGURE 14-8

Acme and other threads:

- The normal thread load is inclined to the axis because of the thread angle 2α and the lead angle λ.
- Since the lead angles λ are small, this inclination can be neglected and only the effect of the thread angle α considered. See figure 8-7a.



- Thus the friction term in the torque equation T_R must be divided by $\cos\alpha$, for raising the load, or for tightening a screw or bolt, this yields

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha}\right)$$

- This equation is an approximate equation because λ is neglected.

• Acme thread is not as efficient as the square thread, because of the additional friction due to the wedging action, but it is often preferred because it is easier to machine

A third component of torque must be applied in power screw applications

- When it loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. See figure 8-7b.
- If f_c is a friction of collar friction, then

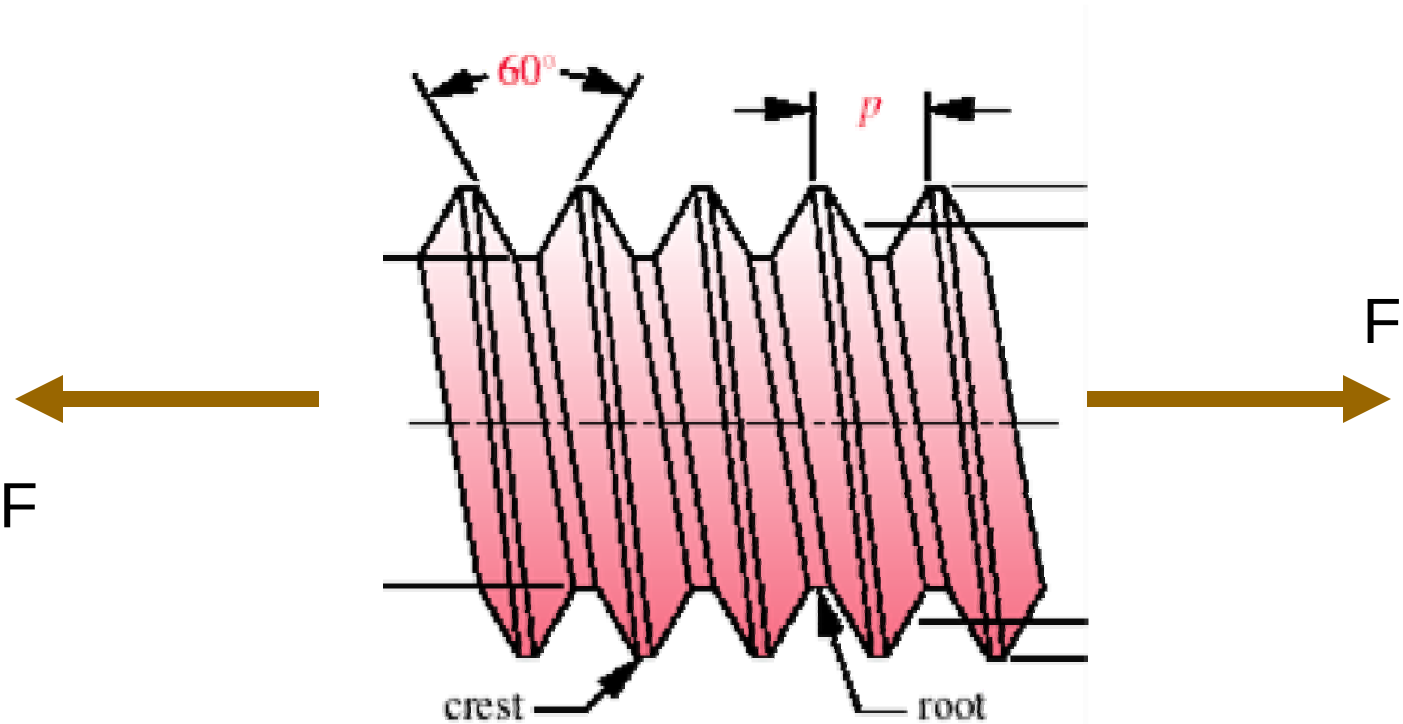
$$T_c = \frac{F f_c d_c}{2}$$

- For large collars, the torque should probably be computed in a manner similar to that employed for disk clutches.

Stresses in Power Screws

- Stresses in Threads
  - Body Stresses
    - Axial
    - Torsion
  - Thread Stresses
    - Bearing
    - Bending
  - Buckling

Tensile Stress



$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2}$$

Torsional Stress

depends on friction at screw-nut interface

For screw and nut,

- if totally locked (rusted together), the screw experiences all of torque
- if frictionless, the screw experiences none of the torque

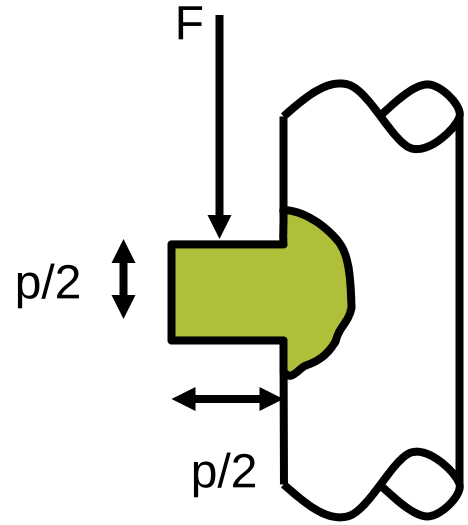
$$\tau = \frac{Tr}{J} = \frac{16\,T}{\pi d_r^3}$$

For power screw,

- if low collar friction, the screw experiences nearly all of torque
- if high collar friction, the nut experiences most of the torque

Thread Stresses – Bearing

$$\sigma_B = -\frac{F}{\pi d_m n_t p/2} = -\frac{2F}{\pi d_m n_t p}$$



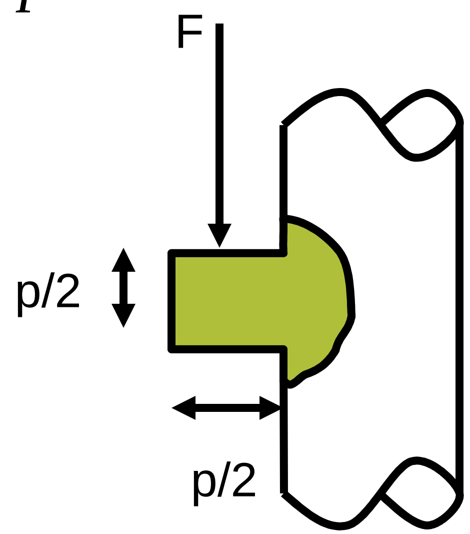
$A_{bearing} = (p/2)(\pi d_m n_t)$

where n_t is the number of engaged threads.

Thread Stresses – Bending

$$M = \frac{Fp}{4} \quad , \quad I = \frac{1}{12}\pi d_r (p/2)^3 \quad ,\& \quad c = p/4$$

$$\sigma_b = \frac{Mc}{I} = \frac{6F}{\pi d_r p n_t}$$



For both bearing and bending, F and n_t are dependent on how well load is shared among teeth, therefore

use F_{actual}=0.38F and n_t=1 (derived from experiments)

Thread Stresses – Transverse shear stresses

The transverse shear stress τ at the center of the root of the thread due to load F is

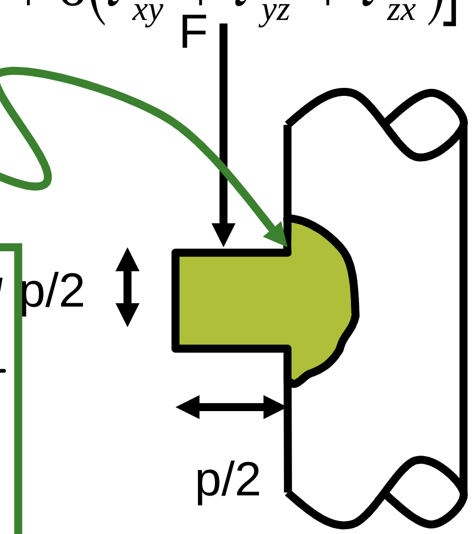
$$\tau = \frac{3V}{2A} = \frac{3}{2}\frac{F}{\pi d_r n_t p/2} = \frac{3F}{\pi d_r n_t p} \qquad (8\text{–}12)$$

At top of tooth the transverse shear stress is zero

- The Von Mises σ’ at the top of the root “plane” is found by first identifying the orthogonal normal stresses and the shear stresses.

Mohr's Circle- Von Mises σ' at the top of the root "plane"

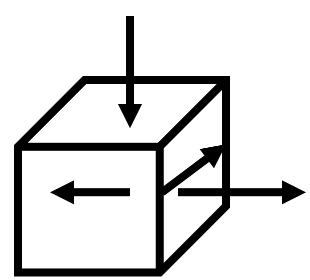
$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{0.5}$$

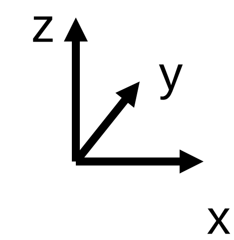


$$\sigma_x = \frac{6F}{\pi d_r n_t p} \qquad \tau_{xy} = \frac{16T}{\pi d_r^3}$$

$$\sigma_y = 0 \qquad \tau_{yz} = 0$$

$$\sigma_z = -\frac{4F}{\pi d_r^2} \qquad \tau_{xz} = 0$$





Buckling

K = radius of gyration

$$S_R = \frac{l}{k} = \frac{l}{\sqrt{\frac{I}{A}}}$$

use d_r

$$(S_R)_D = \pi\sqrt{\frac{2E}{S_y}}$$

$$\boxed{S_R \le (S_R)_D \quad \text{use Johnson}}$$

$$\boxed{S_R > (S_R)_D \quad \text{use Euler}}$$

$$\boxed{\frac{P_{CR}}{A} = S_y - \frac{1}{E}\left(\frac{S_y S_R}{2\pi}\right)^2}$$

$$\boxed{P_{CR} = \frac{\pi^2 EI}{l^2}}$$

- The engaged threads cannot share the load equally. Some experiments show that the first engaged thread carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load.

- In estimating thread stresses by the equations above, substituting $0.38F$ for F and setting n_t to 1 will give the largest level of stresses in the thread-nut combination.

Example: 8-1

- A square-thread power screw has a major diameter of 32mm and a pitch of 4mm with double threads. The given data include $f = f_c = 0.08$, $d_c = 40$mm, and F = 6.4kN per screw.
- Determine:

1. The thread depth, thread width, pitch diameter, minor diameter, and lead.
2. The torque required to raise and lower the load
3. The efficiency during lifting the load
4. The body stresses, torsional and compressive
5. The bearing stress
6. The thread stress bending at the root, shear at the root, and Von Mises stress

Solution

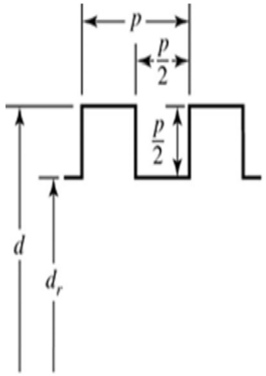
1. Thread depth= $p/2=4/2=2mm$

   Thread width=$p/2=4/2=2mm$

   Pitch diameter $d_m=d-p/2=32-4/2=30mm$

   Minor diameter $d_r=d-p=32-4=28mm$

   Lead $l=np=2(4)=8mm$



Solution…

2. Using equation (8-1) and (8-6), the torque required to turn the screw against the load is:

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right) + \frac{Ff_c d_c}{2} = 15.94 + 10.24 = 26.18 N.m$$

Using equation (8-2) and (8-6), the load-lowering torque is:

$$T_L = \frac{Fd_m}{2}\left(\frac{l - \pi f d_m}{\pi d_m + fl}\right) + \frac{Ff_c d_c}{2} = -0.466 + 10.24 = 9.77 N.m$$

3. The overall efficiency in raising the load is:

$$e = \frac{Fl}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

4. The body shear stress τ due to torsional moment T_R at the outside of the screw body is:

$$\tau = \frac{16T}{\pi d_r^3} = 6.07\,MPa$$

The axial nominal normal stress σ is

$$\sigma = -\frac{4F}{\pi d_r^2} = -10.39\,MPa$$

Solution…

5. The bearing stress is, with one thread carrying $0.38F$

$$\sigma_B = -\frac{2(0.38F)}{\pi d_m (1) p} = -12.9 MPa$$

6. The thread-root bending stress σ_b with one thread carrying $0.38F$ is

$$\sigma_b = \frac{Mc}{I} = \frac{6F}{\pi d_r n_t p} = 41.5 MPa$$

$$\sigma_x = 41.5MPa \quad , \quad \tau_{xy} = 0$$

$$\sigma_y = 0 \quad , \quad \tau_{yz} = 6.07MPa$$

$$\sigma_z = -10.39MPa \quad , \quad \tau_{zx} = 0$$

$$\sigma' = \frac{1}{\sqrt{2}}\left[(41.5-0)^2 + (0-(-10.39))^2 + (-10.39-41.5)^2 + 6\left(6.07^2\right)\right]^{0.5} = 48.7MPa$$

Table 8–4

Screw Bearing Pressure p_b

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material	Safe p_b, MPa	Notes
Steel	Bronze	17.2–24.1	Low speed
Steel	Bronze	11.0–17.2	≤50 mm/s
	Cast iron	6.9–17.2	≤40 mm/s
Steel	Bronze	5.5–9.7	100–200 mm/s
	Cast iron	4.1–6.9	100–200 mm/s
Steel	Bronze	1.0–1.7	≥250 mm/s

Table 8–5

Coefficients of Friction f for Threaded Pairs

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material			
	Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

Table 8–6

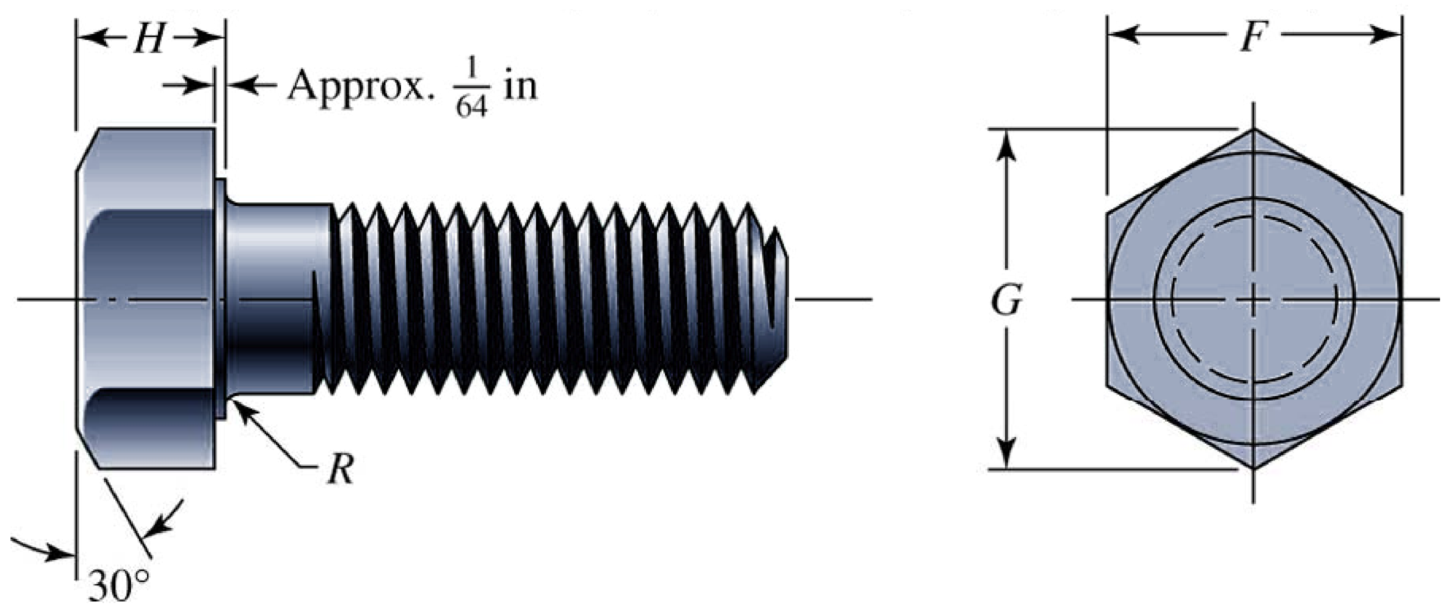
Thrust-Collar Friction Coefficients

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

Threaded Fasteners

- A standard hexagon-head bolt
  - stress concentration occurs at the fillet & at the start of the threads
  - Table A-29 gives the dimension for the standard hexagon-head bolt.



- The diameter of the washer face is the same as the width across flats of the hexagon
- The threads length of metric bolts is

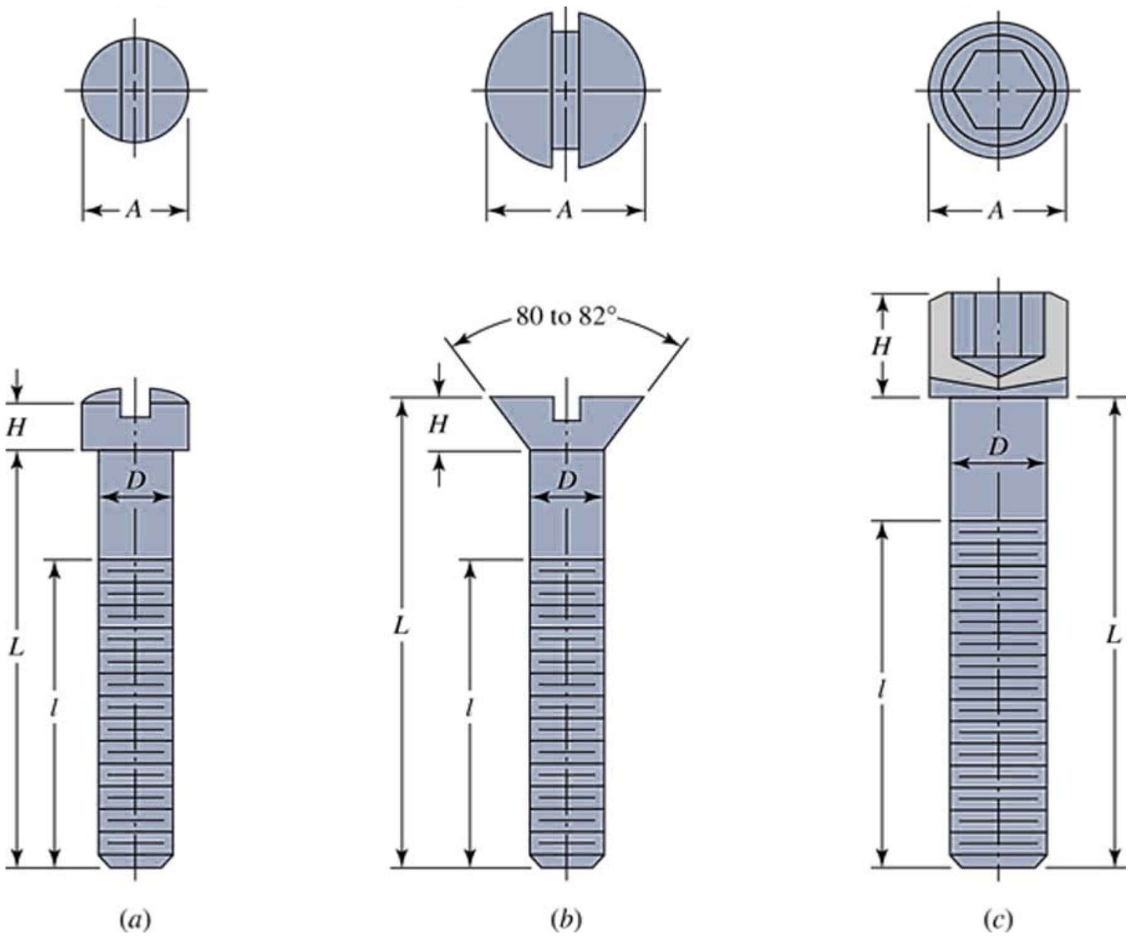
$$L_T = \begin{cases} 2D + 6 & L \le 125 \qquad D \le 48 \\ 2D + 12 & 125 < L \le 200 \\ 2D + 25 & L > 200 \end{cases}$$

All dimensions in mm

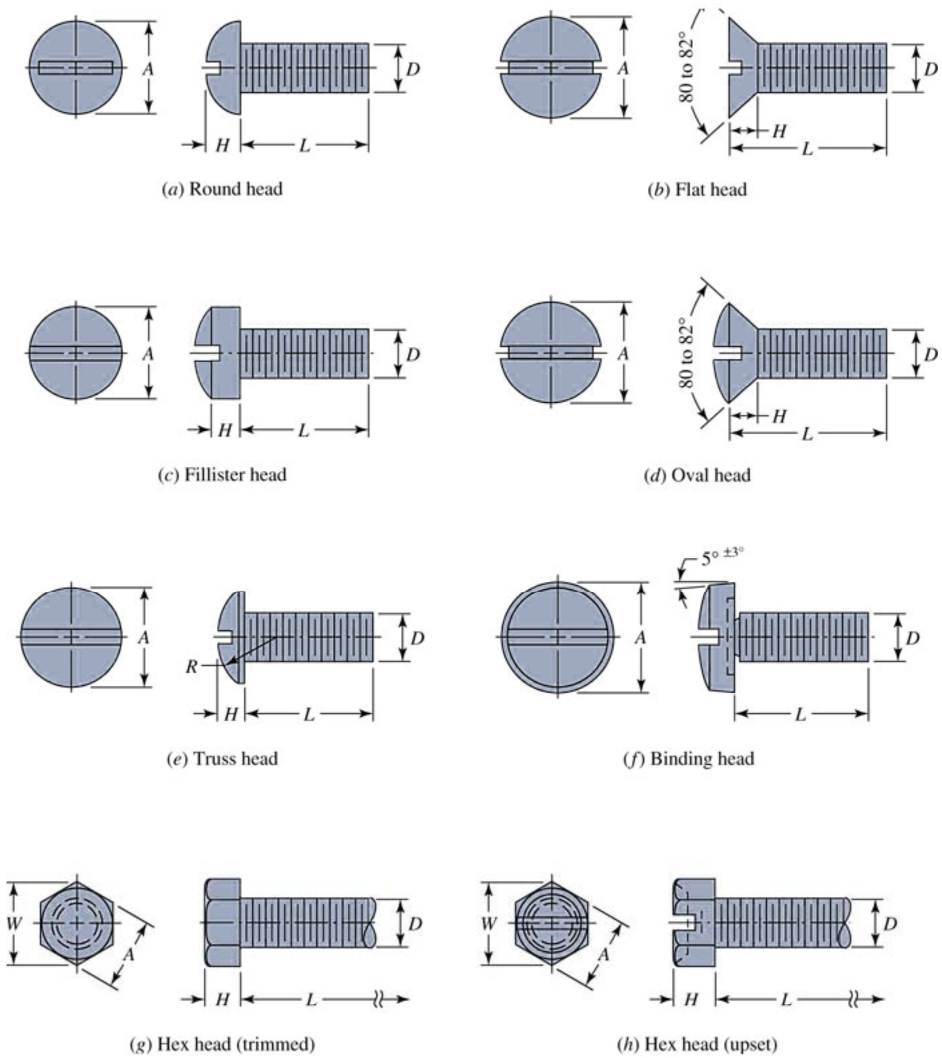
- Ideal bolt length is one in which only one or two threads project from the nut after it is tightened.
- Bolt holes may have burrs and sharp edges after drilling. These could bite into the fillet and increase stress concentration. Therefore, washers must always be used under the bolt head to prevent this.
- Sometimes it is necessary to use washers under the nut too.

- The purpose of a bolt is to clamp two or more parts together.
- The clamping load stretches or elongates the bolt;
  - The load is obtained by twisting the nut until the bolt has elongated almost to the elastic limit.
  - If the nut does not loosen, this bolt tension remains as preload or clamping force.

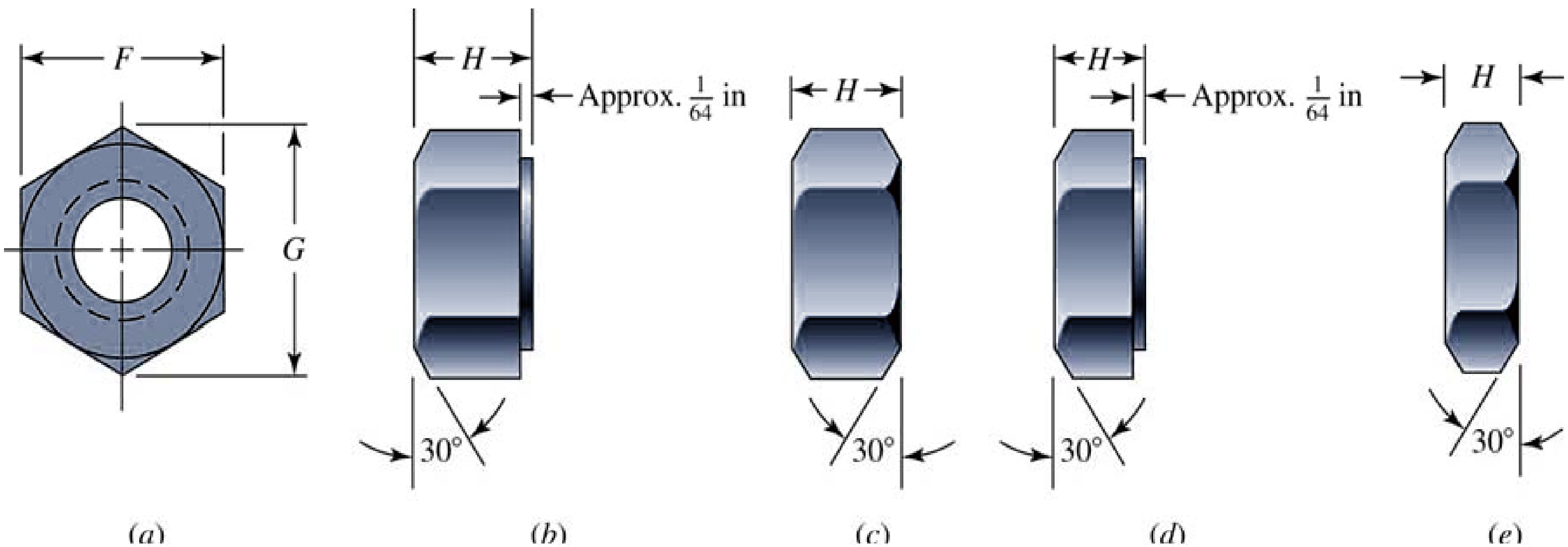
The head of a hexagon-lead cap screw



- It is thinner than that of a hexagon heat bolt.
- Table A-30, gives the dimension for the hexagon-lead cap screw.
- A variety of machine-screw head styles are shown if figure 8-11.



- Several styles of hexagonal nuts are illustrated in fig 8-12. And their dimensions are given in table A-28.
- The material of the nut is selected carefully to match that of the bolt.
- During tightening, the first thread of the nut tends to take the entire load, but yielding occurs, with some strengthening due to the cold work that takes place, and the load is eventually divided over about 3 nut threads. For this reason you should never reuse nuts; in fact, it can be dangerous to do so.



Tension Connections-The fastener **Stiffness**

- A section through a tension-loaded bolted joint is illustrated in Figure 8-13.

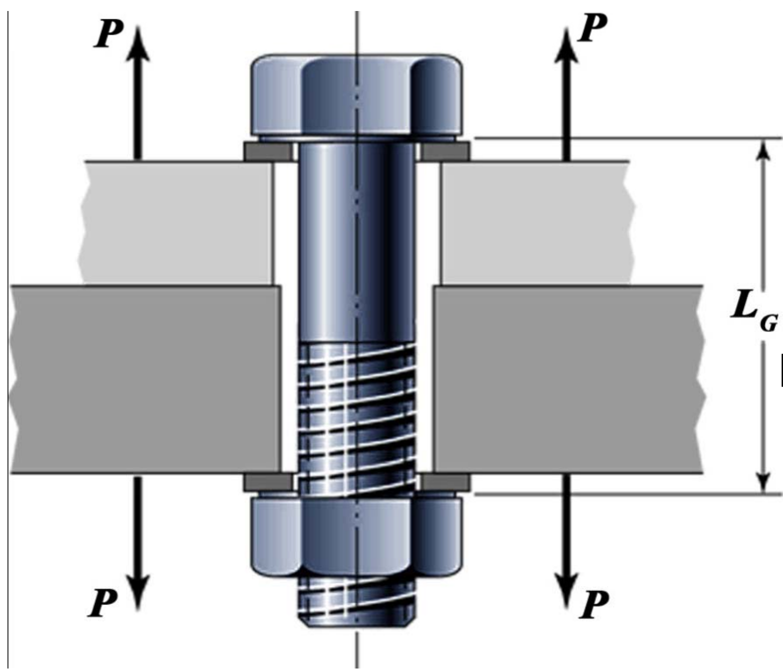


Fig 8.13

- The purpose of the bolt is to clamp the two or more parts together.
- Twisting the nut stretches the bolt to produce the clamping force. This clamping force is called the Pretension or Bolt Preload. It exists in the connection after the nut has been properly tightened no matter whether the external tensile load *P* is exerted or not.
- Since the members are being clamped together, the clamping force which produces tension in the bolt induces compression in the members.
- The spring constant, or stiffness constant, of an elastic member such as a bolt, is the ratio between the force applied to the member and the deflection produced by that force.
- Thus,
  - The defection is defined by $$\delta = \frac{Fl}{EA}$$
  - The spring rate $$k = \frac{EA}{l}$$

- The grip L_G of a connection is the total thickness of the clamped material. In the fig 8-13, the grip is the sum of the thicknesses of both members and both washers.
- For Hexagon-head cap screws the effective grip is shown in figure 8-14 and is given in table 8.7

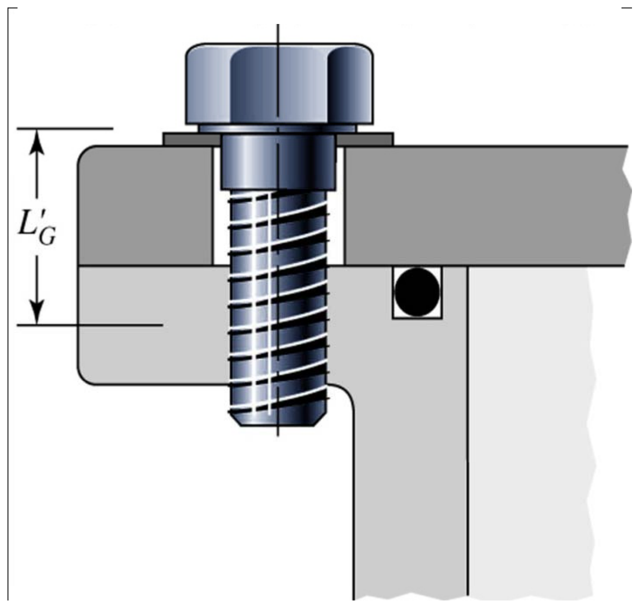
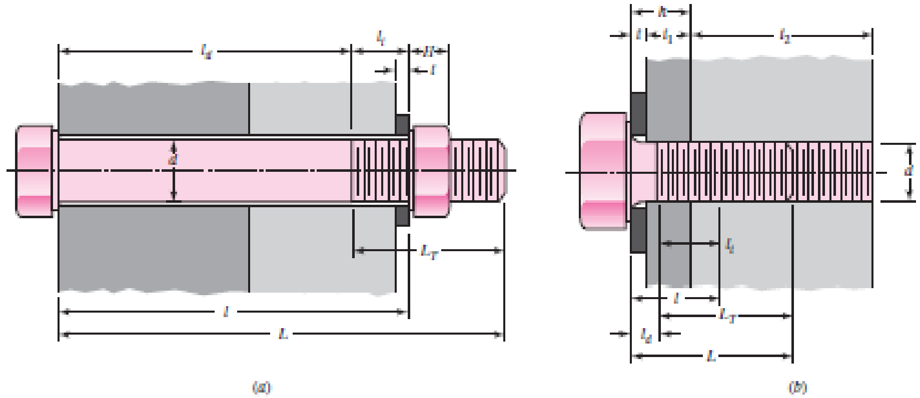


Fig 8.14

Table 8–7

Suggested Procedure for Finding Fastener Stiffness



Given fastener diameter d and pitch p in mm or number of threads per inch

Washer thickness: t from Table A–32 or A–33

Nut thickness [Fig. (a) only]: H from Table A–31

Grip length:

For Fig. (a): l = thickness of all material squeezed between face of bolt and face of nut

For Fig. (b): $l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$

Fastener length (round up using Table A–17*):

For Fig. (a): $L > l + H$

For Fig. (b): $L > h + 1.5d$

Threaded length L_T: Inch series:

$$L_T = \begin{cases} 2d + \frac{1}{4}\text{ in}, & L \leq 6\text{ in} \\ 2d + \frac{1}{2}\text{ in}, & L > 6\text{ in} \end{cases}$$

Metric series:

$$L_T = \begin{cases} 2d + 6\text{ mm}, & L \leq 125\text{ mm}, d \leq 48\text{ mm} \\ 2d + 12\text{ mm}, & 125 < L \leq 200\text{ mm} \\ 2d + 25\text{ mm}, & L > 200\text{ mm} \end{cases}$$

Length of unthreaded portion in grip: $l_d = L - L_T$

Length of threaded portion in grip: $l_t = l - l_d$

Area of unthreaded portion: $A_d = \pi d^2/4$

Area of threaded portion: A_t from Table 8–1 or 8–2

Fastener stiffness: $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

*Bolts and cap screws may not be available in all the preferred lengths listed in Table A–17. Large fasteners may not be available in fractional inches or in millimeter lengths ending in a nonzero digit. Check with your bolt supplier for availability.

- The stiffness of the portion of a bolt or screw within the clamped zone will generally consist of two parts:

  Unthreaded shank portion + threaded portion

- The stiffness constant of the bolt is equivalent to the stiffness of the two springs in series:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Rightarrow \quad k = \frac{k_1 k_2}{k_1 + k_2}$$

- Thus, spring rates of the threaded and unthreaded portions of the bolt in the clamped zone are respectively:

$$k_t = \frac{A_t E}{l_t} \quad , \quad k_d = \frac{A_d E}{l_d}$$

Where

A_t= tensile-stress area (table 8-1, 8-2)

l_t= length of threaded portion of grip

A_d= major-diameter area of fastener

l_d=length of unthreaded portion in grip

- Substituting these stiffness, we get:

$$k_b = \frac{k_d k_t E}{k_d l_t + k_t l_d} \qquad\qquad k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

- Where k_b is the estimated effective stiffness of the bolt or cap screw in the clamped zone.
- For short fasteners, the one in figure 8-14 the unthreaded area is small and so the first of the expression k_t is used to find k_b.
- For long fasteners, the threaded area is relatively small, and so the second expression k_d is used to find k_b.

Joins-Member stiffness (Tension Connection-the Members)

- In previous section, we determined the stiffness of the fastener in the clamped zone.
- In this section, we will study the stiffness of the members in the clamped zone.
- There may be more than two members included in the grip of the fastener.
- Thus, Total spring rate of the member:

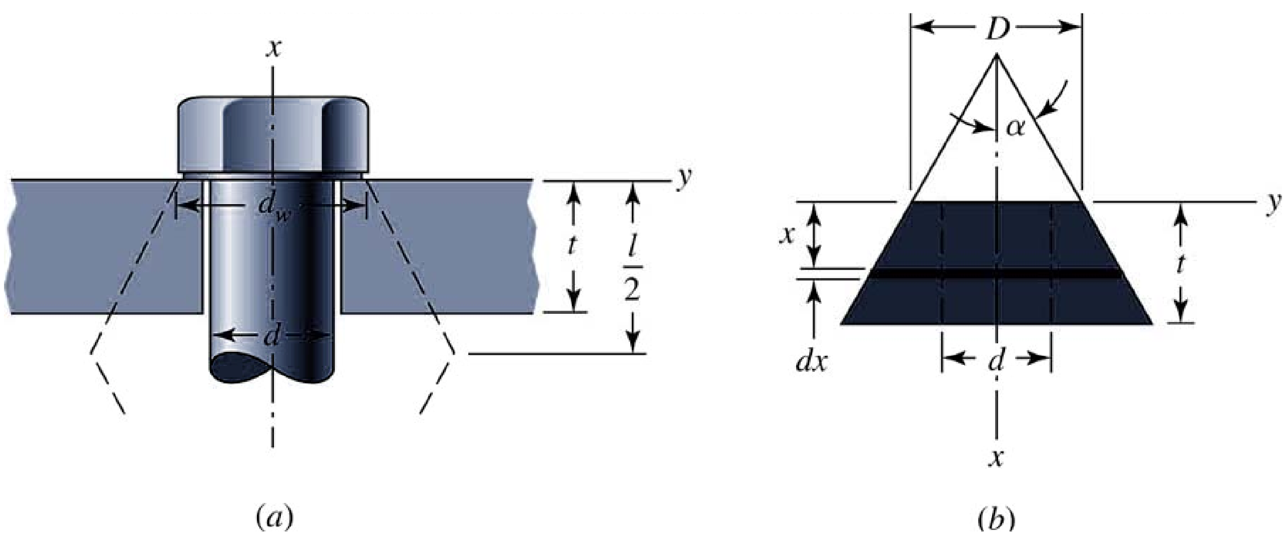
$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots. + \frac{1}{k_i} \qquad 8.18$$

- If one of the members is a soft gasket

$$\Rightarrow \qquad k_{\text{gasket}} < \text{others members}$$

- Thus, we can neglect all stiffness and only the gasket stiffness used.
- If there is no gasket, the stiffness of the members is rather difficult to obtain, except by experimentation, because the compression spreads out between the bolt head and the nut and hence the area is not uniform.

- Some cases in which this area can be determined can be shown in Figure 8-15. Ito illustrates the general cone geometry using a half-apex angle α.



- The construction of an element of the cone of thickness *dx* subjected to a compressive force *P* is:

$$d\delta = \frac{Pdx}{EA}$$

- The area of the element is

$$A = \pi\left(r_o^2 - r_i^2\right) = \pi\left[\left(x\tan\alpha + \frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

- Substituting the 2ed equation into the 1st and integrating the equation gives:

$$\delta = \frac{P}{\pi E}\int_0^t \frac{dx}{\left[x\tan\alpha + \left(D+d\right)/2\right]\left[x\tan\alpha + \left(D-d\right)/2\right]}$$

With $\alpha = 30°$, this becomes

$$k = \frac{0.5774\pi E d}{\ln \dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \tag{8–20}$$

Equation (8–20), or (8–19), must be solved separately for each frustum in th joint. Then individual stiffnesses are assembled to obtain k_m using Eq. (8–18).

- Using a table of integrals, the spring rate or the stiffness is:

$$k = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \dfrac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$

- If the members of the joint have the same Young's modulus E with symmetrical frusta back to back, then they act as two identical springs in series.
- Thus, from equation 8-18, $k_m = k/2$, and using the grip as $l = 2t$ and d_w as the diameter of the washer face, we find the spring rate of the members to be:

$$k_m = \frac{\pi E d \tan\alpha}{2\ln \dfrac{(l\tan\alpha + d_w - d)(d_w + d)}{(l\tan\alpha + d_w + d)(d_w - d)}}$$

- The diameter of the washer face is about 50 percent greater than the fastener diameter for standard hexagon-head bolts and cap screws. Thus we can simplify k_m equation by letting d_w=1.5d and if we use α=30°

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)}$$

Earlier in the section use of $\alpha = 30^\circ$ was recommended for hardened steel, cast iron,

$$\frac{k_m}{Ed} = A \exp(Bd/l) \tag{8–23}$$

with constants A and B defined in Table 8–8. For standard washer faces and members of the same material, Eq. (8–23) offers a simple calculation for member stiffness k_m. For departure from these conditions, Eq. (8–20) remains the basis for approaching the problem.

Material Used	Poisson Ratio	Elastic GPa	Modulus Mpsi	A	B
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14

EXAMPLE 8–2 Two $\frac{1}{2}$-in-thick steel plates with a modulus of elasticity of $30(10^6)$ psi are clamped by washer-faced $\frac{1}{2}$-in-diameter UNC SAE grade 5 bolts with a 0.095-in-thick washer under the nut. Find the member spring rate k_m using the method of conical frusta, and compare the result with the finite element analysis (FEA) curve-fit method of Wileman et al.

Solution The grip is $0.5 + 0.5 + 0.095 = 1.095$ in. Using Eq. (8–22) with $l = 1.095$ and $d = 0.5$ in, we write

$$k_m = \frac{0.5774\pi 30(10^6)0.5}{2\ln\left[5\dfrac{0.5774(1.095) + 0.5(0.5)}{0.5774(1.095) + 2.5(0.5)}\right]} = 15.97(10^6)\text{ lbf/in}$$

From Table 8–8, $A = 0.787\,15$, $B = 0.628\,73$. Equation (8–23) gives

$$k_m = 30(10^6)(0.5)(0.787\,15)\exp[0.628\,73(0.5)/1.095]$$
$$= 15.73(10^6)\text{ lbf/in}$$

For this case, the difference between the results for Eqs. (8–22) and (8–23) is less than 2 percent.

EXAMPLE 8–2

As shown in Fig. 8–17*a*, two plates are clamped by washer-faced $\frac{1}{2}$ in-20 UNF $\times$ $1\frac{1}{2}$ in SAE grade 5 bolts each with a standard $\frac{1}{2}$ N steel plain washer.

(*a*) Determine the member spring rate k_m if the top plate is steel and the bottom plate is gray cast iron.

(*b*) Using the method of conical frusta, determine the member spring rate k_m if both plates are steel.

(*c*) Using Eq. (8–23), determine the member spring rate k_m if both plates are steel. Compare the results with part (*b*).

(*d*) Determine the bolt spring rate k_b.

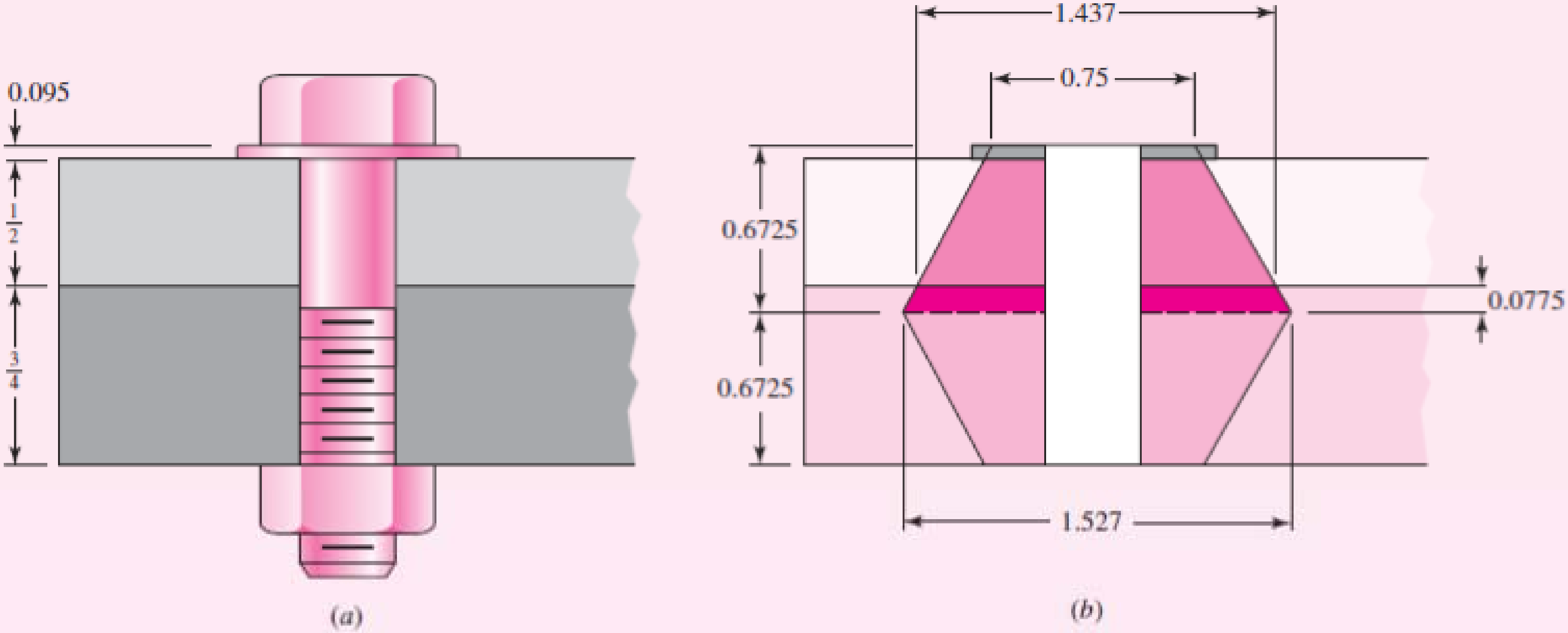
Solution

From Table A–32, the thickness of a standard $\frac{1}{2}$ N plain washer is 0.095 in.

(*a*) As shown in Fig. 8–17*b*, the frusta extend halfway into the joint the distance

$$\frac{1}{2}(0.5 + 0.75 + 0.095) = 0.6725 \text{ in}$$

Dimensions in inches.



The distance between the joint line and the dotted frusta line is 0.6725 − 0.5 − 0.095 = 0.0775 in. Thus, the top frusta consist of the steel washer, steel plate, and 0.0775 in of the cast iron. Since the washer and top plate are both steel with $E = 30(10^6)$ psi, they can be considered a single frustum of 0.595 in thick. The outer diameter of the frustum of the steel member at the joint interface is 0.75 + 2(0.595) tan 30° = 1.437 in. The outer diameter at the midpoint of the entire joint is 0.75 + 2(0.6725) tan 30° = 1.527 in. Using Eq. (8–20), the spring rate of the steel is

$$k_1 = \frac{0.5774\pi(30)(10^6)0.5}{\ln\left\{\dfrac{[1.155(0.595)+0.75-0.5](0.75+0.5)}{[1.155(0.595)+0.75+0.5](0.75-0.5)}\right\}} = 30.80(10^6)\ \text{lbf/in}$$

For the upper cast-iron frustum

$$k_2 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln\left\{\dfrac{[1.155(0.0775)+1.437-0.5](1.437+0.5)}{[1.155(0.0775)+1.437+0.5](1.437-0.5)}\right\}} = 285.5(10^6)\ \text{lbf/in}$$

For the lower cast-iron frustum

$$k_3 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln\left\{\dfrac{[1.155(0.6725)+0.75-0.5](0.75+0.5)}{[1.155(0.6725)+0.75+0.5](0.75-0.5)}\right\}} = 14.15(10^6)\ \text{lbf/in}$$

The three frusta are in series, so from Eq. (8–18)

$$\frac{1}{k_m} = \frac{1}{30.80(10^6)} + \frac{1}{285.5(10^6)} + \frac{1}{14.15(10^6)}$$

Answer This results in $k_m = 9.378\ (10^6)$ lbf/in.

(*b*) If the entire joint is steel, Eq. (8–22) with $l = 2(0.6725) = 1.345$ in gives

Answer
$$k_m = \frac{0.5774\pi(30.0)(10^6)0.5}{2\ln\left\{5\left[\dfrac{0.5774(1.345)+0.5(0.5)}{0.5774(1.345)+2.5(0.5)}\right]\right\}} = 14.64(10^6)\ \text{lbf/in.}$$

(*c*) From Table 8–8, $A = 0.787\,15$, $B = 0.628\,73$. Equation (8–23) gives

Answer $k_m = 30(10^6)(0.5)(0.787\,15)\exp[0.628\,73(0.5)/1.345] = 14.92(10^6)$ lbf/in

For this case, the difference between the results for Eqs. (8–22) and (8–23) is less than 2 percent.

(*d*) Following the procedure of Table 8–7, the threaded length of a 0.5-in bolt is $L_T = 2(0.5) + 0.25 = 1.25$ in. The length of the unthreaded portion is $l_d = 1.5 - 1.25 = 0.25$ in. The length of the unthreaded portion in grip is $l_t = 1.345 - 0.25 = 1.095$ in. The major diameter area is $A_d = (\pi/4)(0.5^2) = 0.196\,3$ in^2. From Table 8–2, the tensile-stress area is $A_t = 0.159\,9$ in^2. From Eq. (8–17)

Answer
$$k_b = \frac{0.196\,3(0.159\,9)30(10^6)}{0.196\,3(1.095) + 0.159\,9(0.25)} = 3.69(10^6)\ \text{lbf/in}$$

Bolt Strength:

- Table 8-11 list the classes and specifications of most the standard metric threaded fasteners.
- The proof load of a bolt is the maximum load (force) that a bolt can withstand without acquiring a permanent set. Proof strength is the quotient of the proof load and the tensile-stress area.
- Although proof strength and yield strength have something in common, the yield strength is usually the higher of the two because it is band on a 0.2 percent permanent deformation.

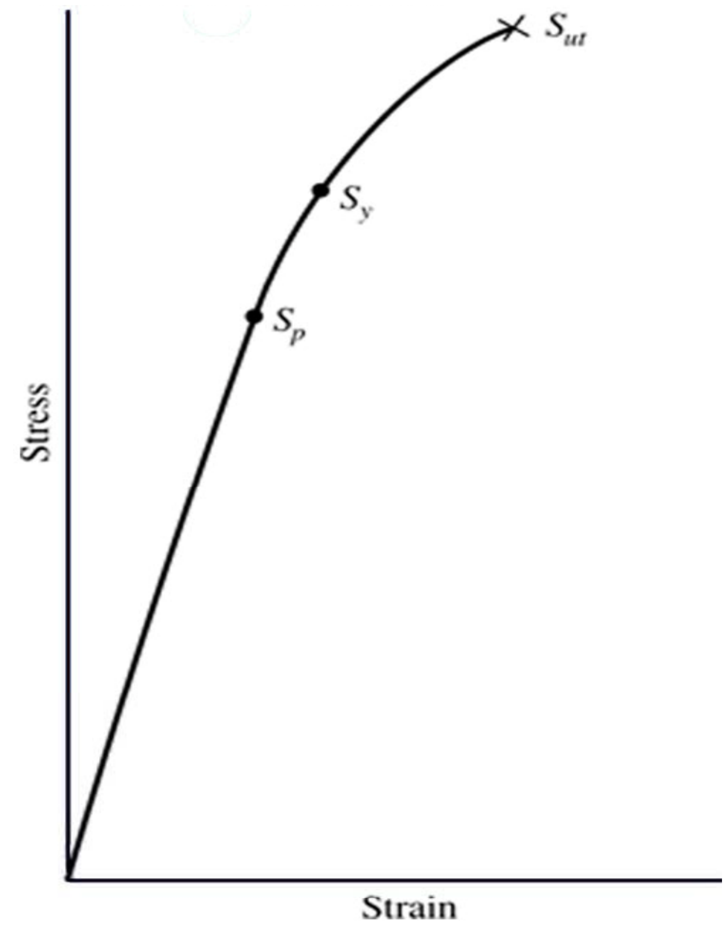


Table 8–11

Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs*

Property Class	Size Range, Inclusive	Minimum Proof Strength,† MPa	Minimum Tensile Strength,† MPa	Minimum Yield Strength,† MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	4.6
4.8	M1.6–M16	310	420	340	Low or medium carbon	4.8
5.8	M5–M24	380	520	420	Low or medium carbon	5.8
8.8	M16–M36	600	830	660	Medium carbon, Q&T	8.8
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	9.8
10.9	M5–M36	830	1040	940	Low-carbon martensite, Q&T	10.9
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	12.9

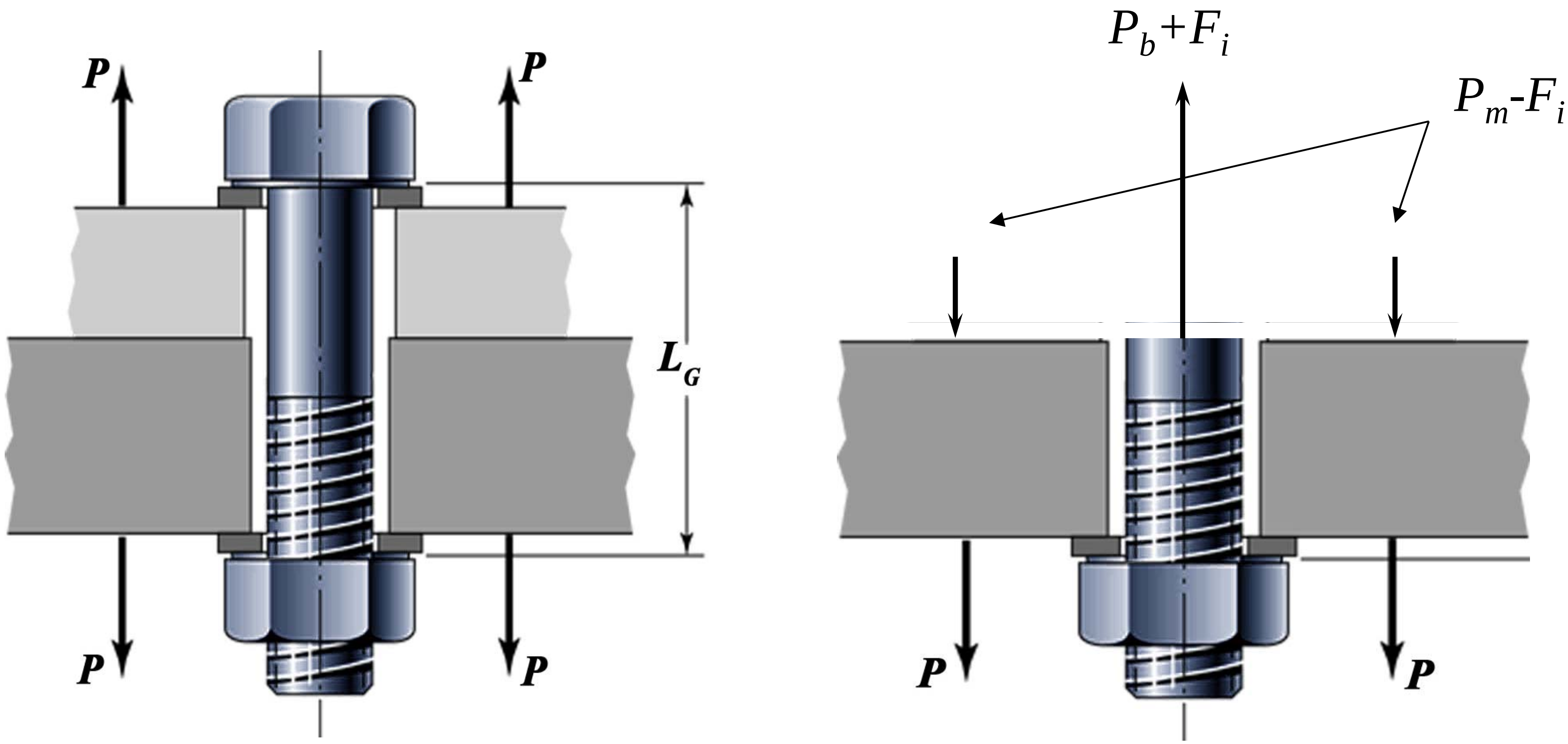
Tension Joints-The External Load

- What happens when an external tensile load P is applied to a bolted connection?

- The load P is tension, and it causes the connection to stretch, or elongate, through some distance δ.

- It is assumed that the clamping force (the preload F_i) has been correctly applied by tightening the nut before P is applied.

- To determine what portion of the externally applied load is carried by the bolt and what portion by the connected parts in the assembly, we can first draw the free body diagram:



- The Symbols that are going to be used are:

  F_i = preload

  P = external tensile load per bolt

  P_b = portion of P taken by bolt

  P_m = portion of P taken by members

  $F_b = P_b + F_i$ =resultant bolt load

  $F_m = P_m - F_i$ =resultant load on members

  C = fraction of external load P carried by bolt

  1-C = fraction of external load P carried by members

  N = Number of bolts in the joint (if N bolts equally share the total external load then $P = P_{\text{total}} / N$)

- Applying the condition of equilibrium for the forces

$$P = P_b + P_m \qquad (1)$$

- Where

  P_b is due to the increased bolt (tensile) force

  P_m represents the decreased clamping (compression) force between the members

- The deformation of the bolt and the members are the same and can be defined by:

$$\delta = \frac{P_b}{k_b} \quad and \quad \delta = \frac{P_m}{k_m} \qquad (2)$$

- The compatibility condition is then:

$$\frac{P_m}{k_m} = \frac{P_b}{k_b} \qquad (3)$$

- from (1) and (3) we have:

$$
\begin{aligned}
P_b &= \frac{k_b P}{k_b + k_m} = CP \\
P_m &= P - P_b = (1 - C)P
\end{aligned}
\qquad (4)
$$

Where $C = k_b / (k_b + k_m)$ is called the stiffness constant of the joint.

- The total forces on the bolt and the members are:

$$F_b = P_b + F_i = CP + F_i \qquad F_m < 0 \qquad (5)$$
$$F_m = P_m - F_i = (1 - C)P - F_i \qquad F_m < 0$$

The results are valid only as long as some clamping load remains in the members

- The ratios C and 1-C are the coefficients of P in equation (5). It describes the proportion of the external load taken by the bolt and by the members respectively.

- In all cases, the members take over %80 of the external load. Think how important this when fatigue loading is present.

- Making L_G longer causes the members to take an even greater percentage of the external load.

Torque Requirements (Relating Bolt Torque to Bolt Tension)

- The most important factor determining the preload in a bolt is the torque required to tighten the bolt.

- The torque may be applied manually by means of a wrench that has a dial attachment indicating the magnitude of the torque being enforced.

- Pneumatic or air wrenches give more consistent results than a manual torque wrench and are employed extensively.

- An expression relating applied torque to initial tension can be obtained using the equation developed for power screws. Observe that the load F of a screw is equivalent to F_i for a bolt and that collar friction in the jack corresponds to friction on the flat surface of the nut or under the screw head. Thus

$$T = \frac{F_i d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha}\right) + \frac{F_i f_c d_c}{2} \qquad (6)$$

- Since $\tan\lambda = l/\pi d_m$, we divide the numerator and denominator of the first term by πd_m and get

$$T = \frac{F_i d_m}{2}\left(\frac{\tan\lambda + f\sec\alpha}{1 - f\tan\lambda\sec\alpha}\right) + \frac{F_i f_c d_c}{2} \qquad (7)$$

- The diameter of the washer face of a hexagonal nut is the same as the width across flats and equal to 1.5 times the nominal size. Therefore the mean collar diameter is $d_c = (d+1.5d)/2 = 1.25d$.

- Equation (7) can now be arranged to give:

$$T = \left[\left(\frac{d_m}{2d}\right)\left(\frac{\tan\lambda + f\sec\alpha}{1 - f\tan\lambda\sec\alpha}\right) + 0.625 f_c\right] F_i d = K F_i d \qquad (8)$$

Where K is the torque coefficient

- The coefficient of friction depends upon the surface smoothness, accuracy, and degree of lubrication.
- On the average, both f and f_c are about 0.15.
- For $f = f_c$=0.15, K=0.2, no matter what size bolts are employed and no matter whether the threads are coarse or fine.

Table 8–15

Torque Factors K for Use with Eq. (8–27)

Bolt Condition	K
Nonplated, black finish	0.30
Zinc-plated	0.20
Lubricated	0.18
Cadmium-plated	0.16
With Bowman Anti-Seize	0.12
With Bowman-Grip nuts	0.09

EXAMPLE 8–3 A $\frac{3}{4}$ in-16 UNF $\times$ $2\frac{1}{2}$ in SAE grade 5 bolt is subjected to a load P of 6 kip in a tension joint. The initial bolt tension is $F_i = 25$ kip. The bolt and joint stiffnesses are $k_b = 6.50$ and $k_m = 13.8$ Mlbf/in, respectively.

(*a*) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(*b*) Specify the torque necessary to develop the preload, using Eq. (8–27).

(*c*) Specify the torque necessary to develop the preload, using Eq. (8–26) with $f = f_c = 0.15$.

Solution From Table 8–2, $A_t = 0.373$ in^2.

(*a*) The preload stress is

Answer

$$\sigma_i = \frac{F_i}{A_t} = \frac{25}{0.373} = 67.02 \text{ kpsi}$$

The stiffness constant is

$$C = \frac{k_b}{k_b + k_m} = \frac{6.5}{6.5 + 13.8} = 0.320$$

From Eq. (8–24), the stress under the service load is

Answer

$$\sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = C\frac{P}{A_t} + \sigma_i$$

$$= 0.320\frac{6}{0.373} + 67.02 = 72.17 \text{ kpsi}$$

From Table 8–9, the SAE minimum proof strength of the bolt is $S_p = 85$ kpsi. The preload and service load stresses are respectively 21 and 15 percent less than the proof strength.

Answer

$$T = KF_i d = 0.2(25)(10^3)(0.75) = 3750 \text{ lbf} \cdot \text{in}$$

(*c*) The minor diameter can be determined from the minor area in Table 8–2. Thus $d_r = \sqrt{4A_r/\pi} = \sqrt{4(0.351)/\pi} = 0.6685$ in. Thus, the mean diameter is $d_m = (0.75 + 0.6685)/2 = 0.7093$ in. The lead angle is

$$\lambda = \tan^{-1}\frac{l}{\pi d_m} = \tan^{-1}\frac{1}{\pi d_m N} = \tan^{-1}\frac{1}{\pi(0.7093)(16)} = 1.6066^\circ$$

For $\alpha = 30^\circ$, Eq. (8–26) gives

$$T = \left\{\left[\frac{0.7093}{2(0.75)}\right]\left[\frac{\tan 1.6066^\circ + 0.15(\sec 30^\circ)}{1 - 0.15(\tan 1.6066^\circ)(\sec 30^\circ)}\right] + 0.625(0.15)\right\} 25(10^3)(0.75)$$

$$= 3551 \text{ lbf} \cdot \text{in}$$

which is 5.3 percent less than the value found in part (*b*).

Statically Loaded Tension Joint with Preload

- Equations (5) represent the forces in a bolted joint with preload.
- The tensile stress σ_b in the bolt can be found as:

$$\sigma_b = \frac{CP}{A_t} + \frac{F_i}{A_t} = (S_p) / n_p \qquad (9)$$

- A means of ensuring a safe joint requires that the external load be smaller than that needed to cause the joint separate.

$$n_p = \frac{S_p A_t}{CP + F_i}$$

Yielding factor of safety guarding against static stress exceeding the proof strength

- Let $n_L P$ be the value of external load that would cause bolt failure and limiting value of σ_b be the proof strength S_p. Thus equation (9) becomes:

$$\frac{C n_L P}{A_t} + \frac{F_i}{A_t} = S_p \qquad (10)$$

- Thus the load factor n_L is:

$$n_L = \frac{S_p A_t - F_i}{CP} \qquad (11)$$

- $n_L > 1$ in equation (11) $\Rightarrow \sigma_b < S_p$

- Another means of ensuring a safe joint is to require that the external load be smaller than that needed to cause the joint to separate.
- If separation does occur, then entire external load will be imposed on the bolt.
- Let P_0 be the value of the external load that would cause joint separation.
- At separation F_m=0, thus from second equation in (5):

$$(1 - C)P_0 - F_i = 0 \qquad (12)$$

- Let the factor of safety against joint separation be:

$$n_0 = \frac{P_0}{P} \tag{13}$$

- *P_o External loads that would cause joint separation*
- Substituting $P_0 = n_0$P in equation (12) we find

$$n_0 = \frac{F_i}{P\left(1 - C\right)} \tag{14}$$

Here n_0 is a load factor guarding against joint separation and P is the maximum load applied to the joint.

- The bolt strength is the main factor in the design and analysis of bolted connections. The proof load F_p is the load that a bolt can carry without developing a permanent deformation.
- For both static and fatigue loading that the following be used for preload:

$$F_i = \begin{cases} 0.75F_p & reused \quad connections \\ 0.9F_p & permanent \quad connections \end{cases} \qquad (15)$$

- Where F_p is the proof load, obtained from equation

$$F_P = A_t S_P \qquad (16)$$

- Here S_p is the proof strength obtained from Tables (8-9 to 8-11). For other material, an approximate value is S_P=0.85S_y.

Gasketed Joints

- Sometimes a sealing or gasketing material must be placed between the parts connected.
- Gaskets are made of materials that are soft relative to other joint parts.
- The gasket pressure p is found by dividing the force in the member by the gasket area per bolt (N).

$$p = -\frac{F_m}{A_g / N} \qquad (17)$$

- For a load factor n, equation (11) can be written as:

$$F_m = (1 - C)nP - F_i \qquad (18)$$

- Substituting (18) into (17) gives the gasket pressure as:

$$p = \frac{N}{A_g}\left[F_i - nP\left(1 - C\right)\right] \tag{19}$$

- Note: to maintain the uniformity of pressure, bolts should not be spaced more than six bolt diameters apart. But to maintain wrench clearance, bolts should be spaced at least three diameters apart. So, a rough rule for bolt spacing when the bolts are arranged around a circle is

$$3 \le \frac{\pi D_b}{Nd} \le 6 \tag{20}$$

Where D_b is the diameter of the bolt circle and N is the number of bolts

Fatigue Loading

- Most of the time, the type of fatigue loading encountered in the analysis of bolted joints is one in which the externally applied load fluctuates between zero and some maximum force *P*.
- This would be the situation in a pressure cylinder, for example where a pressure either exists or does not exist
- For such cases:

$$F_{\max}=F_b \text{ and } F_{\min}=F_i$$

- Thus,

$$F_a=(F_{\max} - F_{\min})/2=(F_b - F_i)/2$$

- Dividing this by A_t yields the alternating component of the bolt stress:

$$\sigma_a = \frac{F_b - F_i}{2A_t} = \frac{\left(CP + F_i\right) - F_i}{2A_t} = \frac{CP}{2A_t} \qquad (21)$$

- The mean stress is equal to the alternating component plus the minimum stress, $\sigma_i = F_i / A_t$

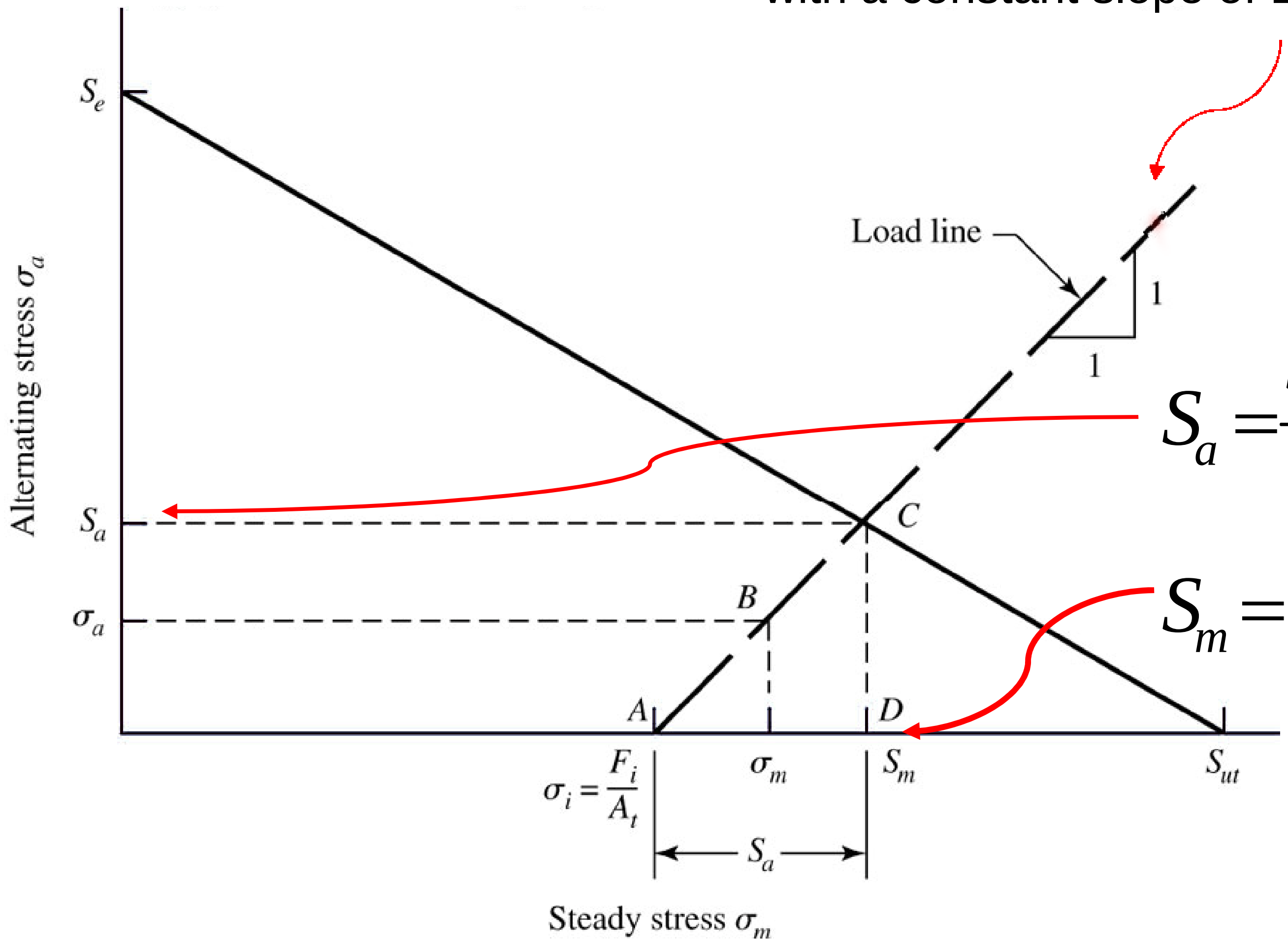
$$\sigma_m = \frac{CP}{2A_t} + \frac{F_i}{A_t} \qquad (22)$$

- On the designer's fatigue diagram as shown in figure 8-20, the load line is:

$$\sigma_m = \sigma_a + \sigma_i \qquad (23)$$

Goodman fatigue diagram

The stress on the bolt starts from the preload stress and increases with a constant slope of 1.



$$S_a = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut} + S_e}$$

$$S_m = S_a + \sigma_i$$

- The strength components S_a and S_m of the fatigue failure locus. Using Goodman failure criteria:

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \qquad (24)$$

- Substitute equation (23) with σ as S for into (24) we have:

$$S_a = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut} + S_e} \qquad (25)$$

$$S_m = S_a + \sigma_i$$

- In this section, K_f is applied for both σ_a and σ_m

- The factor of safety guarding against fatigue is given by

$$n_f = \frac{S_a}{\sigma_a} \quad or \quad n_f = \frac{2S_e(S_{ut}A_t - F_i)}{CP(S_{ut} + S_e)} \qquad (26)$$

- When there is no preload, $F_i = 0$, $C = 1$, thus

$$n_{f0} = \frac{2S_e S_{ut} A_t}{P(S_{ut} + S_e)} \qquad (27)$$

Table 8–16

Fatigue Stress-Concentration Factors K_f for Threaded Elements

SAE Grade	Metric Grade	Rolled Threads	Cut Threads	Fillet
0 to 2	3.6 to 5.8	2.2	2.8	2.1
4 to 8	6.6 to 10.9	3.0	3.8	2.3

- Preload is beneficial for resisting fatigue when n_f / n_{fo} is greater than unity. Thus,

$$F_i \leq (1\text{-}C)S_{ut}A_t$$

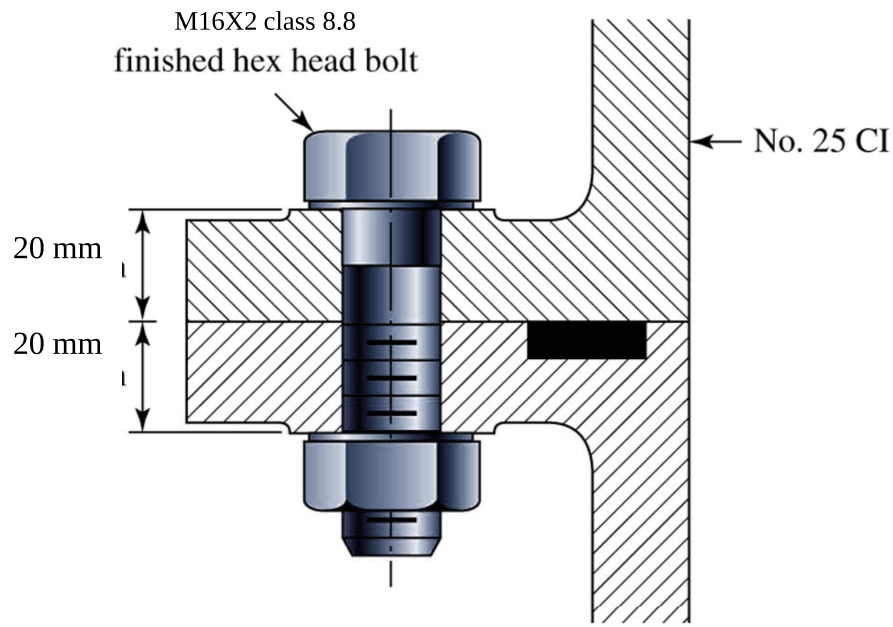
- After solving equation of n_f , you should also check the possibility of yielding using the proof strength.

$$n_p = \frac{S_p}{\sigma_m + \sigma_a} \qquad (28)$$

Example

- As shown in figure is a cross-section taken from a pressure cylinder. A total of *N* bolts are to be used to resist a separating force of 160 kN. The members are of No. 25 cast iron. The bolt is M16X2 class 8.8.
- Find:

1. The stiffnesses k_b and k_m
2. The stiffness constant of the joint C
3. The minimum number of bolts required using a factor of safety 2 and accounting for the fact that the bolts may be reused when the joint is taken apart.
4. What is the new bolt preload



Solution: 1. Finding the stiffnesses k_b and k_m

- The stiffness of the bolt k_b is:

  From M16X2 $\Rightarrow$ nominal major diameter $D = d = 16$ mm, Pitch P = 2 mm

  From table A-31 with M16 $\Rightarrow$ $H = 14.8$ mm for Regular Hexagonal head.

  The total length of the bolt $L = L_G + H$ + (2 threads after the nuts)

  Thus,

  $L = 2(20) + 14.8 + 2(P) = 40+14.8+2(2) = 58.8$ mm

  From table A-17, choose L=60 mm

- From equation 8-14 or table 8-7

  $L_T = 2D + 6 = 2(16) + 6 = 38\text{mm}$

  $l_d = L - L_T = 60-38 = 22\text{mm}$

  $l_t = L_G - l_d = 40-22 = 18\text{mm}$

  From table 8-1, $A_t = 157\ \text{mm}^2$

  $A_d = \pi D^2/4 = \pi(16)^2/4 = 201.06\ \text{mm}^2$

  Bolt is steel $\Rightarrow E$=200GPa

- Therefore, the stiffness of the bolt is:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = 892.6 MN/m$$

- From table A-24 for no. 25 cast iron we will use E =11.5 kpsi = 11.5(6.88) = 79.12 MPa

  Hence, the stiffness of the member k_m:

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = 1271.12 MN/m$$

2. The stiffness constant of the joint *C*

$$C = \frac{k_b}{k_b + k_m} = 0.413$$

3. The number of bolts N

- From table 8-11, for property class 8.8 and M16 $\Rightarrow S_p = 600$MPa
  $\Rightarrow F_p = A_t S_p = (157)(600) = 94200$N
- For reused $F_i = 0.75 F_p = 0.75(94200) = 70650$N
- Therefore:

$$n = 2 = \frac{S_p A_t - F_i}{C(P/N)} = \frac{(600)(157) - 70650}{(0.413)(160000/N)} \Rightarrow N = 5.6$$

- Take $N = 6$ bolts $\Rightarrow n = 2.14$, which is greater than the required value. So we choose six bolts use the recommended tightening preload

4. The new bolt preload

- For N = 6, the new bolt preload can be calculated by:

$$F_i = S_P A_t - nC\left(P / N\right) = 33467\,N$$

EXAMPLE 8-4 Figure 8–19 is a cross section of a grade 25 cast-iron pressure vessel. A total of N bolts are to be used to resist a separating force of 36 kip.

(*a*) Determine k_b, k_m, and C.

(*b*) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.

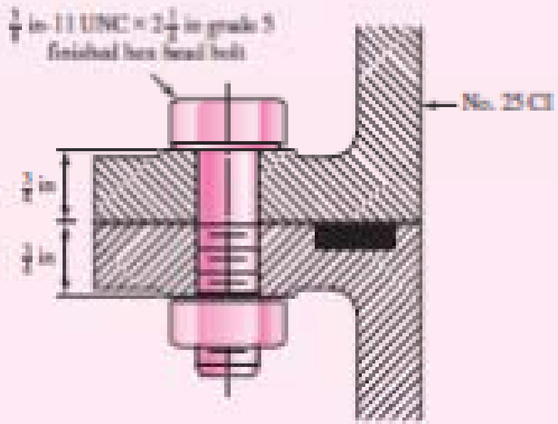
(*c*) With the number of bolts obtained in part (*b*), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

Solution (*a*) The grip is $l = 1.50$ in. From Table A–31, the nut thickness is $\frac{35}{64}$ in. Adding two threads beyond the nut of $\frac{2}{11}$ in gives a bolt length of

$$L = \frac{35}{64} + 1.50 + \frac{2}{11} = 2.229 \text{ in}$$

From Table A–17 the next fraction size bolt is $L = 2\frac{1}{2}$ in. From Eq. (8–13), the thread length is $L_T = 2(0.625) + 0.25 = 1.50$ in. Thus, the length of the unthreaded portion

| Figure 8–19



in the grip is $l_d = 2.25 - 1.50 = 0.75$ in. The threaded length in the grip is $l_t = l - l_d = 0.75$ in. From Table 8–2, $A_t = 0.226$ in^2. The major-diameter area is $A_d = \pi(0.625)^2/4 = 0.3068$ in^2. The bolt stiffness is then

Answer

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.3068(0.226)(30)}{0.3068(0.75) + 0.226(0.75)}$$

$$= 5.21 \text{ Mlbf/in}$$

From Table A–24, for no. 25 cast iron we will use $E = 14$ Mpsi. The stiffness of the members, from Eq. (8–22), is

Answer

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(14)(0.625)}{2\ln\left[5\dfrac{0.5774(1.5) + 0.5(0.625)}{0.5774(1.5) + 2.5(0.625)}\right]}$$

$$= 8.95 \text{ Mlbf/in}$$

If you are using Eq. (8–23), from Table 8–8, $A = 0.778\,71$ and $B = 0.616\,16$, and

$$k_m = EdA\exp(Bd/l)$$

$$= 14(0.625)(0.778\,71)\exp[0.616\,16(0.625)/1.5]$$

$$= 8.81 \text{ Mlbf/in}$$

which is only 1.6 percent lower than the previous result.

From the first calculation for k_m, the stiffness constant C is

Answer

$$C = \frac{k_b}{k_b + k_m} = \frac{5.21}{5.21 + 8.95} = 0.368$$

(*b*) From Table 8–9, $S_p = 85$ kpsi. Then, using Eqs. (8–31) and (8–32), we find the recommended preload to be

$$F_i = 0.75A_tS_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

For N bolts, Eq. (8–29) can be written

$$n_L = \frac{S_pA_t - F_i}{C(P_{\text{total}}/N)} \tag{1}$$

or

$$N = \frac{Cn_L P_{\text{total}}}{S_pA_t - F_i} = \frac{0.368(2)(36)}{85(0.226) - 14.4} = 5.52$$

Answer Six bolts should be used to provide the specified load factor.

(*c*) With six bolts, the load factor actually realized is

Answer

$$n_L = \frac{85(0.226) - 14.4}{0.368(36/6)} = 2.18$$

From Eq. (8–28), the yielding factor of safety is

Answer

$$n_p = \frac{S_pA_t}{C(P_{\text{total}}/N) + F_i} = \frac{85(0.226)}{0.368(36/6) + 14.4} = 1.16$$

From Eq. (8–30), the load factor guarding against joint separation is

Answer

$$n_0 = \frac{F_i}{(P_{\text{total}}/N)(1 - C)} = \frac{14.4}{(36/6)(1 - 0.368)} = 3.80$$

Example

- An M30X3.5 ISO 8.8 bolt is used in a joint at recommended preload and the joint is subject to a repeated tensile fatigue load of P=80kN per bolt. The joint constant is C=0.33. Find the load factors and the factor of safety guarding against a fatigue failure based on the Goodman theory.

Solution

- From table 8-1
- $D = d = 30$ mm $\Rightarrow$ P=3.5 mm, $A_t = 561\text{mm}^2$
- From table 8-11
- For 8.8 class and M30X3.5 $\Rightarrow S_p = 600$ MPa, $S_y = 660$ MPa, $S_{ut} = 830$ MPa
- From table 8-17
- For ISO 8.8 $\Rightarrow S_e$=129 MPa

$$n_f = \frac{S_a}{\sigma_a}$$

$$S_a = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut} + S_e}$$

$$\sigma_i = \frac{F_i}{A_t}$$

$$F_i = 0.75F_p = 0.75\left(S_p A_t\right) = 0.75(600)(561) = 252450N$$

$$\therefore \quad \sigma_i = \frac{252450}{561\left(10^{-6}\right)} = 450MPa$$

$$\therefore \quad S_a = \frac{129\left(830 - 450\right)}{830 + 129} = 51.16MPa$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{(0.33)(80000)}{2(561)(10^{-6})}$$

$$n_f = \frac{51.16}{23.53} = 2.17$$

- The load factor n and the factor of safety against joint separation:

$$n = \frac{S_p A_t - F_i}{CP} = \frac{(600)(561) - 252450}{(0.33)(80000)} = 3.2$$

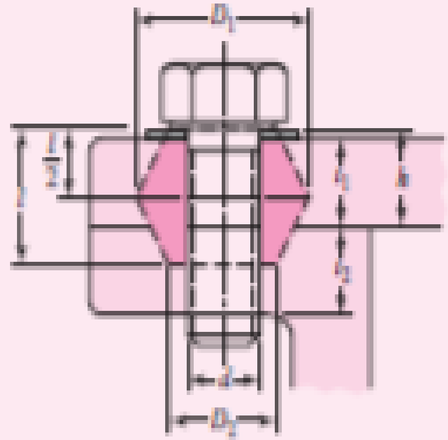
$$n_0 = \frac{F_i}{P(1-C)} = \frac{252450}{(80000)(1-0.33)} = 4.71$$

EXAMPLE 8-5

Figure 8–21 shows a connection using cap screws. The joint is subjected to a fluctuating force whose maximum value is 5 kip per screw. The required data are: cap screw, 5/8 in-11 NC, SAE 5; hardened-steel washer, $t_w = \frac{1}{16}$ in thick; steel cover plate, $t_1 = \frac{5}{8}$ in, $E_s = 30$ Mpsi; and cast-iron base, $t_2 = \frac{5}{8}$ in, $E_{ci} = 16$ Mpsi.

(*a*) Find k_b, k_m, and C using the assumptions given in the caption of Fig. 8–21.

(*b*) Find all factors of safety and explain what they mean.



(*a*) For the symbols of Figs. 8–15 and 8–21, $h = t_1 + t_w = 0.6875$ in, $l = h + d/2 = 1$ in, and $D_2 = 1.5d = 0.9375$ in. The joint is composed of three frusta; the upper two frusta are steel and the lower one is cast iron.

For the upper frustum: $t = l/2 = 0.5$ in, $D = 0.9375$ in, and $E = 30$ Mpsi. Using these values in Eq. (8–20) gives $k_1 = 46.46$ Mlbf/in.

For the middle frustum: $t = h - l/2 = 0.1875$ in and $D = 0.9375 + 2(l - h)$ tan 30° $= 1.298$ in. With these and $E_s = 30$ Mpsi, Eq. (8–20) gives $k_2 = 197.43$ Mlbf/in.

The lower frustum has $D = 0.9375$ in, $t = l - h = 0.3125$ in, and $E_{ci} = 16$ Mpsi. The same equation yields $k_3 = 32.39$ Mlbf/in.

Substituting these three stiffnesses into Eq. (8–18) gives $k_m = 17.40$ Mlbf/in. The cap screw is short and threaded all the way. Using $l = 1$ in for the grip and $A_t = 0.226$ in^2 from Table 8–2, we find the stiffness to be $k_b = A_t E/l = 6.78$ Mlbf/in. Thus the joint constant is

$$C = \frac{k_b}{k_b + k_m} = \frac{6.78}{6.78 + 17.40} = 0.280$$

Figure 8–21

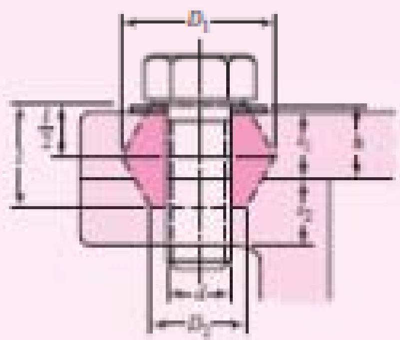
Pressure-cone frustum member model for a cap screw. For this model the significant sizes are

$$l = \begin{cases} h + t_2/2 & t_2 < d \\ h + d/2 & t_2 \geq d \end{cases}$$

$D_1 = d_w + l \tan \alpha = 1.5d + 0.577l$

$D_2 = d_w = 1.5d$

where l = effective grip. The solutions are for $\alpha = 30°$ and $d_w = 1.5d$.



(*b*) Equation (8–30) gives the preload as

$$F_i = 0.75F_p = 0.75A_tS_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

where from Table 8–9, $S_p = 85$ kpsi for an SAE grade 5 cap screw. Using Eq. (8–28), we obtain the load factor as the yielding factor of safety is

Answer

$$n_p = \frac{S_pA_t}{CP + F_i} = \frac{85(0.226)}{0.280(5) + 14.4} = 1.22$$

This is the traditional factor of safety, which compares the maximum bolt stress to the proof strength.

Using Eq. (8–29),

Answer

$$n_L = \frac{S_pA_t - F_i}{CP} = \frac{85(0.226) - 14.4}{0.280(5)} = 3.44$$

This factor is an indication of the overload on P that can be applied without exceeding the proof strength.

Next, using Eq. (8–30), we have

Answer

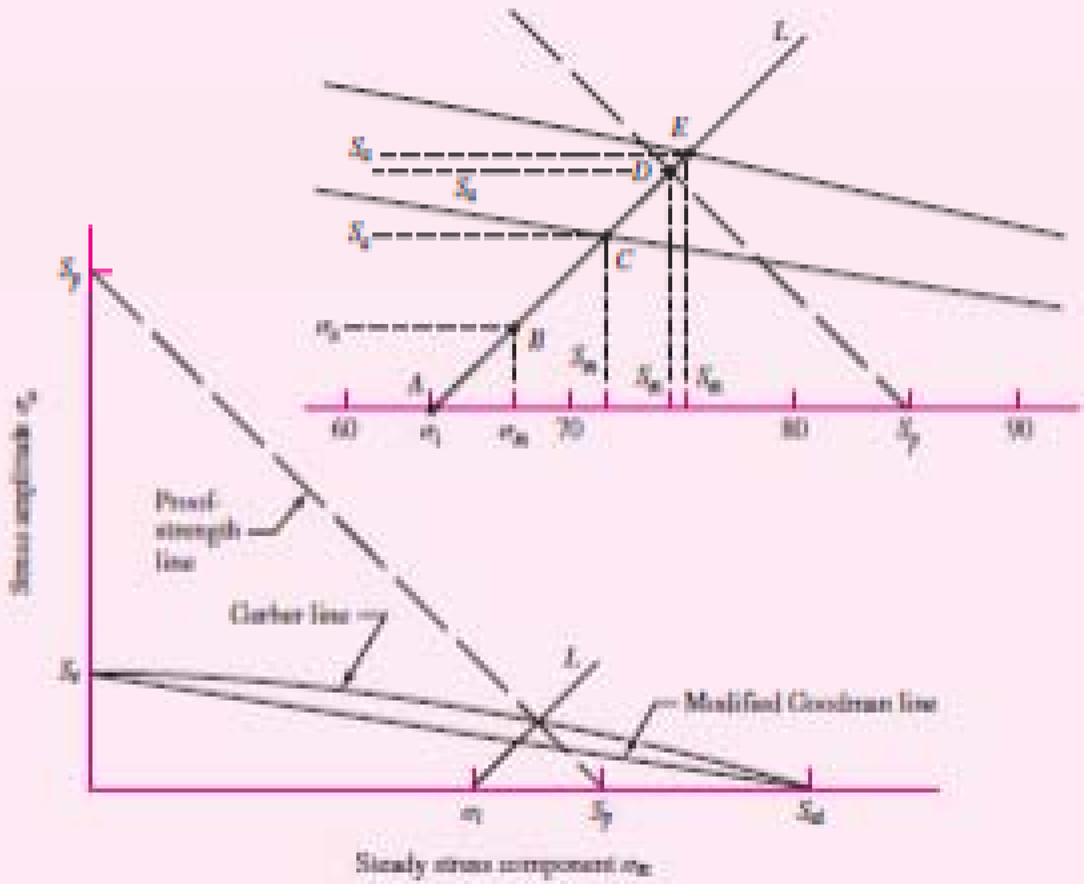
$$n_0 = \frac{F_i}{P(1-C)} = \frac{14.4}{5(1-0.280)} = 4.00$$

If the force P gets too large, the joint will separate and the bolt will take the entire load. This factor guards against that event.

For the remaining factors, refer to Fig. 8–22. This diagram contains the modified Goodman line, the Gerber line, the proof-strength line, and the load line. The intersection

Figure 8–22

Designer's fatigue diagram for preloaded bolts, drawn to scale, showing the modified Goodman line, the Gerber line, and the Langer proof-strength line, with an exploded view of the area of interest. The strengths used are $S_p = 85$ kpsi, $S_e = 18.6$ kpsi, and $S_{ut} = 120$ kpsi. The coordinates are A, $\sigma_i = 63.72$ kpsi; B, $\sigma_a = 3.10$ kpsi, $\sigma_m = 66.82$ kpsi; C, $S_a = 7.55$ kpsi, $S_m = 71.29$ kpsi; D, $S_a = 10.64$ kpsi, $S_m = 74.36$ kpsi; E, $S_a = 11.32$ kpsi, $S_m = 75.04$ kpsi.



of the load line L with the respective failure lines at points C, D, and E defines a set of strengths S_a and S_m at each intersection. Point B represents the stress state σ_a, σ_m. Point A is the preload stress σ_i. Therefore the load line begins at A and makes an angle having a unit slope. This angle is 45° only when both stress axes have the same scale.

The factors of safety are found by dividing the distances AC, AD, and AE by the distance AB. Note that this is the same as dividing S_a for each theory by σ_a.

The quantities shown in the caption of Fig. 8–22 are obtained as follows:

Point A

$$\sigma_i = \frac{F_i}{A_t} = \frac{14.4}{0.226} = 63.72 \text{ kpsi}$$

Point B

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.280(5)}{2(0.226)} = 3.10 \text{ kpsi}$$

$$\sigma_m = \sigma_a + \sigma_i = 3.10 + 63.72 = 66.82 \text{ kpsi}$$

Point C

This is the modified Goodman criteria. From Table 8–17, we find $S_e = 18.6$ kpsi. Then, using Eq. (8–45), the factor of safety is found to be

nswer

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{18.6(120 - 63.72)}{3.10(120 + 18.6)} = 2.44$$

Point D
This is on the proof-strength line where

$$S_m + S_a = S_p \qquad (1)$$

In addition, the horizontal projection of the load line *AD* is

$$S_m = \sigma_i + S_a \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously results in

$$S_a = \frac{S_p - \sigma_i}{2} = \frac{85 - 63.72}{2} = 10.64 \text{ kpsi}$$

The factor of safety resulting from this is

Answer

$$n_p = \frac{S_a}{\sigma_a} = \frac{10.64}{3.10} = 3.43$$

which, of course, is identical to the result previously obtained by using Eq. (8–29).

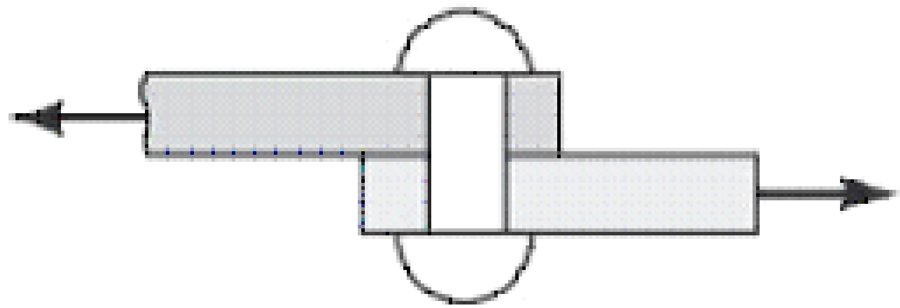
A similar analysis of a fatigue diagram could have been done using yield strength instead of proof strength. Though the two strengths are somewhat related, proof strength is a much better and more positive indicator of a fully loaded bolt than is the yield strength. It is also worth remembering that proof-strength values are specified in design codes; yield strengths are not.

We found $n_f = 2.44$ on the basis of fatigue and the modified Goodman line, and $n_p = 3.43$ on the basis of proof strength. Thus the danger of failure is by fatigue, not by overproof loading. These two factors should always be compared to determine where the greatest danger lies.

Shear Joints

- Riveted and bolted joints loaded in sheer are treated exactly alike in design and analysis.
- In days when driving hot rivets was a common structural joining method, the driving ensured that the rivets filled every hole completely and, upon cooling, providing a clamping preload. This kind of rivet pattern can shear a sheer load.

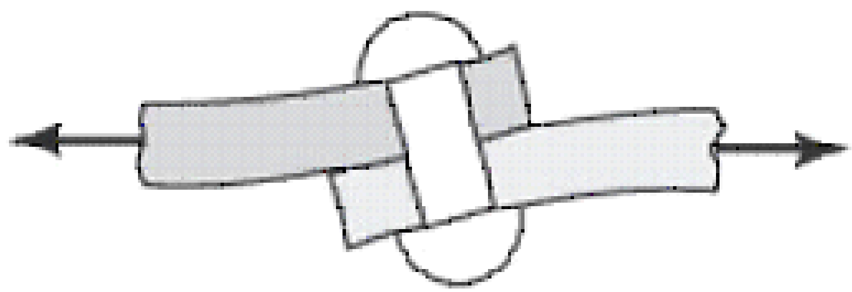
Bolted and Riveted Joints Loaded in Shear:



- In figure 9-24a is shown a riveted connection loaded in shear.
- There are several various means by which this connection might fail:

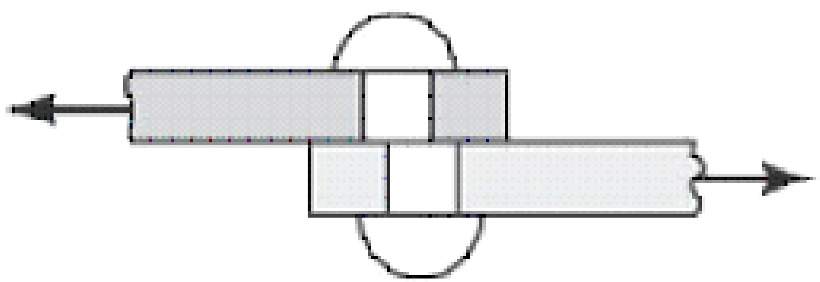
Failure by bending of the rivet or of the riveted members at as shown in figure 9-24-b.

- The bending moment is approximately $M = Ft/2$, where F is the shearing force and t is the grip of the rivet, that is, the total thickness of the connected parts. $\sigma = Mc/I$



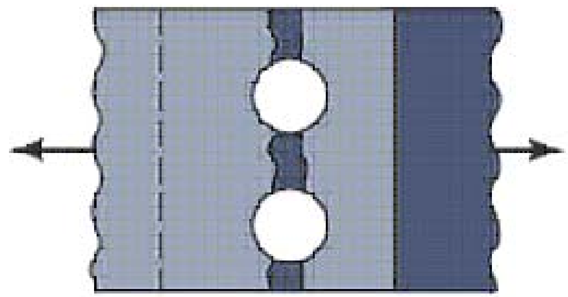
Failure of the rivet by pure shear as shown in figure 9 24-c.

- The shear stress can be calculated using the formula: $\tau = F/A$. Where A is the cross-sectional area of all the rivets in the group. The diameter used in the design is the rivet diameter not the hole.



Rupture of one of the connected members or plates by pure tension. Figure 9-24-d

- The formula $\sigma = F/A$, where A is the net area of the plate (the area reduced by an amount equal to the area of all the rivet holes).
- It is true that the use of a bolt with an initial preload and, sometimes, a rivet will place the area around the hole in compression and thus to nullify effects of stress concentration.
- Unless definite steps are taken to ensure that the preload does not relax, it's on the conservative side to design as if the full stress-concentration effect were presented.



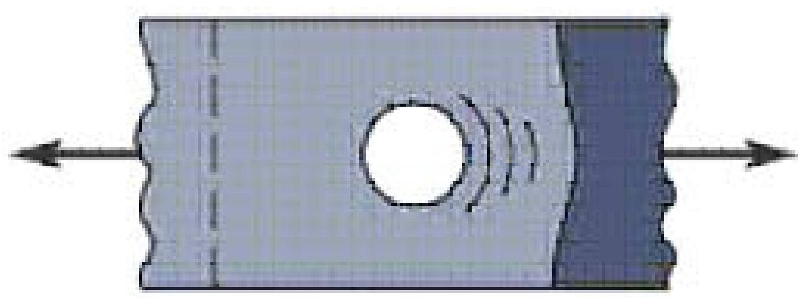
Failure occurs by crushing of the rivet or plate as shown in fig 9-24-e

- It is usually called a bearing stress
- The formula can be calculated using the formula, $\sigma = F/A$, where:

  A = projected area for a single rivet is $(t)(d)$

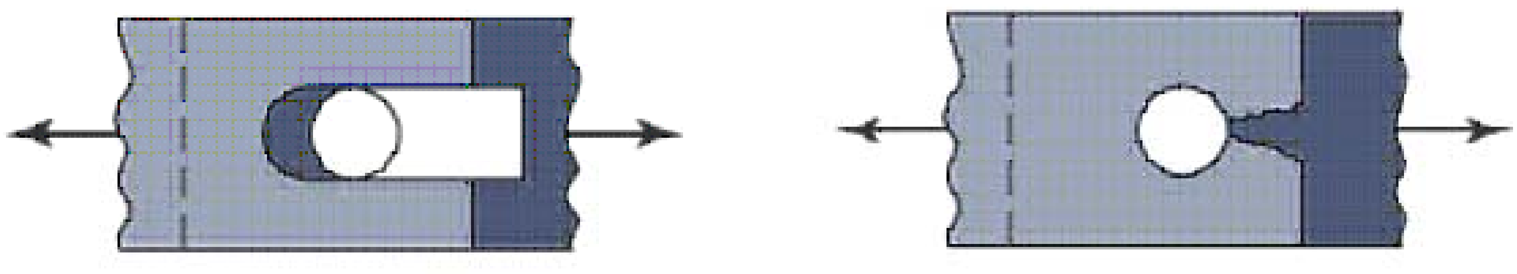
  t = the thickness of the thinnest plate.

  d = the rivet or bolt diameter.



Failure due to edge shearing, or tearing as shown in fig. 9-24-f and g.

- In structure practice this failure is avoided by spacing the rivets at least 1 ½ diameters away from the edge.



- In structure design it is customary to select in advance the number of rivets and their diameters and spacing.
- The strength is then determined for each method of failure.
- If the calculated strength is not satisfactory, a change is made in the diameter, spacing, or number of rivets used to bring the strength in line with expected loading conditions.

EXAMPLE 8–6 The bolted connection shown in Figure 8–24 uses SAE grade 5 bolts. The members are hot-rolled AISI 1018 steel. A tensile shear load $F = 4000$ lbf is applied to the connection. Find the factor of safety for all possible modes of failure.

Solution Members: $S_y = 32$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = (0.577)92 = 53.08$ kpsi

Shear of bolts

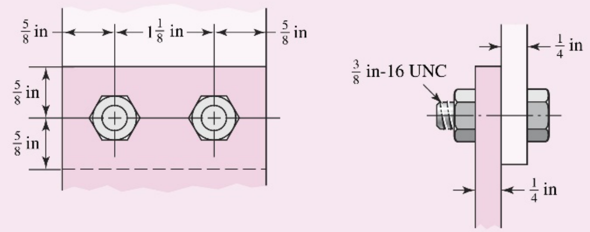
$$A_s = 2\left[\frac{\pi(0.375)^2}{4}\right] = 0.221 \text{ in}^2$$

$$\tau = \frac{F_s}{A_s} = \frac{4}{0.221} = 18.1 \text{ kpsi}$$

Answer

$$n = \frac{S_{sy}}{\tau} = \frac{53.08}{18.1} = 2.93$$

Figure 8–24



Bearing on bolts

$$A_b = 2(0.25)(0.375) = 0.188 \text{ in}^2$$

$$\sigma_b = \frac{-4}{0.188} = -21.3 \text{ kpsi}$$

Answer

$$n = \frac{S_y}{|\sigma_b|} = \frac{92}{|-21.3|} = 4.32$$

Bearing on members

Answer

$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{32}{|-21.3|} = 1.50$$

Tension of members

$$A_t = (2.375 - 0.75)(1/4) = 0.406 \text{ in}^2$$

$$\sigma_t = \frac{4}{0.406} = 9.85 \text{ kpsi}$$

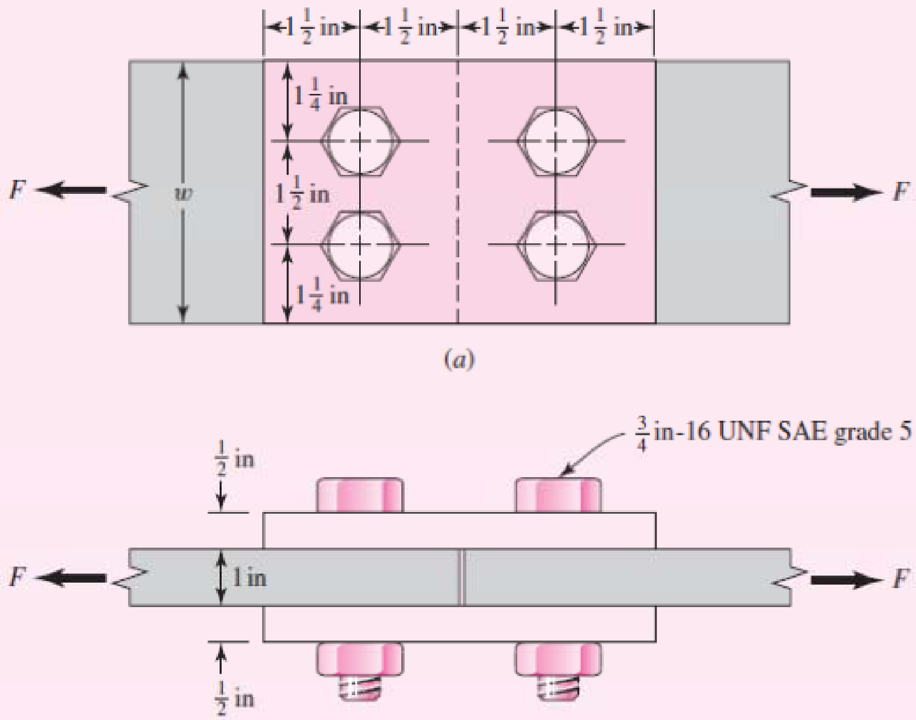
Answer

$$n = \frac{S_y}{A_t} = \frac{32}{9.85} = 3.25$$

EXAMPLE 8–6 Two 1- by 4-in 1018 cold-rolled steel bars are butt-spliced with two $\frac{1}{2}$- by 4-in 1018 cold-rolled splice plates using four $\frac{3}{4}$ in-16 UNF grade 5 bolts as depicted in Fig. 8–24. For a design factor of $n_d = 1.5$ estimate the static load F that can be carried if the bolts lose preload.

Solution From Table A–20, minimum strengths of $S_y = 54$ kpsi and $S_{ut} = 64$ kpsi are found for the members, and from Table 8–9 minimum strengths of $S_p = 85$ kpsi and $S_{ut} = 120$ kpsi for the bolts are found.

re 8–24



$F/2$ is transmitted by each of the splice plates, but since the areas of the splice plates are half those of the center bars, the stresses associated with the plates are the same. So for stresses associated with the plates, the force and areas used will be those of the center plates.

Bearing in bolts, all bolts loaded:

$$\sigma = \frac{F}{2td} = \frac{S_p}{n_d}$$

$$F = \frac{2tdS_p}{n_d} = \frac{2(1)\left(\frac{3}{4}\right)85}{1.5} = 85 \text{ kip}$$

Bearing in members, all bolts active:

$$\sigma = \frac{F}{2td} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{2td(S_y)_{\text{mem}}}{n_d} = \frac{2(1)\left(\frac{3}{4}\right)54}{1.5} = 54 \text{ kip}$$

Shear of bolt, all bolts active: If the bolt threads do not extend into the shear planes for four shanks:

$$\tau = \frac{F}{4\pi d^2/4} = 0.577\frac{S_p}{n_d}$$

$$F = 0.577\pi d^2 \frac{S_p}{n_d} = 0.577\pi(0.75)^2 \frac{85}{1.5} = 57.8 \text{ kip}$$

If the bolt threads extend into a shear plane:

$$\tau = \frac{F}{4A_r} = 0.577 \frac{S_p}{n_d}$$

$$F = \frac{0.577(4)A_r S_p}{n_d} = \frac{0.577(4)0.351(85)}{1.5} = 45.9 \text{ kip}$$

Edge shearing of member at two margin bolts: From Fig. 8–25,

$$\tau = \frac{F}{4at} = \frac{0.577(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{4at0.577(S_y)_{\text{mem}}}{n_d} = \frac{4(1.125)(1)0.577(54)}{1.5} = 93.5 \text{ kip}$$

Tensile yielding of members across bolt holes:

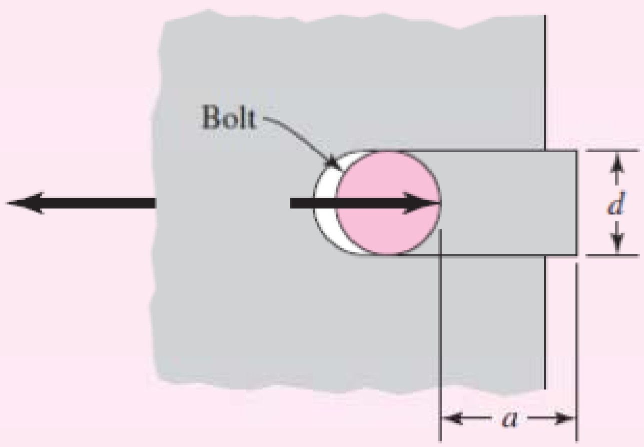
$$\sigma = \frac{F}{\left[4 - 2\left(\frac{3}{4}\right)\right]t} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{\left[4 - 2\left(\frac{3}{4}\right)\right]t(S_y)_{\text{mem}}}{n_d} = \frac{\left[4 - 2\left(\frac{3}{4}\right)\right](1)54}{1.5} = 90 \text{ kip}$$

Member yield:

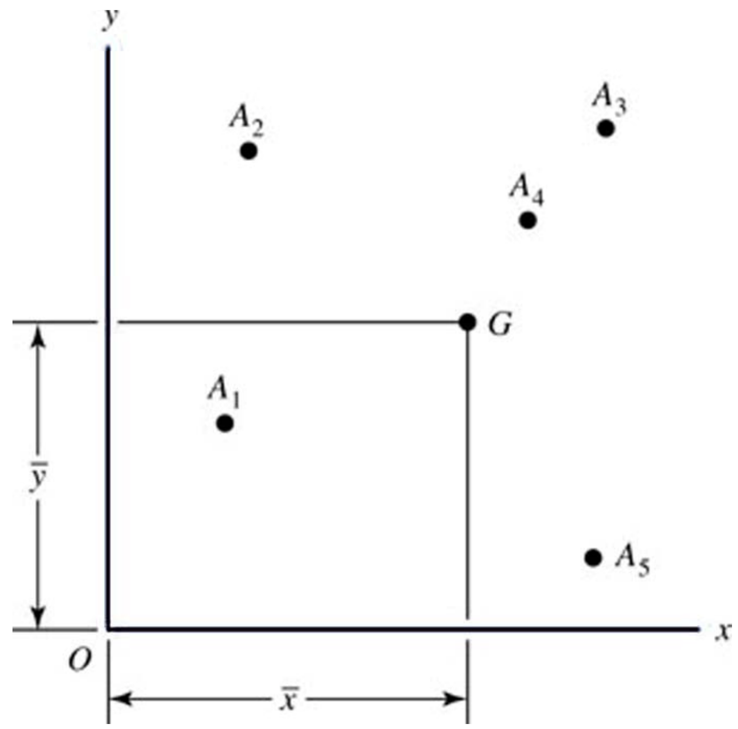
$$F = \frac{wt(S_y)_{\text{mem}}}{n_d} = \frac{4(1)54}{1.5} = 144 \text{ kip}$$

On the basis of bolt shear, the limiting value of the force is 45.9 kip, assuming the threads extend into a shear plane. However, it would be poor design to allow the threads to extend into a shear plane. So, assuming a *good* design based on bolt shear, the limiting value of the force is 57.8 kip. For the members, the bearing stress limits the load to 54 kip.



Centroids of bolt groups

- In figure 8.23 let A_1 to A_5 be the respective cross-sectional areas of a groups of five pins, or hot-driven rivets, or tight-filtering shoulder bolts.

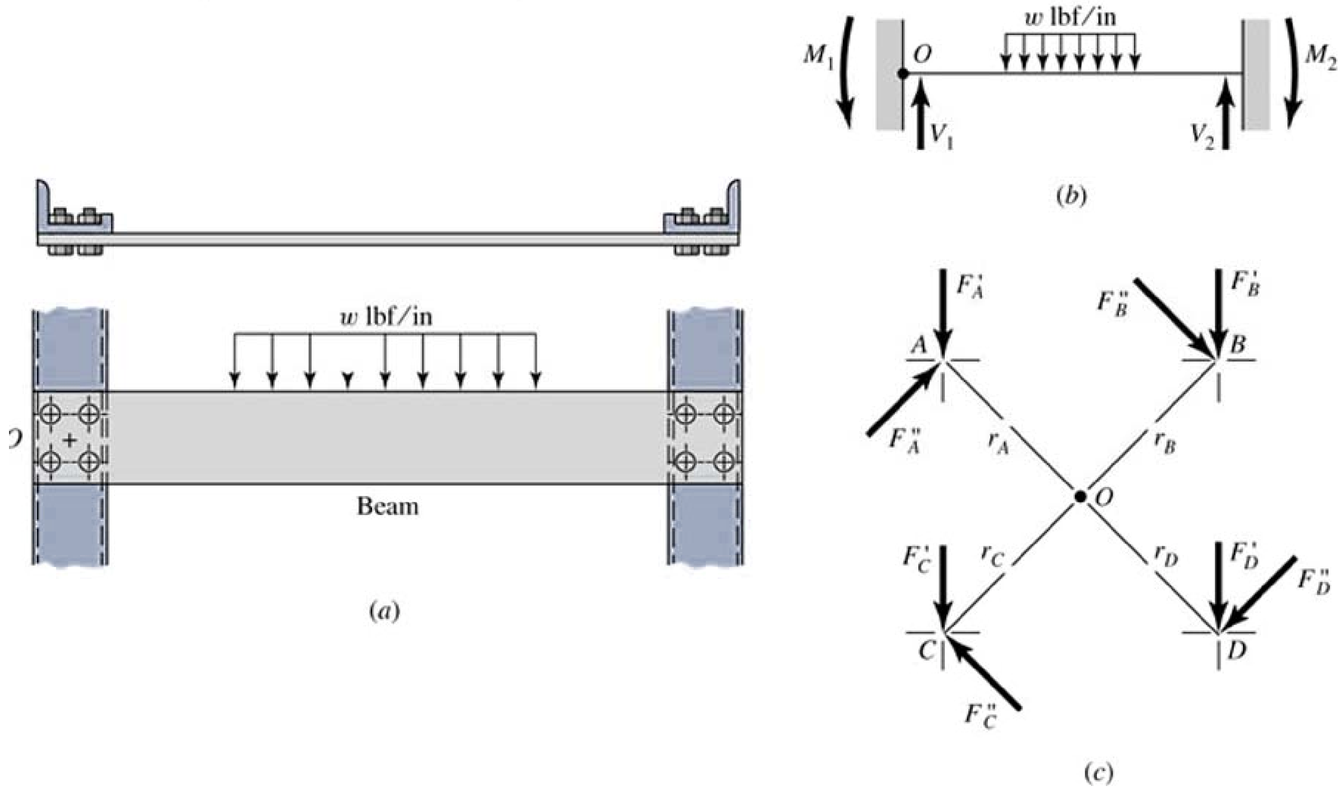


- Under this assumption the rotational pivot point lies at the centroid of the cross-sectional area pattern of the pins, rivets, or bolts.
- Using static, the centroid *G* can be found by:

$$\bar{x} = \frac{\sum_{1}^{5} A_i x_i}{\sum_{1}^{5} A_i} \quad \textit{and} \quad \bar{y} = \frac{\sum_{1}^{5} A_i y_i}{\sum_{1}^{5} A_i} \qquad (8\text{-}49)$$

Shear of bolts and rivets due to eccentric loading

- An example of eccentric loading of fasteners is shown in figure 8-24
- The beam is fastened to vertical numbers at the ends with specially prepared load-sharing bolts.
- The centers of the bolts at the left end of the beam are drawn to a large scale in figure (8-24-c).



- Point *0* represents the centroid of the group.
- The result at forces acting on the pins with a net force and moment equal and opposite to the reaction loads V_1 and M_1 acting at *0*.

The total load taken by each bolt can be found by the following 3 steps

1. Each bolt takes $F' = V_1 / n$,

   Where n = number of bolts in the group.

   F' = direct load or primary shear.

2. The moment load or secondary shear:

- It is the additional load on each bolt due to the moment M.
- If r_A, r_B, r_C, etc, are the radial distances from the centroid to the center of each bolt. Thus,

$$M = F_A'' r_A + F_B'' r_B + F_C'' r_C + \ldots \qquad (1)$$

Where F'' is the moment load

- Therefore

$$\frac{F''_A}{r_A} = \frac{F''_B}{r_B} = \frac{F''_C}{r_C} \qquad (2)$$

- Solving equations (2) and (2) simultaneously, we obtain:

$$F''_n = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \ldots} \qquad (8.50)$$

Where *n* refers to the particular bolt whose load is to be found

3. Find the resultant load of both vertical and moment loads

Example

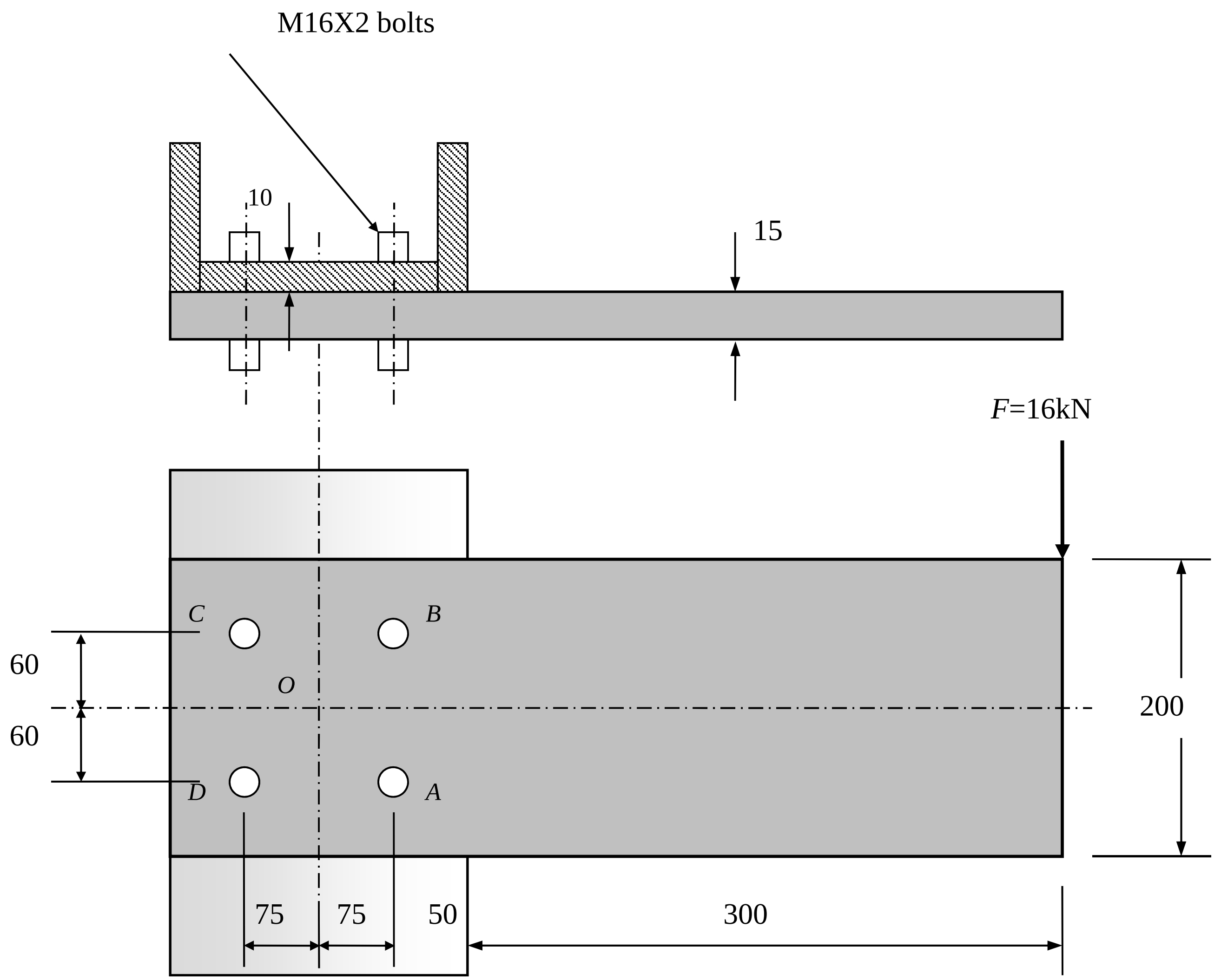
- As shown in figure a 15X200 mm rectangular steel bar cantilevered to a 250 mm steel channel using four bolts. On the basis of the external load of 16kN, find:

  a) The resultant load on each bolt

  b) The maximum bolt shear stress

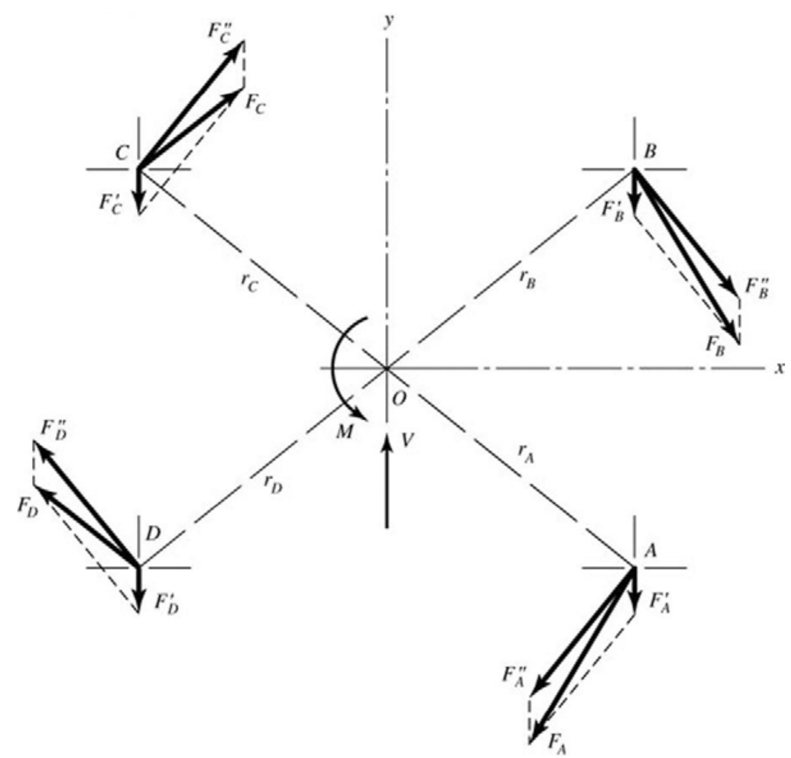
  c) The maximum bearing stress

  d) The bending stress through bolts *A* and *B*.


M16X2 bolts
10
15
F=16kN
C
B
O
D
A
60
60
200
75
75
50
300

Solution

- By symmetric, the location of the centroid is located at point *O*.
- The free body diagram shows that the shear reaction *V* would pass through *O* and the moment reaction *M* would be about *O*.



- V=16kN and M = (16)(0.425) = 6.8kN.m

  The distance from the centroid to the center of each bolt is

  $r = ((60)^2 + (75)^2)^{0.5} = 96$mm
- The primary shear load per bolt is

  $F` = V/n$ = 16/4 = 4kN
- Since the distance between the bolts are equal, hence the secondary shear forces can be calculated using equation 8-50:

$$F_1'' = \frac{Mr}{4r^2} = \frac{M}{4r} = \frac{6800}{4(0.096)} = 17.71kN$$

- The primary and secondary shear forces are plotted also in the pervious figure.

a) The result load on each bolt can be found using the parallelogram rule

- Therefore, the resultant load on each bolts are:

$$F_A = F_B = 21.0 \text{ kN}$$
$$F_C = F_D = 13.8 \text{ kN}$$

b) The maximum Load on each bolt

- Bolts A and B carries the largest shear load, therefore our calculation will be for bolts A and B
- Where the shear acts on the bolt?

To answer this question, we need to find first the length of the bolt:

$L=L_G + H + 2P$

$L_G = 15+10 = 25\text{mm}$

H: From table A-31, H=14.8mm,

$P = 2$

$\therefore\ L = (15+10) + (14.8) + 2(2) = 43.8\text{ mm}$

From table A-17, L= 45mm

- From equation 8-14, L_T = 2D+6 = 2(16)+6 = 38mm
- ∴ the unthreaded length l_d= L- L_T = 45-38=7mm
- ∴ the threaded length l_t=L_G-l_d=25-7=18mm
- You can see now that the unthreaded length is less than the thickness of the plate (15 thickness). Therefore the shear stress area is based on the threaded section and hence will be represented by the miner diameter for the bolt: from table 8-1, A_r =144mm^2.
- ∴ τ =F/A_r = 21000/144(10-6) = 145.83MPa

c) The maximum bearing stress:

- From the figure we can see that the channel is thinner than the bar, so the largest bearing stress is due to the pressing of the bolt against the channel web:

$$\sigma_b = \frac{F}{A_b} = \frac{F_A}{td} = \frac{21000}{(10)(16)} = 131.25MPa$$

d) The bending stress through bolts *A* and *B*.

$$M = 16(300 + 50) = 5600\,N.m$$

$$I = I_{bar} - 2\left(I_{holes} + d^2 A\right) =$$

$$\frac{15(200)^3}{12} - 2\left[\frac{15(16)^3}{12} + (60)^2(15)(16)\right] = 8.26(10^6)mm^4$$

$$\sigma = \frac{Mc}{I} = \frac{5600(100)}{8.26(10^6)} = 67.8MPa$$

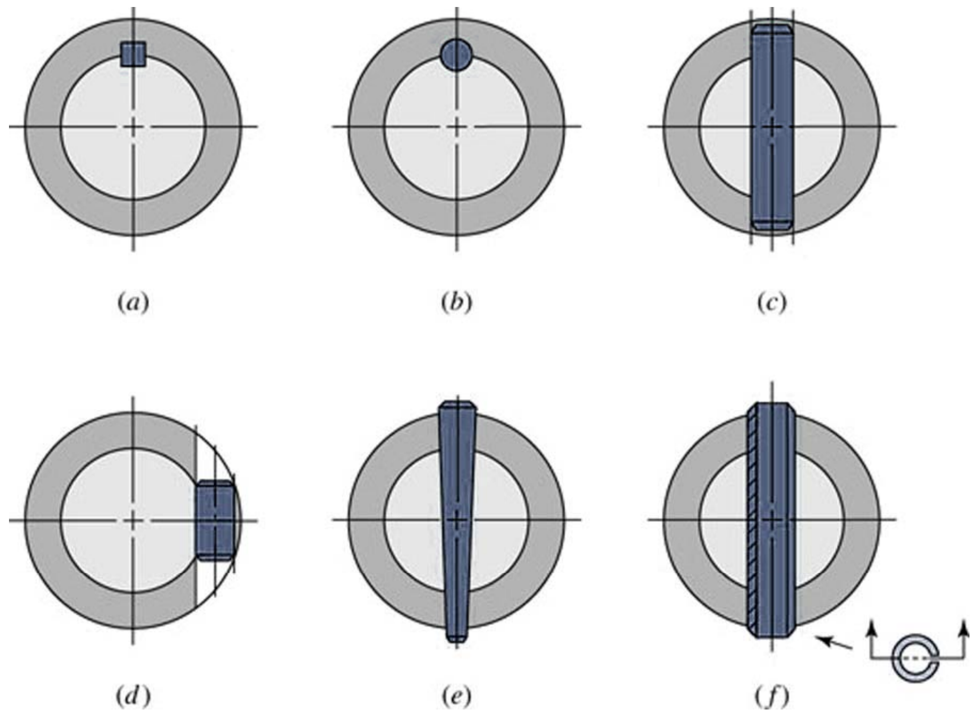
Keys and Pins

- Keys and pins are used on shafts to secure rotating elements, such as gears, pulleys, or other wheels.

- Keys are used to enable the transmission of torque from the shaft to the shaft-support element.

- Pins are used for axial positioning and for the transfer of torque or thrust or both.

Figure 8-28 shows a variety of keys and pins.



(*a*) Squre key; (*b*) Round key;

(*c*) and (*d*) Round pins;

(*e*) Taper pin; (*f*) Split tubular spring pin