

## Lecture ①

- Textbook: Loss Models From Data to decisions  
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## Ch 2

### • Random Variables

- Phenomena are occurrences that can be observed. An experiment is an observation of a given phenomenon under specified conditions. The result of an experiment is called an outcome; an event is a set of one or more possible outcomes. A stochastic phenomenon is a phenomenon for which an associated experiment has more than one possible outcome. An event associated with a stochastic phenomenon is said to be contingent. ...  
Probability is a measure of likelihood from zero to one. ...  
A random variable is a function that assigns a numerical value to every possible outcome.

### • Key Functions and Four Models

#### Defn ①

The cumulative distribution function (distribution fn)  $F(x)$  is defined as:

$$F(x) = \text{pr}(X \leq x)$$

set ①  $0 \leq F(x) \leq 1 \quad \forall x$

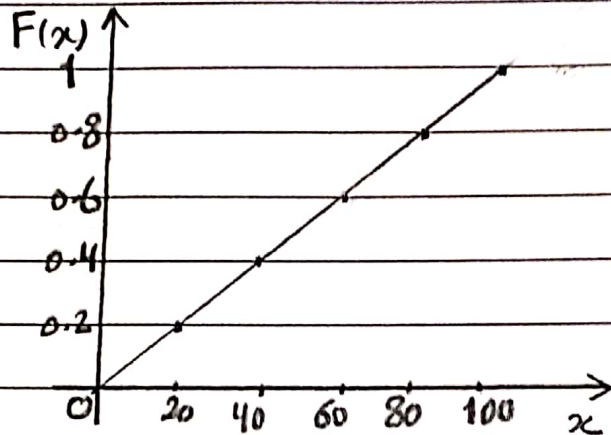
②  $F(x)$  is nondecreasing continuous fn

③  $\lim_{x \rightarrow -\infty} F(x) = 0$  and  $\lim_{x \rightarrow \infty} F(x) = 1$

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Model ①: The age at death of a randomly selected birth.

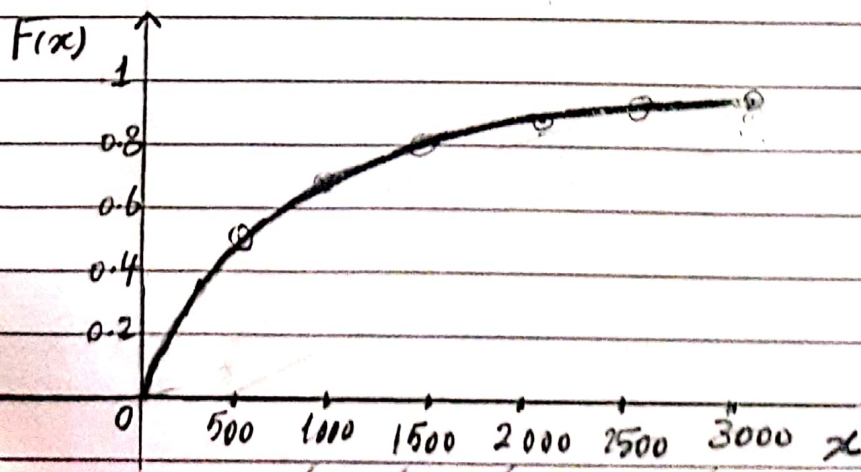
$$F_1(x) = \begin{cases} 0, & x < 0 \\ 0.01x, & 0 \leq x < 100 \\ 1, & x \geq 100 \end{cases}$$



Model ②: The # of dollars paid on a randomly selected automobile bodily injury claim.

سواء كان مبلغ التعويض أكبر من 2000 دولار أم لا، فإن التعويض لا يتجاوز 3000 دولار.

$$F_2(x) = \begin{cases} 0, & x < 0 \\ 1 - \left(\frac{2000}{x+2000}\right)^3, & x \geq 0 \end{cases}$$



Model ③: The # of automobile bodily injury claims in one year from a randomly selected insured automobile

العدد الإجمالي لادعاءات إصابة الجسد من السيارات المؤمنة في سنة واحدة من سيارة مختارة عشوائياً

$$F_3(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \leq x < 1 \\ 0.75, & 1 \leq x < 2 \\ 0.87, & 2 \leq x < 3 \\ 0.95, & 3 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$

Model ④: the total dollars in medical malpractice claims paid in one year owing to events at a randomly selected hospital

المبلغ الإجمالي بالدولار المدفوع في السنة الواحدة من دعاوى سوء الممارسة الطبية بسبب أحداث في مستشفى مختارة عشوائياً

$$F_4(x) = \begin{cases} 0, & x < 0 \\ 1 - 0.3 e^{-0.00001x}, & x \geq 0 \end{cases}$$

Note that: the distn fn is continuous everywhere except 0

Defn ②

the support of a random variable is the set of numbers that are possible values of the random variable.

Defn ③: A random variable is called discrete if the support contains at most a countable number of values. It is called continuous if the distn fn is continuous and is differentiable everywhere with the possible exception of a countable number of values. It is called mixed if it is not discrete and is continuous everywhere with the exception of at least one value and at most a countable number of values.

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Ex ① For each of the four models, determine the support and indicate which type of Random Variable it is.

Ans:

For model ① the support is  $[0, 100]$ , the R.V. is Continuous

For model ② " " "  $[0, \infty)$ , " " " "

For model ③ " " "  $\{0, 1, 2, 3, 4\}$ , " " is discrete

For model ④ " "  $[0, \infty)$ , the Random Variable is mixed. The distribution fn in model ④ is

Continuous everywhere except at 0, where it jumps  $F(0) = 0.7$ .

Most of the probability is at zero (0.7) because in most of the year nothing is paid. The remaining 0.3 of probability is distributed over positive values.