

11.3 Grouped Data

The likelihood function for grouped data is given by

$$L(\theta) = \prod_{j=1}^k [F(c_j|\theta) - F(c_{j-1}|\theta)]^{n_j}$$

and its logarithm is

$$l(\theta) = \sum_{j=1}^k n_j \ln [F(c_j|\theta) - F(c_{j-1}|\theta)]$$

where c_0 is the smallest possible observation (often zero) and c_k is the largest possible observation (often infinity), and n_j be the number of observations in the interval $(c_{j-1}, c_j]$

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Using Data set C from Ch 10, reproduced here as Table 11.2, determine the maximum likelihood estimate for an exponential dist.

Ans:

Table 11.2 Data Set C

Payment Range	Number of payments
0 - 7,500	99
7,500 - 17,500	42
17,500 - 32,500	29
32,500 - 67,500	28
67,500 - 125,000	17
125,000 - 300,000	9
over 300,000	3

$$X \sim \text{exp}(\theta)$$

$$\rightarrow F(x) = 1 - e^{-x/\theta}$$

The loglikelihood fn is

$$l(\theta) = 99 \ln [F(7500) - F(0)] + 42 \ln [F(17500) - F(7500)] + \dots + 3 \ln [1 - F(300,000)]$$



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$$l(\theta) = 99 \ln(1 - e^{-7500/\theta}) + 42 \ln(e^{-7500/\theta} - e^{-17500/\theta}) + \dots$$
$$+ 3 \ln e^{-300,000/\theta}$$

* We can find the maximum likelihood estimate of θ and the value of the log likelihood function by using **MATLAB**.
See the program in the attached file (UEx 11.2 p. 232 Ex 11.3 p. 235 L2a-21.v)

H.W. Assignment N # (3) Mid II pb 11.1 p. 232