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lecture 23

Mathematical Statistics

ch 11 p. 240

لا يكتب في هذا الهامش

Ex 11.6 p. 240

For Data Set A given below, assume that the seven drivers with five or more accidents all had exactly five accidents. Determine the maximum likelihood estimate for a poisson distribution and for a binomial distribution with $m=8$.

Data Set A

Number of accidents (x)	Number of drivers (n_x)
0	81,714
1	11,306
2	1,618
3	250
4	40
5 or more	7

$n = 94,935$

Ans:

In general, for a discrete distribution with complete data, the likelihood function is

$$L(\theta) = \prod_{j=1}^{\infty} [p(x_j|\theta)]^{n_j}$$

where x_j is one of the observed values, $p(x_j|\theta)$ is its corresponding prob., and n_x is the number of times (عدد مرات) x that was observed in the sample.

$n_x \rightarrow$ frequency (تكرار)

* For poisson distribution

$$L(\lambda) = \prod_{x=0}^{\infty} \left(\frac{e^{-\lambda}\lambda^x}{x!}\right)^{n_x} = \prod_{x=0}^{\infty} \frac{e^{-n_x\lambda}\lambda^{x n_x}}{(x!)^{n_x}}$$

$$\Rightarrow \ell(\lambda) = \sum_{x=0}^{\infty} (-n_x\lambda + x n_x \ln\lambda - n_x \ln x!)$$

$$\ell(\lambda) = -\lambda\sum_{x=0}^{\infty} n_x + \ln\lambda \sum_{x=0}^{\infty} x n_x - \sum_{x=0}^{\infty} n_x \ln x!$$

$$\ell(\lambda) = -n\lambda + n\bar{x}\ln\lambda - \sum_{x=0}^{\infty} n_x \ln x!$$

$$\therefore \ell'(\lambda) = -n + \frac{n\bar{x}}{\lambda}$$

mean $= \dfrac{\sum_i x_i f_i}{\sum_i f_i}$

To get $\hat{\lambda}$ equating $\ell'(\lambda)$ with zero

$$\frac{n\bar{x}}{\lambda} - n = 0 \Rightarrow \boxed{\hat{\lambda} = \bar{x}} \quad (1)$$

$$\Rightarrow \bar{x} = \frac{\sum_x x n_x}{\sum_x n_x} = \frac{\sum_x x n_x}{n}$$

* For the binomial distribution $m, q \Rightarrow X \sim Binomial(m,q)$

$$L(q) = \prod_{x=0}^{m}\left[\binom{m}{x} q^{x}(1-q)^{m-x}\right]^{n_x}$$

$$L(q) = \prod_{x=0}^{m} \frac{(m!)^{n_x} q^{x n_x} (1-q)^{(m-x)n_x}}{(x!)^{n_x}[(m-x)!]^{n_x}}$$

$$\ell(q) = \sum_{x=0}^{m}[\cancel{n_x \ln m!} + x n_x \ln q + (m-x) n_x \ln(1-q)]$$

$$- \sum_{x=0}^{m}[\cancel{n_x \ln x!} + \cancel{n_x \ln (m-x)!}]$$

$$\ell'(q) = \sum_{x=0}^{m}\left[\frac{x n_x}{q} - \frac{(m-x)n_x}{1-q}\right]$$

$$\sum_x x n_x = n\bar{x}$$

$$\Rightarrow \ell'(q) = \frac{n\bar{x}}{q} - \frac{mn - n\bar{x}}{1-q} = 0$$

$$\therefore \frac{n\bar{x} - \cancel{n\bar{x}q} - mnq + \cancel{nq\bar{x}}}{q(1-q)} = 0$$

$$\cancel{n}\bar{x} = m\cancel{n}q$$

$$\therefore \boxed{\hat{q} = \frac{\bar{x}}{m}} \quad (2)$$

$$\therefore \bar{x} = \frac{\sum_x x n_x}{n}$$

$$\bar{x} = [81,714(0) + 11,306(1) + 1,618(2) + 250(3) + 40(4) + 7(5)] / 94,935$$

$$\therefore \bar{x} = 0.16313$$

(1) $\Rightarrow \hat{\lambda} = \bar{x} = 0.16313$ for poisson distn

, and

(2) $\Rightarrow \hat{q} = \frac{\bar{x}}{m} = \frac{0.16313}{8} = 0.02039$ for Binomial distn