Lecture (28)

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~ Greatest Accuracy Credibility ~

* Introduction

We return to the basic problem. For a particular policyholder, we have observed n exposure units of past claims $X = (X_1, X_2, \dots, X_n)^T$. We have a manual rate μ (we no longer use M for the manual rate) applicable to this policyholder, $E(X) = \bar{X} = n^{-1}(X_1 + X_2 + \dots + X_n)$ could be quite different from μ. This difference raises the question of whether next year's net premium (per exposure unit) should be based on μ, on $\bar{X}$, or on a combination of the two.

To proceed, let us assume that the risk level of each policyholder in the rating class may be characterized by a risk parameter θ, where θ varies by policyholder. $\pi(\theta) = pr(\Theta \le \theta)$, $\pi(\theta)$ represents the probability that a policyholder picked at random from the rating class has a risk parameter less than or equal to θ.

* EX 17.2 P. 403

There are two types of drivers. Good drivers make up 75% of the population and in one year have zero claims with probability 0.7, one claim with probability 0.2 and two claims with probability 0.1. Bad drivers make up the other 25% of the population and have zero, one, or two claims with probabilities 0.5, 0.3 and 0.2, respectively. Describe this process and how it relates to an unknown risk parameter.

Ans:

When a driver buys our insurance policy, we do not know if the individual is a good or bad driver. So the risk parameter Θ can be one of two values, $\Theta = G$ for good drivers and $\Theta = B$ for bad drivers. The probability model for the number of claims, and risk parameter Θ is given in the following Table.

x	$pr(X=x \mid \Theta=G)$	$pr(X=x \mid \Theta=B)$	θ	$pr(\Theta=\theta)$
0	0.7	0.5	G	0.75
1	0.2	0.3	B	0.25
2	0.1	0.2		

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The Bayesian Methodology

Let the claims or losses X_js, $j=1,2,\dots,n,n+1$ which are identically distributed have a conditional probability function

$$f_{X_j|\Theta}(x_j|\theta), \quad j=1,2,\dots,n,n+1$$

We are interested in the conditional distribution of X_{n+1} given $\Theta=\theta$, in order to predict the claims experience X_{n+1} of the same policyholder whose value of θ has been assumed not to have changed.
Also, we are interested to calculate the conditional distn of X_{n+1} given $X=x$ which is called predictive distn. The predictive distn is the relevant distn for risk analysis, management, and decision making.

For X_js that are independent conditional on $\Theta=\theta$, we have

$$f_{X,\Theta}(x,\theta) = f(x_1,\dots,x_n|\theta)\,\pi(\theta)$$

$$= \left[\prod_{j=1}^{n} f_{X_j|\Theta}(x_j|\theta)\right]\pi(\theta) \quad (1)$$

∴ The joint distribution of X is thus the marginal distn given by

$$f_X(x) = \int \left[\prod_{j=1}^{n} f_{X_j|\Theta}(x_j|\theta)\right]\pi(\theta)\,d\theta \quad (2)$$

, the conditional density of X_{n+1} given $X=x$ is the joint distn of $(X_1,\dots,X_{n+1})$ divided by $f_X(x)$

$$f_{X_{n+1}|X}(x_{n+1}|x) = \frac{1}{f_X(x)}\int\left[\prod_{j=1}^{n+1} f_{X_j|\Theta}(x_j|\theta)\right]\pi(\theta)\,d\theta \quad (3)$$

and the posterior density of Θ given X is

$$\pi_{\Theta|X}(\theta|x) = \frac{f_{X,\Theta}(x,\theta)}{f_X(x)} = \frac{1}{f_X(x)}\left[\prod_{j=1}^{n} f_{X_j|\Theta}(x_j|\theta)\right]\pi(\theta)$$

For posterior distn (4)

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(4) $\Rightarrow \quad \left[\prod_{j=1}^{n} f_{X_j|\Theta}(x_j|\theta)\right]\pi(\theta) = \pi_{\Theta|X}(\theta|X)\, f_X(X)$ (5)

Substitute (5) in (3), we get

$$f_{X_{n+1}|X}(x_{n+1}|X) = \frac{1}{\cancel{f_X(X)}} \int \frac{\cancel{f_X(x)}\, \pi_{\Theta|X}(\theta|X)}{\cancel{\pi(\theta)}}\, f_{X_{n+1}|\Theta}(x_{n+1}|\theta)\, \cancel{\pi(\theta)}\, d\theta$$

$$\therefore \quad f_{X_{n+1}|X}(x_{n+1}|X) = \int f_{X_{n+1}|\Theta}(x_{n+1}|\theta)\, \pi_{\Theta|X}(\theta|X)\, d\theta \qquad (6)$$

For predictive distn

i.e the Conditional distn of X_{n+1} given X can be Considered as a mixture distn, with the mixing distn being the posterior distn $\pi_{\Theta|X}(\theta|X)$.

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(EX 17.2 Continued) For a particular policyholder, suppose that we have observed $x_1=0$ and $x_2=1$. Determine the predictive distn of $X_3|X_1=0, X_2=1$ and the posterior distn of $\Theta|X_1=0, X_2=1$

Ans:

For the posterior distn

(4) $\Rightarrow$ the posterior probabilities are given by

$$\pi(G|0,1) = \frac{f(0|G)\, f(1|G)\, \pi(G)}{f(0,1)}$$

where $f_X(0,1) = \sum_{\theta} f_{X_1|\Theta}(0|\theta)\, f_{X_2|\Theta}(1|\theta)\, \pi(\theta)$ From (1)

$$= 0.7(0.2)(0.75) + 0.5(0.3)(0.25) = 0.1425$$

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$$\therefore \pi(G|0,1) = \frac{0.7(0.2)(0.75)}{0.1425} = 0.736842$$

Similarly, $\pi(B|0,1) = \frac{0.5(0.3)(0.25)}{0.1425} = 0.263158$

* For the predictive distⁿ

⑥ ⇒ The predictive probabilities are given by

Note that
$X = (X_1, X_2)^T$
$x = (0,1)^T$

$$f_{X_3|X}(0|0,1) = \sum_{\theta} f(0|\theta)\,\pi(\theta|0,1)$$
$$= f(0|G)\,\pi(G|0,1) + f(0|B)\,\pi(B|0,1)$$
$$= 0.7(0.736842) + 0.5(0.263158) = 0.647368$$

$$f_{X_3|X}(1|0,1) = \sum_{\theta} f(1|\theta)\,\pi(\theta|0,1)$$
$$= f(1|G)\,\pi(G|0,1) + f(1|B)\,\pi(B|0,1)$$
$$= 0.2(0.736842) + 0.3(0.263158) = 0.226316$$

and
$$f_{X_3|X}(2|0,1) = \sum_{\theta} f(2|\theta)\,\pi(\theta|0,1)$$
$$= f(2|G)\,\pi(G|0,1) + f(2|B)\,\pi(B|0,1)$$
$$= 0.1(0.736842) + 0.2(0.263158) = 0.126316$$

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