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Lecture (29)

Ch17: "Greatest Accuracy Credibility"

* Bayesian premium

To return to the original problem, we have observed $X = x$ for a particular policyholder and we wish to predict X_{n+1} (or its mean)

Defn The Bayesian premium is the expected value of the hypothetical means, with expectation taken over the posterior distribution $\pi_{\Theta|X}(\theta|x)$.

$\therefore$ i.e $$E(X_{n+1}|X=x) = \int \mu_{n+1}(\theta)\, \pi_{\Theta|X}(\theta|x)\, d\theta \quad (7)$$

we can prove this relation as follows:

$$\because E(X_{n+1}|X=x) = \int x_{n+1}\, f_{X_{n+1}|X}(x_{n+1}|x)\, dx_{n+1}$$

From (6) of the predictive distn

$$\therefore E(X_{n+1}|X=x) = \int x_{n+1}\left[\int f_{X_{n+1}|\Theta}(x_{n+1}|\theta)\, \pi_{\Theta|X}(\theta|x)\, d\theta\right] dx_{n+1}$$

$$= \int\left[\int x_{n+1}\, f_{X_{n+1}|\Theta}(x_{n+1}|\theta)\, dx_{n+1}\right] \pi_{\Theta|X}(\theta|x)\, d\theta$$

$$\therefore E(X_{n+1}|X=x) = \int \mu_{n+1}(\theta)\, \pi_{\Theta|X}(\theta|x)\, d\theta$$

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EX 17.9 p. 412

(EX 17.7 Continued) Determine the Bayesian premium

Ans:

By using (7), we can write

$$E(X_3|0,1) = \mu_3(G)\,\pi(G|0,1) + \mu_3(B)\,\pi(B|0,1)$$

where $\mu_3(G)$ and $\mu_3(B)$ are the hypothetical means given by

$\mu_3(G) = 0(0.7)+1(0.2)+2(0.1) = 0.4$ and $\mu_3(B) = 0(0.5)+1(0.3)+2(0.2) = 0.7$

Revise EX 17.2

& $\because \pi(G|0,1) = 0.736842$ and $\pi(B|0,1) = 0.263158$ Revise EX 17.7

$\therefore E(X_3|0,1) = 0.4(0.736842) + 0.7(0.263158) = 0.4789$

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- The Bühlmann Model

The simplest credibility model, the Bühlmann model specifies that, for each policyholder (Conditional on Θ), past losses $X_1, X_2, \ldots, X_n$ have the same mean and variance and are i.i.d. (identical distributed) conditional on Θ.

Thus, define

$$\mu(\theta) = E(X_j \mid \Theta = \theta) \qquad (1)$$

and

$$v(\theta) = Var(X_j \mid \Theta = \theta) \qquad (2)$$

where $\mu(\theta)$ is called the hypothetical mean, whereas $v(\theta)$ is called the process variance.

Consequently, the expected value of the hypothetical means μ, the expected value of the process variance v and the variance of the hypothetical means a can be defined as follows:

$$\mu = E[\mu(\Theta)] \quad \text{(Collective premium)} \qquad (3)$$

$$v = E[v(\Theta)] \qquad (4)$$

and

$$a = Var[\mu(\Theta)] \qquad (5)$$

Thus the Bühlmann credibility premium is given by

$$P_c = Z\bar{X} + (1-Z)\mu \qquad (6)$$

where $Z = \dfrac{n}{n+k}$ (Bühlmann credibility factor) (7)

and

$$k = \frac{v}{a} = \frac{E[Var(X_j \mid \Theta)]}{Var[E(X_j \mid \Theta)]} \qquad (8)$$

Note that the Bühlmann credibility premium that is given by (6) is a weighted average of the sample mean $\bar{X}$ and the collective premium μ.

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EX 17.14 p.420

(EX 17.9 Continued) Determine the Bühlmann estimate of $E(X_3|0,1)$.

Ans: We have,

$\mu(G) = E(X_j|G) = 0.4$, $\mu(B) = E(X_j|B) = 0.7$

$\pi(G) = 0.75$, $\pi(B) = 0.25$

Revise EX 17.2 & EX 17.9

$\therefore \mu = E[\mu(\Theta)]$

$\therefore \mu = \sum_{\theta} \mu(\theta)\,\pi(\theta)$

$\mu = \mu(G)\pi(G) + \mu(B)\pi(B)$

$\therefore \mu = 0.4(0.75) + 0.7(0.25) = 0.475$,

$\because a = Var[\mu(\Theta)]$

$\therefore a = \sum_{\theta} [\mu(\theta)]^2 \pi(\theta) - \mu^2$

$= \mu^2(G)\pi(G) + \mu^2(B)\pi(B) - (0.475)^2$

$\therefore a = 0.16(0.75) + 0.49(0.25) - (0.475)^2 = 0.016875$

For the process variance, $v(\theta) = Var(X_j|\Theta=\theta)$

$\Rightarrow v(G) = Var(X_j|G)$

$\therefore v(G) = 0^2(0.7) + 1^2(0.2) + 2^2(0.1) - (0.4)^2 = 0.44$

$v(B) = Var(X_j|B)$

$\therefore v(B) = 0^2(0.5) + 1^2(0.3) + 2^2(0.2) - (0.7)^2 = 0.61$,

$\because v = E[v(\Theta)]$

$\therefore v = \sum_{\theta} v(\theta)\pi(\theta)$

$v = v(G)\pi(G) + v(B)\pi(B)$

$\therefore v = 0.44(0.75) + 0.61(0.25) = 0.4825$

$\therefore k = \frac{v}{a} = \frac{0.4825}{0.016875} = 28.5926$ (from (8))

$\bar{X} = \frac{0+1}{2} = 0.5$, $\mu = 0.475$

$\Rightarrow Z = \frac{n}{n+k} = \frac{2}{2+28.5926} = 0.0654$ by using (7) and by using formula (6), we get the Bühlmann estimate of $E(X_3|0,1)$

$E(X_3|0,1) = 0.0654(0.5) + 0.9346(0.475) = 0.4766$

the expected next value

which is very closed to the result of Bayesian premium estimate See EX 17.9