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Lecture 4

Ch 3

Basic Distributional Quantities

Defn For a given value of d with $pr(X>d)>0$, the excess loss variable is

$$Y^P = X - d, \quad \text{given that } X>d$$

Its expected value,

$e_X(d) = e(d) = E(Y^P) = E(X-d \mid X>d)$ is called the mean excess loss function.

Notes:
- when X is a payment variable, the mean excess loss is the expected amount paid, given that there has been a payment in excess of a deductible of d.
- when X is the age at death, the mean excess loss is the expected remaining time until death, given that the person is alive at age d.

⇛ The kth moment of the excess loss variable is determined from

$$e_X^k(d) = \frac{\int_d^\infty (x-d)^k f(x)\,dx}{1-F(d)} \quad \text{for cont. r.v} \quad (1)$$

$$= \frac{\sum_{x_j>d} (x_j-d)^k p(x_j)}{1-F(d)} \quad \text{for discrete r.v} \quad (2)$$

The 1st moment in continuous case is of the form

$$e_X(d) = \frac{\int_d^\infty (x-d) f(x)\,dx}{1-F(d)}$$

$$e_X(d) = \frac{-\int_d^\infty (x-d)\,dS(x)}{1-F(d)} \quad (3)$$

$\int u\,dv = uv - \int v\,du$

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$$e_X(d) = \frac{\cancel{-(x-d)S(x)\Big|_d^{\infty}}^{\,0} + \int_d^{\infty} S(x)\,dx}{S(d)}$$

$$\therefore\; e_X(d) = E(Y^P) = \frac{\int_d^{\infty} S(x)\,dx}{S(d)} \qquad (3)'$$

Note that: the excess loss variable $Y^P = X - d$ is also called left truncated and shifted variable.

Def$\underline{n}$

The left censored and shifted variable is

$$Y^L = (X-d)_+ = \begin{cases} 0, & X \le d \\ X-d, & X > d \end{cases}$$

$\Rightarrow$ The kth moment of the left censored and shifted variable is determined from

$$E\left[(X-d)_+^k\right] = \int_d^{\infty} (x-d)^k f(x)\,dx \qquad \text{for Cont. r.v} \quad (4)$$

$$= \sum_{x_j > d} (x_j - d)^k p(x_j) \qquad \text{// discrete r.v} \quad (5)$$

(1), (2), (4), (5)

$$\Rightarrow E\left[(X-d)_+^k\right] = e^k(d)\,[1 - F(d)]$$

$$\text{or} \quad E[Y^L] = E(Y^P)\,[1 - F(d)] \qquad (6)$$

Note: Y^P this variable includes payment per payment

Y^L " " " payment per loss

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- To show the difference between the excess loss variable and the left censored and shifted variable see Fig 3.2 and Fig 3.3 p.24

3.4 p.23 EX An automobile insurance policy with no coverage modifications has the following possible losses, with probabilities in parentheses:
100(0.4), 500(0.2), 1000(0.2), 2500(0.1), and 10,000(0.1).
Determine the probability mass functions and expected values for the excess loss and left censored and shifted variables, where the deductible is set at 750.

Ans:

prob. of exceeding the deductible

$= pr(X > 750) = 1 - F(750)$, $\boxed{d = 750}$

$= 0.2 + 0.1 + 0.1 = 0.4$

* For $Y^P = X - d$ excess loss variable

$x_i - d$:	250	1750	9250
$\frac{p(x_j)}{1-F(d)}$:	0.2/0.4	0.1/0.4	0.1/0.4
	0.5	0.25	0.25

$$e_X(d) = E(Y^P) = E(X - d \mid X > d)$$
$$= 250(0.5) + 1750(0.25) + 9250(0.25)$$
$$= \boxed{2,875}$$

- For $Y^L = (X - d)_+$ left censored and shifted variable $\Rightarrow (X - d)_+ = 0 \quad X \leq 750$, $(X - d)_+ = X - 750 \quad X > 750$

$x_j - d$:	0	250	1750	9250
$p(x_j)$:	0.6	0.2	0.1	0.1

$$E(Y^L) = E[(X - d)_+]$$
$$= 0(0.6) + 250(0.2) + 1750(0.1) + 9250(0.1)$$
$$= \boxed{1150}$$

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Not that

$$E\left[(X-d)_+^k\right] = e^k(d)\left[1-F(d)\right]$$

$$\Rightarrow E\left[(X-d)_+\right] = e(d)\left[1-F(d)\right] = e(d)S(d) \quad (7)$$

excess loss variable

left censored and shifted variable

In pervious Example, clearly

$$1150 = 0.4(2875)$$

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