Lecture 7

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Applications

* pb 3.21 p.30

A portfolio contains 16 independent risks, each with a gamma distribution with parameters $\alpha = 1$ and $\theta = 250$. Give an expression using the incomplete gamma function for the probability that the sum of the losses exceeds 6000. Then approximate this probability using the central limit theorem.

solved in Lecture 6

Hint For $X \sim \text{gamma}(\underset{\text{scale parameter}}{\theta}, \underset{\text{shape parameter}}{\alpha})$, $E(X) = \theta\alpha$, $Var(X) = \alpha\theta^2$

• pb 3.22 p.30

The severities of individual claims have a pareto distribution with parameters $\alpha = 8/3$ and $\theta = 8000$. Use the central limit theorem to approximate the probability that the sum of 100 independent claims will exceed 600,000.

Ans:

$$X \sim \text{pareto}(\overset{\alpha}{8/3}, \overset{\theta}{8{,}000})$$

kth moment, $E(X^k) = \dfrac{\theta^k \, k!}{(\alpha-1)\cdots(\alpha-k)}$ Revise p.465

$$\mu = E(X) = \frac{\theta}{\alpha - 1}$$

$$\therefore \mu = E(X) = \frac{8000}{8/3 - 1} = \frac{3}{5}(8{,}000) = 4{,}800$$

$$E(X^2) = \frac{\theta^2 \, 2!}{(\alpha-1)(\alpha-2)} = \frac{2(8000)^2}{5/3\,(2/3)} = 115{,}200{,}000$$

$$Var(X) = E(X^2) - \mu^2 = 115{,}200{,}000 - (4{,}800)^2$$
$$= 92160000$$

For the sum of random variables

$$S_k = X_1 + X_2 + \cdots + X_k \quad \text{where } X_1, X_2, \ldots, X_k \text{ are independent r.v.s}$$

$$E(S_{100}) = 100(4800) = 480{,}000$$ By using Central limit theorem

Also,

$$Var(S_{100}) = 100 \times 92160000$$
$$= 9216000000$$

$\Rightarrow$ standard deviation for the sum S_{100} is

$$\sqrt{Var(S_{100})} = 96{,}000$$

$$pr(S_{100} > 600{,}000)$$
$$= 1 - \Phi\left(\frac{600{,}000 - 480{,}000}{96{,}000}\right)$$
$$= 1 - \Phi(1.25) \simeq 0.106$$

- By using MATLAB

$$pr(S_{100} > 600{,}000) = 1 - \text{normcdf}(\overset{x}{600{,}000}, \overset{\mu}{480{,}000}, \overset{\sigma}{96{,}000})$$
$$= 0.1056$$

Note: pb 3.23 p.31 H.W

* pb 3.24 p.31

A sample of 1000 health insurance contracts on adults produced a sample mean of 1300 for the annual benefits paid with a standard deviation of 400. It is expected that 2500 contracts will be issued next year. Use the central limit theorem to estimate the probability that benefit payments will be more than 101 % of the expected amount.

Ans: # Contracts = 2500 , X for benefit payments

The sum of 2500 contracts has an approximate normal dist.n with mean $2500(1300) = 3{,}250{,}000$ and standard deviation $\sqrt{2500(400)^2} = 20{,}000$

Ans:

$$pr(X > 1.01(3{,}250{,}000))$$?? المطلوب

$$= pr(X > 3{,}282{,}500)$$

$\Rightarrow$

$$pr\left(Z > \frac{3{,}282{,}500 - 3{,}250{,}000}{20{,}000}\right)$$
$$= pr(Z > 1.625)$$
$$= 0.052$$

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