Differential Equations of Order One

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1. Family of Curves
2. Existence of a Unique Solution
3. Separable Equations

Example:

$$(x-c)^2+(y-c)^2=2c^2. \tag{1}$$

represents a family of circles with their centers on $y=x$. If we assume that c in the equation (1) is arbitrary constant, then by using the elimination of arbitrary constant, then result equation is called *differential equation of the family of curve (1)*.

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So we get

$$\frac{x^2+y^2}{x+y}=2c, y\neq -x.$$

Q(1): Find a differential equation satisfied by the family of parabolas having their virtices at the origin and their foci (focus) on the y-axis.
Q(2): Find the differential equation of the family of circles having their centers on the y-axis.

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Q: Find the largest region of the xy-plane for which the following initial value problems have unique solutions:

(a) $\sqrt{x^2-4}\,y' = 1+\sin(x)\ln(y)$, with initial condition $y(3) = 4$.

(b) $\ln(x-2).\frac{dy}{dx} = \sqrt{y-2}$, with initial condition $y(\frac{5}{2}) = 4$.

(c) $\sqrt{\frac{x}{y}}\,y' = \cos(x+y); y \neq 0$, with initial condition $y(1) = 1$.

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$$F(x)dx + G(y)dy = 0, \tag{3}$$

the variables separated here and we can find a solution immediately.

Q(i): Find a solution for each of the following:

(a) $2x(y^2+y)dx + (x^2-1)ydy = 0; y \neq 0.$

(b) $(xy+1)dx = (x^2y^2+x^2+y^2+1)dy.$

Q(ii): Solve the following IVP

$$e^y\frac{dy}{dx} = \cos(2x) + 2e^y\sin^2(x) - 1; y(\frac{\pi}{2}) = \ln(2).$$