MATH107 Vectors and Matrices

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3-5/11/16

Notations and Algebra Matrices

1- Matrix: A matrix is rectangular array of objects, written in rows and columns. These objects can be numbers or functions. We write a matrix as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \text{or} \quad \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

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Examples:

(i) $\begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$ is 2×2 matrix.

(ii) $\begin{bmatrix} 0 & 1 & 2 \\ 9 & 7 & 3 \\ 3 & 5 & 1 \end{bmatrix}$ is 3×3 matrix.

(ii) $\begin{bmatrix} 1 & x & x^2 & e^x \\ x+1 & \sin(x) & -x & 8 \\ 2^x & 0 & 15 & (x^3+5)^{100} \end{bmatrix}$ is 3×4 matrix.

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Exercise: Can we find a matrix which is square, row and column at the same time??.

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Example: $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

7- Diagonal Matrix: A square matrix with all its non-diagonal entries zero is called diagonal matrix. Example: $\begin{bmatrix} 500 & 0 & 0 \\ 0 & 10975^{13} & 0 \\ 0 & 0 & 2^{2^2} \end{bmatrix}$

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9- Transpose of a Matrix: A transpose of a matrix is obtained by interchanging between rows and corresponding columns. The transpose of a matrix A is denoted by A^t. Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad A^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

*** Properties of the Transpose of a Matrix:**

1. $(A^t)^t = A$.
2. $(AB)^t = B^t A^t$.
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Example:

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11- Skew-Symmetric Matrix: A square matrix is skew-symmetric , if $A^t = -A$. Example:

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -5 \\ 3 & 5 & 0 \end{bmatrix}, \quad A^t = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}, \quad A^t = -A.$$

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12- Equality of matrices: Two matrices are equal, if they have the same size and the corresponding entries are equal.

Example: Write down the system of equations, if matrices A and B are equal

$$A = \begin{bmatrix} x-2 & y-3 \\ x+y & z+3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3+z \\ z & y \end{bmatrix}.$$

Solution: First we note that they the same size 2×2. If $A = B$, then:

$$\begin{aligned} x \qquad\qquad &= 3 \\ y - z &= 6 \\ x + y - z &= 0 \\ -y + z &= -3. \end{aligned}$$

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Example: Find $A+B$, where $A=\begin{bmatrix}2 & 1\\ 3 & 4\\ 4 & 5\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ 2 & -5\\ 3 & 4\end{bmatrix}$.

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$$A+B=\begin{bmatrix}2+1 & 1-1\\ 3+2 & 4-5\\ 4+3 & 5+4\end{bmatrix}=\begin{bmatrix}3 & 0\\ 5 & -1\\ 7 & 9\end{bmatrix}.$$

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Scalar Multiplication: If a matrix multiplied by a scalar α, then each entry is multiplied by scalar α. Examples:

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 1 \\ 4 & 1 & 4 \end{bmatrix}, \quad 2A = \begin{bmatrix} 4 & 6 & 4 \\ 2 & 4 & 2 \\ 8 & 2 & 8 \end{bmatrix}, \quad kA = \begin{bmatrix} 2.k & 3.k & 2.k \\ 1.k & 2.k & 1.k \\ 4.k & 1.k & 4.k \end{bmatrix}.$$

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Matrix Multiplication: Let A be a $n \times m$ matrix and B is a $k \times p$. Then the necessary condition for AB to be exists is $m = k$ (for BA, we must have $p = n$). Note that the multiplication is not abelian i.e. $AB \neq BA$.

Example: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} x \\ y \end{pmatrix}$. Then,

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

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Example: Let A be the matrix

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Compute $A^3, A^{-3}, A^2 - 2A + I$.

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Elementary Matrix:

An $n \times n$ matrix is called *elementary matrix* if it can be obtained from $n \times n$ identity matrix by performing a single row operation.

Example:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\mathbf{-3R_3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix} = E_1$$

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Note: When A is multiplied from the left by elementary matrix E, the effect is same as to perform an elementary row operation on A. Let A be a 3×4 matrix,

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix}$$

and E be 3×3 elementary matrix obtained by row operation $3R_1 + R_3$ from an identity matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}, \mathbf{3R_1 + R_3}$$

Note: When A is multiplied from the left by elementary matrix E, the effect is same as to perform an elementary row operation on A. Let A be a 3×4 matrix,

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix}$$

and E be 3×3 elementary matrix obtained by row operation $3R_1 + R_3$ from an identity matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}, \mathbf{3R_1 + R_3}$$

Method for finding inverse of a Matrix

To find an inverse row of matrix A, we perform a sequence of elementary row operations that reduce.

$$\left[A \middle| I\right] \text{ to } \left[I \middle| A^{-1}\right]$$

Example 1:

Find inverse matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ by using Elementary matrix method.

solution:

$$\left[A \middle| I\right] = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \xrightarrow{\mathbf{-2R_1+R_2}} \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] \xrightarrow{\mathbf{-R_2}} \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

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Example 3:

$x_1 + 3x_2 + x_3 = 4$
$2x_1 + 2x_2 + x_3 = -1$
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$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

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solution:

$$\left[A\middle|I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbf{R_2-2R_1, R_3-2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\mathbf{4R_3-3R_2}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

$$\xrightarrow{\mathbf{R_1+R_3, -R_2+R_3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 4 & 0 & 0 & -4 & 4 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

Example 3:

$x_1 + 3x_2 + x_3 = 4$

$2x_1 + 2x_2 + x_3 = -1$

$2x_1 + 3x_2 + x_3 = 3$

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

solution:

$$\left[A \middle| I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbf{R_2 - 2R_1, R_3 - 2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\mathbf{4R_3 - 3R_2}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

$$\xrightarrow{\mathbf{R_1 + R_3, -R_2 + R_3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 4 & 0 & 0 & -4 & 4 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

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solution:

$$\left[A\middle|I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbf{R_2-2R_1, R_3-2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

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solution:

$$\left[A \middle| I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbf{R_2 - 2R_1, R_3 - 2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\mathbf{4R_3 - 3R_2}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

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solution:

$$\left[A \middle| I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbf{R_2 - 2R_1, R_3 - 2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\mathbf{4R_3 - 3R_2}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

$$\xrightarrow{\mathbf{R_1 + R_3, -R_2 + R_3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 4 & 0 & 0 & -4 & 4 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

$$\xrightarrow{\frac{1}{4}\mathbf{R_2}, -\mathbf{R_3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right]$$

$$\xrightarrow{-\mathbf{3R_2}+\mathbf{R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] = \left[I \middle| A^{-1}\right]$$

$$A^{-1} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

Solution is $x_1 = -6$ $x_2 = 4$ $x_3 = -7$

$$\xrightarrow{\frac{1}{4}\mathbf{R_2},-\mathbf{R_3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right]$$

$$\xrightarrow{-3\mathbf{R_2}+\mathbf{R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] = \left[I \middle| A^{-1}\right]$$

$$A^{-1} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

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$$\xrightarrow{-\mathbf{3R_2}+\mathbf{R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] = \left[I \middle| A^{-1}\right]$$

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