# MATH107 Vectors and Matrices 

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## Notations and Algebra Matrices

1- Matrix: A matrix is rectangular array of objects, written in rows and columns. These objects can be numbers or functions. We write a matrix as follows:


2- Size of Matrix: If a matrix $A$ has $m$ rows and $n$ columns, then we say $A$ is " $m$ by $n$ matrix" and we write it as " $m \times n$ "

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(ii) $\left[\begin{array}{lll}0 & 1 & 2 \\ 9 & 7 & 3 \\ 3 & 5 & 1\end{array}\right]$ is $3 \times 3$ matrix.
(ii) $\left[\begin{array}{cccc}1 & x & x^{2} & e^{x} \\ x+1 & \sin (x) & -x & 8 \\ 2^{x} & 0 & 15 & \left(x^{3}+5\right)^{100}\end{array}\right]$ is $3 \times 4$ matrix.

3- Square Matrix: When $m=n$, then the matrix is square matrix. Example $\left[\begin{array}{cc}2 & 0 \\ 3 & -1\end{array}\right]$ is $2 \times 2$ square matrix.

4- Row Matrix: When $m=1$, then the matrix is called row matrix. Example: $\left[\begin{array}{lllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\end{array}\right]$

5- Column Matrix: When $n=1$, then the matrix is called column matrix. Example: $\left[\begin{array}{l}1 \\ 2 \\ 3 \\ 4 \\ 5\end{array}\right]$
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6- Zero Matrix: A zero matrix is a matrix whose all entries are zero. Example: $0=$ 7- Diagonal Matrix: A square matrix with all its non-diagonal entries zero is called diagonal matrix. Example: $\left[\begin{array}{ccc}500 & 0 & 0 \\ 0 & 10975^{13} & 0 \\ 0 & 0 & 2^{2^{2}}\end{array}\right]$

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## interchanging between rows and corresponding columns. The transpose of a matrix $A$ is denoted by $A^{t}$. Example:

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(2) $(A B)^{t}=B^{t} A^{t}$.
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2 & 0 & -5 \\
3 & 5 & 0
\end{array}\right], \quad A^{t}=\left[\begin{array}{ccc}
0 & 2 & 3 \\
-2 & 0 & 5 \\
-3 & -5 & 0
\end{array}\right], \quad A^{t}=-A
$$

## 12- Equality of matrices: Two matrices are equal, if they have the

 same size and the corresponding entries are equal.
## Example: Write down the system of equations, if matrices $A$ and $B$ are

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Solution: First we note that they the same size $2 \times 2$. If $A=B$, then:

$y-z=6$


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| ---: | :--- |
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-y+z & =-3
\end{array}
$$

## 12- Addition of matrices:



$$
A+B=\left[\begin{array}{ll}
2+1 & 1-1 \\
3+2 & 4-5 \\
4+3 & 5+4
\end{array}\right]=\left[\begin{array}{cc}
3 & 0 \\
5 & -1 \\
7 & 9
\end{array}\right]
$$

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## Scalar Multiplication: If a matrix multiplied by a scalar $\alpha$, then each

 entry is multiplied by scalar $\alpha$. Examples:$$
A=\left[\begin{array}{lll}
2 & 3 & 2 \\
1 & 2 & 1 \\
4 & 1 & 4
\end{array}\right], \quad 2 A=\left[\begin{array}{lll}
4 & 6 & 4 \\
2 & 4 & 2 \\
8 & 2 & 8
\end{array}\right], \quad k A=\left[\begin{array}{ccc}
2 . k & 3 . k & 2 . k \\
1 . k & 2 . k & 1 . k \\
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## Matrix Multiplication:

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A B=\left(\begin{array}{ll}
a & b \\
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\end{array}\right)\binom{x}{y}=\binom{a x+b y}{c x+d y}
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$B A$ is not exists!!

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## Solution:

$$
\begin{aligned}
& A \\
& 2 \times 3
\end{aligned} \quad \begin{gathered}
B \\
3 \times 4
\end{gathered} \quad C \text { 2×4 }
$$

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\left.\begin{array}{cccc}
A & \times & B & = \\
2 \times 3 & C \\
3 \times 4 & & 2 \times 4
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## Solution:

$$
\begin{array}{lc}
A & \times \\
2 \times 3 & B \\
2 \times 4 & C \\
2 \times 4
\end{array}
$$

$$
C=A B=\left(\begin{array}{llll}
c_{11} & c_{12} & c_{13} & c_{14} \\
c_{21} & c_{22} & c_{23} & c_{24}
\end{array}\right)
$$

$$
c_{11}=1(4)+2(0)+4(2)=12
$$

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\begin{gathered}
c_{11}=1(4)+2(0)+4(2)=12 \\
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$$
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c_{11}=1(4)+2(0)+4(2)=12, \\
c_{12}=1(1)+2(-1)+4(7)=27, \\
c_{13}=1(4)+2(3)+4(5)=30,
\end{gathered}
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$$

$$
c_{22}=2(1)+6(-1)+0(7)=-4
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& c_{11}=1(4)+2(0)+4(2)=12 \\
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Example: Find $A B$, where $A=\left(\begin{array}{lll}1 & 2 & 4 \\ 2 & 6 & 0\end{array}\right), B=\left(\begin{array}{cccc}4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2\end{array}\right)$.

## Solution:

$$
\begin{array}{lccl}
A & \times & B & = \\
2 \times 3 & & 3 \times 4
\end{array}
$$

$$
C=A B=\left(\begin{array}{llll}
c_{11} & c_{12} & c_{13} & c_{14} \\
c_{21} & c_{22} & c_{23} & c_{24}
\end{array}\right)
$$

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0 & -1 & 3 & 1 \\
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\end{array}\right)=\left(\begin{array}{cccc}
12 & 27 & 30 & 13 \\
8 & -4 & 26 & 12
\end{array}\right) .
$$

## Inverse of a $2 \times 2$ Matrix

The inverse of a $2 \times 2$ matrix $A$ is a $2 \times 2$ matrix $A^{-1}$, such that
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Example: The inverse of $A=\left[\begin{array}{ll}2 & 3 \\ 5 & 4\end{array}\right]$ is $A^{-1}=\frac{1}{-7}\left[\begin{array}{cc}4 & -3 \\ -5 & 2\end{array}\right]$

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## Properties of inverse

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(1) $A^{-1} A=A A^{-1}=I$.
(2) If $A$ and $B$ are invertible matrices of the same size, then $A B$ is also invertible and $(A B)^{-1}=B^{-1} A^{-1}$.

## Power of a Matrix:

(1) $A^{0}=I$.

$A^{r} A^{s}=A^{r+s}$$\left(\Delta^{r}\right)^{s}-\Delta^{r s}$
(3) $\left(A^{-1}\right)^{-1}=A$.


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(9) $A^{r} A^{s}=A^{r+s}$.
(0) $\left(A^{r}\right)^{s}=A^{r s}$.
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(1) $\left(A^{n}\right)^{-1}=\left(A^{-1}\right)^{n}, n \geq 0$.

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(9) $A^{r} A^{s}=A^{r+s}$.
(1) $\left(A^{r}\right)^{s}=A^{r s}$.
(1) $\left(A^{-1}\right)^{-1}=A$.
(3) $\left(A^{n}\right)^{-1}=\left(A^{-1}\right)^{n}, n \geq 0$.
(3) $(k A)^{-1}=\frac{1}{k} A^{-1}$.

## Example: Let $A$ be the matrix

$$
\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right] .
$$

Compute $A^{3}, A^{-3}, A^{2}-2 A+I$.

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\begin{equation*}
A^{2}=A A \tag{array}
\end{equation*}
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4 & 1
\end{array}\right]=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
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12 & 1
\end{array}\right] \\
& A^{3}=A^{2} A=\left[\begin{array}{ll}
4 & 0
\end{array}\right]\left[\begin{array}{l}
2
\end{array}\right.
\end{aligned}
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4
\end{array}\right.
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4 & 1
\end{array}\right]=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right] \\
& A^{3}=A^{2} A=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]=\left[\begin{array}{c}
8 \\
28
\end{array}\right.
\end{aligned}
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Compute $A^{3}, A^{-3}, A^{2}-2 A+I$.

$$
\begin{aligned}
A^{2} & =A A=\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right] \\
A^{3} & =A^{2} A=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
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\begin{aligned}
A^{2} & =A A=\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right] \\
A^{3} & =A^{2} A=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]=\left[\begin{array}{cc}
8 & 0 \\
28 & 1
\end{array}\right] \\
A^{-3} & =\left(A^{3}\right)^{-1}
\end{aligned}
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A^{2} & =A A=\left[\begin{array}{ll}
2 & 0 \\
4 & 1
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4 & 1
\end{array}\right]=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right] \\
A^{3} & =A^{2} A=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]=\left[\begin{array}{cc}
8 & 0 \\
28 & 1
\end{array}\right] \\
A^{-3} & =\left(A^{3}\right)^{-1}=\frac{1}{8}\left[\begin{array}{cc}
1 & 0 \\
-28 & 8
\end{array}\right]
\end{aligned}
$$

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A^{2} & =A A=\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]\left[\begin{array}{ll}
2 & 0 \\
4 & 1
\end{array}\right]=\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right] \\
A^{3} & =A^{2} A=\left[\begin{array}{cc}
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12 & 1
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1 & 0 \\
-28 & 8
\end{array}\right] \\
A^{2}-2 A+I & =\left[\begin{array}{ll}
4 & 0
\end{array}\right]=\left[\begin{array}{cc}
1 & 0
\end{array}\right]
\end{aligned}
$$

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2 & 0 \\
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4 & 0 \\
12 & 1
\end{array}\right] \\
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4 & 0 \\
12 & 1
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4 & 1
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1 & 0 \\
-28 & 8
\end{array}\right] \\
A^{2}-2 A+I & =\left[\begin{array}{cc}
4 & 0 \\
12 & 1
\end{array}\right]-\left[\begin{array}{cc}
4 & 0 \\
8 & 2
\end{array}\right]+\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]=\left[\begin{array}{l}
1 \\
4
\end{array}\right.
\end{aligned}
$$

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Compute $A^{3}, A^{-3}, A^{2}-2 A+I$.

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A^{2} & =A A=\left[\begin{array}{ll}
2 & 0 \\
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4 & 0 \\
12 & 1
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A^{3} & =A^{2} A=\left[\begin{array}{cc}
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12 & 1
\end{array}\right]-\left[\begin{array}{cc}
4 & 0 \\
8 & 2
\end{array}\right]+\left[\begin{array}{ll}
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\end{array}\right]=\left[\begin{array}{ll}
1 & 0 \\
4 & 0
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\end{aligned}
$$

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An $n \times n$ matrix is called elementary matrix if it can be obtained from $n \times n$ identity matrix by performing a single row operation.

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$$
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{3 R _ { 3 }}}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -3
\end{array}\right]=E_{1}
$$



## Elementary Matrix:

An $n \times n$ matrix is called elementary matrix if it can be obtained from $n \times n$ identity matrix by performing a single row operation. Example:

$$
\begin{gathered}
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{3 R}_{3}}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -3
\end{array}\right]=E_{1} \\
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{2 R}_{\mathbf{3}}+\mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & -2
\end{array}\right]=E_{2} \\
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{\mathrm{R}_{1} \longleftrightarrow \mathrm{R}_{3}}\left[\begin{array}{ccc}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{array}\right]=E_{3}
\end{gathered}
$$

## Elementary Matrix:

An $n \times n$ matrix is called elementary matrix if it can be obtained from $n \times n$ identity matrix by performing a single row operation.
Example:

$$
\begin{gathered}
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{3 R}_{3}}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -3
\end{array}\right]=E_{1} \\
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{2 R}_{3}+\mathbf{R}_{2}}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & -2
\end{array}\right]=E_{2} \\
I=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{1}} \longleftrightarrow \mathbf{R}_{3}}\left[\begin{array}{lll}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{array}\right]=E_{3}
\end{gathered}
$$

Note: When $A$ is multiplied from the left by elementary matrix $E$, the effect is same as to perform an elementary row operation on $A$. Let $A$ be a $3 \times 4$ matrix,

$$
A=\left[\begin{array}{cccc}
1 & 0 & 2 & 3 \\
2 & -1 & 3 & 6 \\
1 & 4 & 4 & 0
\end{array}\right]
$$

and $E$ be $3 \times 3$ elementary matrix obtained by row operation $3 R_{1}+R_{3}$ from an identity matrix

Note: When $A$ is multiplied from the left by elementary matrix $E$, the effect is same as to perform an elementary row operation on $A$. Let $A$ be a $3 \times 4$ matrix,

$$
A=\left[\begin{array}{cccc}
1 & 0 & 2 & 3 \\
2 & -1 & 3 & 6 \\
1 & 4 & 4 & 0
\end{array}\right]
$$

and $E$ be $3 \times 3$ elementary matrix obtained by row operation $3 R_{1}+R_{3}$ from an identity matrix

$$
\begin{gathered}
E=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
3 & 0 & 1
\end{array}\right] \\
E A=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
3 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
1 & 0 & 2 & 3 \\
2 & -1 & 3 & 6 \\
1 & 4 & 4 & 0
\end{array}\right]=\left[\begin{array}{cccc}
1 & 0 & 2 & 3 \\
2 & -1 & 3 & 6 \\
4 & 4 & 10 & 9
\end{array}\right], \mathbf{3} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{3}}
\end{gathered}
$$

## Method for finding inverse of a Matrix

To find an inverse row of matrix $A$, we perform a sequence of elementary row operations that reduce.

$$
[A \mid I] \text { to }\left[I \mid A^{-1}\right]
$$

## Example 1:

Find inverse matrix $A=\left[\begin{array}{ll}1 & 4 \\ 2 & 7\end{array}\right]$ by using Elementary matrix method. solution:

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\end{array}\right]
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\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}
$$



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\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & -1 & -2 & 1
\end{array}\right]
$$

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\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & -1 & -2 & 1
\end{array}\right] \xrightarrow{-\mathbf{R}_{2}}
$$

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\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & -1 & -2 & 1
\end{array}\right] \xrightarrow{-\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & 1 & 2 & -1
\end{array}\right]
$$

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1 & 4 & 1 & 0 \\
2 & 7 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & -1 & -2 & 1
\end{array}\right] \xrightarrow{-\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & 1 & 2 & -1
\end{array}\right] } \\
& \xrightarrow{-4 \mathbf{R}_{2}+\mathbf{R}_{1}}\left[\begin{array}{ll}
1 & -7 \\
\hline
\end{array}\right]
\end{aligned}
$$

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1 & 4 & 1 & 0 \\
2 & 7 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & -1 & -2 & 1
\end{array}\right] \xrightarrow{-\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & 1 & 2 & -1
\end{array}\right] } \\
& \xrightarrow{-4 \mathbf{R}_{2}+\mathbf{R}_{1}}\left[\begin{array}{cc|cc}
1 & 0 & -7 & 4 \\
0 & 1 & 2 & -1
\end{array}\right]=\left[I \mid A^{-1}\right]
\end{aligned}
$$

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1 & 4 & 1 & 0 \\
2 & 7 & 0 & 1
\end{array}\right] \xrightarrow{-\mathbf{2} \mathbf{R}_{1}+\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & -1 & -2 & 1
\end{array}\right] \xrightarrow{-\mathbf{R}_{2}}\left[\begin{array}{cc|cc}
1 & 4 & 1 & 0 \\
0 & 1 & 2 & -1
\end{array}\right]} \\
\xrightarrow{-4 \mathbf{R}_{2}+\mathbf{R}_{1}}\left[\begin{array}{cc|cc}
1 & 0 & -7 & 4 \\
0 & 1 & 2 & -1
\end{array}\right]=\left[I \mid A^{-1}\right] \\
A^{-1}=\left[\begin{array}{cc}
-7 & 4 \\
2 & -1
\end{array}\right]
\end{gathered}
$$

## Example 2:

Find inverse matrix $A=\left[\begin{array}{ccc}3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4\end{array}\right]$ by using Elementary matrix method. solution:

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3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right]
$$

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[A \mid I]=\left[\begin{array}{ccc|ccc}
3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{1} \longleftrightarrow \mathbf{R}_{2}}
$$

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Find inverse matrix $A=\left[\begin{array}{ccc}3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4\end{array}\right]$ by using Elementary matrix method.

## solution:

$$
[A \mid I]=\left[\begin{array}{ccc|ccc}
3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{1}} \longleftrightarrow \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
3 & 4 & -1 & 1 & 0 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right]
$$

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\begin{aligned}
& {[A \mid I]=\left[\begin{array}{ccc|ccc}
3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{1}} \longleftrightarrow \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
3 & 4 & -1 & 1 & 0 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right]} \\
& \xrightarrow{-3 R_{1}+R_{2},-2 R_{1}+R_{3}}
\end{aligned}
$$

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Find inverse matrix $A=\left[\begin{array}{ccc}3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4\end{array}\right]$ by using Elementary matrix method.

## solution:

$$
\begin{aligned}
{[A \mid I]=} & {\left[\begin{array}{ccc|ccc}
3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{1}} \longleftrightarrow \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
3 & 4 & -1 & 1 & 0 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] } \\
& \xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{2}},-\mathbf{2} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
0 & 4 & -10 & 1 & -3 & 0 \\
0 & 5 & -10 & 0 & -2 & 1
\end{array}\right]
\end{aligned}
$$

## Example 2:

Find inverse matrix $A=\left[\begin{array}{ccc}3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4\end{array}\right]$ by using Elementary matrix method.

## solution:

$$
\begin{aligned}
{[A \mid I]=} & {\left[\begin{array}{ccc|ccc}
3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{1}} \longleftrightarrow \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
3 & 4 & -1 & 1 & 0 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] } \\
& \xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{2}},-\mathbf{2} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
0 & 4 & -10 & 1 & -3 & 0 \\
0 & 5 & -10 & 0 & -2 & 1
\end{array}\right] \\
& \xrightarrow{-\mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{cc|ccc}
1 & & 0 & 1 & 0
\end{array}\right]
\end{aligned}
$$

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## solution:

$$
\begin{aligned}
{[A \mid I]=} & {\left[\begin{array}{ccc|ccc}
3 & 4 & -1 & 1 & 0 & 0 \\
1 & 0 & 3 & 0 & 1 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{1}} \longleftrightarrow \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
3 & 4 & -1 & 1 & 0 & 0 \\
2 & 5 & -4 & 0 & 0 & 1
\end{array}\right] } \\
& \xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{2},-\mathbf{2}} \mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
0 & 4 & -10 & 1 & -3 & 0 \\
0 & 5 & -10 & 0 & -2 & 1
\end{array}\right] \\
& \xrightarrow{-\mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
0 & 4 & -10 & 1 & -3 & 0 \\
0 & 1 & 0 & -1 & 1 & 1
\end{array}\right]
\end{aligned}
$$

$$
\xrightarrow{\mathbf{R}_{2} \longleftrightarrow \mathbf{R}_{3}, \frac{-4 \mathbf{R}_{3}+\mathbf{R}_{\mathbf{2}}}{-10}}
$$

$$
\xrightarrow{\mathbf{R}_{2} \longleftrightarrow \mathbf{R}_{3} \xrightarrow{-4 \mathbf{R}_{3}+\mathbf{R}_{2}}-10}\left[\begin{array}{lll|ccc}
1 & 0 & 3 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 1 & 1 \\
0 & 0 & 1 & -1 / 2 & 7 / 10 & 2 / 5
\end{array}\right]=\left[I \mid A^{-1}\right]
$$

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$


## solution:



## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
[A \mid I]=\left[\begin{array}{lll|lll}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right]
$$

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
[A \mid I]=\left[\begin{array}{lll|lll}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{2}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{3}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}}
$$

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
[A \mid I]=\left[\begin{array}{lll|lll}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{2}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{3}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & -3 & -1 & -2 & 0 & 1
\end{array}\right]
$$

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
\begin{gathered}
{[A \mid I]=\left[\begin{array}{ccc|ccc}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{2}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{3}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & -3 & -1 & -2 & 0 & 1
\end{array}\right]} \\
\\
\xrightarrow{\mathbf{4} \mathbf{R}_{\mathbf{3}}-\mathbf{3} \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{cc}
1
\end{array}\right.
\end{gathered}
$$



## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
\begin{gathered}
{[A \mid I]=\left[\begin{array}{lll|lll}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{2}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{3}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & -3 & -1 & -2 & 0 & 1
\end{array}\right]} \\
\\
\xrightarrow{\mathbf{4} \mathbf{R}_{\mathbf{3}}-\mathbf{3} \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 1 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & 0 & -1 & -2 & -3 & 4
\end{array}\right]
\end{gathered}
$$

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
\begin{gathered}
{[A \mid I]=\left[\begin{array}{lll|lll}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{2}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{3}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & -3 & -1 & -2 & 0 & 1
\end{array}\right]} \\
\\
\xrightarrow{\mathbf{4} \mathbf{R}_{\mathbf{3}}-\mathbf{3} \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 1 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & 0 & -1 & -2 & -3 & 4
\end{array}\right]
\end{gathered}
$$

$$
\xrightarrow{\mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{3}},-\mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{3}}}
$$

## Example 3:

$x_{1}+3 x_{2}+x_{3}=4$
$2 x_{1}+2 x_{2}+x_{3}=-1$
$2 x_{1}+3 x_{2}+x_{3}=3$

$$
\left[\begin{array}{lll}
1 & 3 & 1 \\
2 & 2 & 1 \\
2 & 3 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]
$$

solution:

$$
\begin{gathered}
{[A \mid I]=\left[\begin{array}{lll|lll}
1 & 3 & 1 & 1 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 & 0 \\
2 & 3 & 1 & 0 & 0 & 1
\end{array}\right] \xrightarrow{\mathbf{R}_{\mathbf{2}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{3}}-\mathbf{2} \mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 3 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & -3 & -1 & -2 & 0 & 1
\end{array}\right]} \\
\\
\xrightarrow{\mathbf{4} \mathbf{R}_{\mathbf{3}}-\mathbf{3} \mathbf{R}_{\mathbf{2}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 1 & 1 & 0 & 0 \\
0 & -4 & -1 & -2 & 1 & 0 \\
0 & 0 & -1 & -2 & -3 & 4
\end{array}\right] \\
\\
\xrightarrow{\mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{3}},-\mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 4 & 0 & 0 & -4 & 4 \\
0 & 0 & -1 & -2 & -3 & 4
\end{array}\right]
\end{gathered}
$$








Solution is $x_{1}=-6 x_{2}=4 x_{3}=-7$

$$
\xrightarrow{\frac{1}{4} \mathbf{R}_{2},-\mathbf{R}_{3}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right]
$$


$\xrightarrow{\frac{1}{4} \mathbf{R}_{\mathbf{2}},-\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4\end{array}\right]$


$$
\begin{gathered}
\xrightarrow{\frac{1}{4} \mathbf{R}_{2},-\mathbf{R}_{3}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right] \\
\xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right]=\left[I \mid A^{-1}\right]
\end{gathered}
$$



$$
\begin{gathered}
\xrightarrow{\frac{1}{4} \mathbf{R}_{\mathbf{2}},-\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right] \\
\xrightarrow{-\mathbf{3 R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{1}}}\left[\begin{array}{lll|ccc}
1 & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right]=\left[I \mid A^{-1}\right] \\
A^{-1}\left[\begin{array}{cccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right] \\
X=A^{-1} B=\left[\begin{array}{ccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right]\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]=\left[\begin{array}{l}
-1 \\
4 \\
-7
\end{array}\right]
\end{gathered}
$$

$$
\begin{gathered}
\xrightarrow{\frac{1}{4} \mathbf{R}_{\mathbf{2}},-\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right] \\
\xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{1}}}\left[\begin{array}{ccc|ccc}
1 & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right]=\left[I \mid A^{-1}\right] \\
A^{-1}\left[\begin{array}{cccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right] \\
X=A^{-1} B=
\end{gathered}
$$

$$
\begin{gathered}
\xrightarrow{\frac{1}{4} \mathbf{R}_{\mathbf{2},-} \mathbf{R}_{3}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right] \\
\xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{2}}+\mathbf{R}_{1}}\left[\begin{array}{lll|lll}
1 & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right]=\left[I \mid A^{-1}\right] \\
A^{-1}\left[\begin{array}{cccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right] \\
X=A^{-1} B=\left[\begin{array}{ccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right]\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]=\left[\begin{array}{c}
-1 \\
4 \\
-7
\end{array}\right]
\end{gathered}
$$

Solution is

$$
\begin{gathered}
\xrightarrow{\frac{1}{4} \mathbf{R}_{\mathbf{2}},-\mathbf{R}_{\mathbf{3}}}\left[\begin{array}{ccc|ccc}
1 & 3 & 0 & -1 & -3 & 4 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right] \\
\xrightarrow{-\mathbf{3} \mathbf{R}_{\mathbf{2}}+\mathbf{R}_{\mathbf{1}}}\left[\begin{array}{lll|lll}
1 & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 2 & 3 & -4
\end{array}\right]=\left[I \mid A^{-1}\right] \\
A^{-1}\left[\begin{array}{cccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right] \\
X=A^{-1} B=\left[\begin{array}{ccc}
-1 & 0 & 1 \\
0 & -1 & 1 \\
2 & 3 & -4
\end{array}\right]\left[\begin{array}{c}
4 \\
-1 \\
3
\end{array}\right]=\left[\begin{array}{c}
-1 \\
4 \\
-7
\end{array}\right]
\end{gathered}
$$

Solution is $x_{1}=-6 x_{2}=4 x_{3}=-7$

