Limits are now a staple of introductory calculus courses, and pre-calculus courses as well. The formal idea of closeness is meant to settle disagreements on what the limit of a particular function, sequence, or series is. This was problematic at the time, as, without a formal idea of what it meant for an expression or sequence to approach a number, many mathematicians argued for various different limits for some problematic sequences and expressions.  
Cauchy's ideas allowed a single reasonable idea to dominate, and it proliferated due to its compatibility with concretizing the previously rather loose ideas of derivatives and integrals.  
The idea in English may be described as a way to settle a claim. We may only claim that a function f approaches a limit of L if we can show that, for any small interval around L, every value of f(x) around the input value lies within that small interval.  
The epsilon and delta values just formalize how large those intervals may be.

