King Saud University
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PHYS 103 (GENERAL PHYSICS)
CHAPTER 9: Linear Momentum and Collisions

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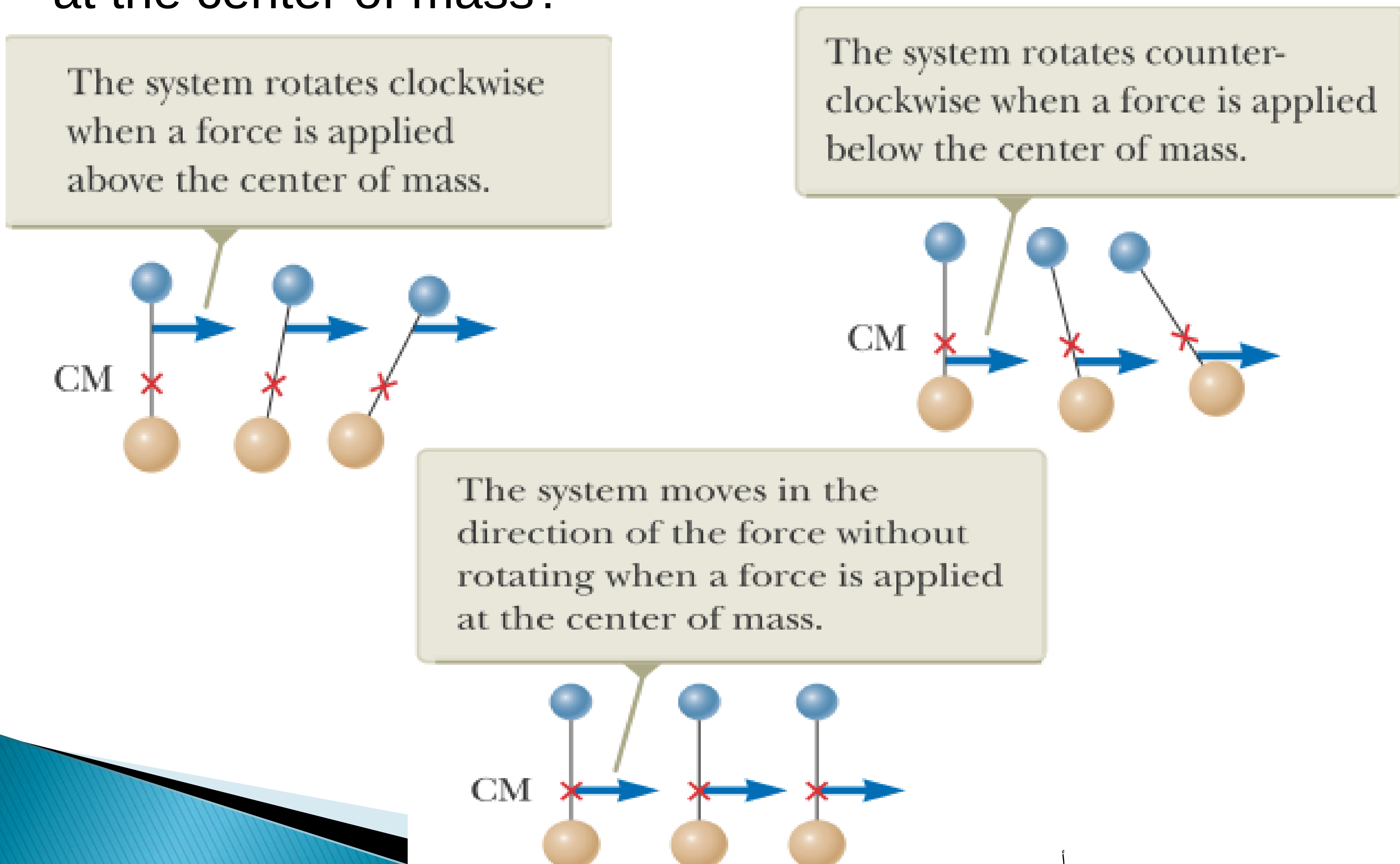
9.6 The Center of Mass

- Consider a system consisting of a pair of particles that have different masses

Connected together light, rigid rod.

- The position of the center of mass of a system can be described as being the average position of the system's mass.
- The center of mass of the system is located somewhere on the line joining the two particles and is closer to the particle having the larger mass.

What happen if a single force is applied at a point on the rod above the center of mass? below the center of mass? at the center of mass?



The center of mass of the pair of particles located on the x axis is given by

$$x_{CM} \equiv \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

The x coordinate of the center of mass of n particles is defined

$$x_{CM} \equiv \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n} = \frac{\sum_i m_ix_i}{\sum_i m_i} = \frac{\sum_i m_ix_i}{M} = \frac{1}{M}\sum_i m_ix_i \quad (9.29)$$

The y and z coordinates of the center of mass are similarly defined by

$$y_{CM} \equiv \frac{1}{M}\sum_i m_iy_i \quad \text{and} \quad z_{CM} \equiv \frac{1}{M}\sum_i m_iz_i$$

The center of mass can be located in three dimensions by its position vector $\vec{\mathbf{r}}_{CM}$.

The components of this vector are x_{CM}, y_{CM}, and z_{CM},

$$\vec{\mathbf{r}}_{CM} = x_{CM}\hat{\mathbf{i}} + y_{CM}\hat{\mathbf{j}} + z_{CM}\hat{\mathbf{k}} = \frac{1}{M}\sum_i m_i x_i\hat{\mathbf{i}} + \frac{1}{M}\sum_i m_i y_i\hat{\mathbf{j}} + \frac{1}{M}\sum_i m_i z_i\hat{\mathbf{k}}$$

$$\vec{\mathbf{r}}_i \equiv x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$$

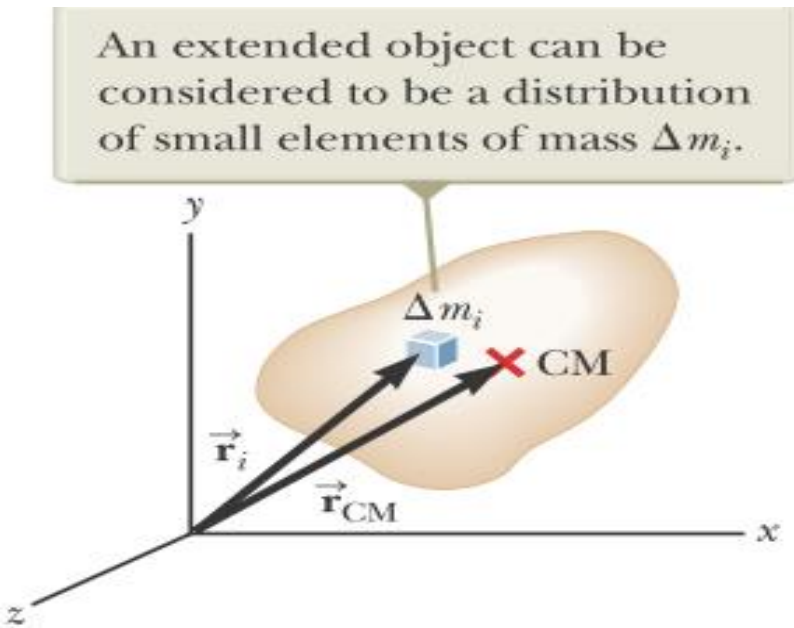
$$\vec{\mathbf{r}}_{CM} \equiv \frac{1}{M}\sum_i m_i\vec{\mathbf{r}}_i$$

If we let the number of elements n approach infinity, the size of each element approaches zero

$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{1}{M} \sum_i x_i \, \Delta m_i = \frac{1}{M} \int x \, dm$$

Likewise, for y_{CM} and z_{CM} we obtain

$$y_{CM} = \frac{1}{M} \int y \, dm \quad \text{and} \quad z_{CM} = \frac{1}{M} \int z \, dm$$



$$\vec{\mathbf{r}}_{CM} = \frac{1}{M} \int \vec{\mathbf{r}} \, dm$$

9.7 Systems of Many Particles

- The linear momentum of a particle

p=mv

- The total linear momentum of the system:

p= $M\vec{\mathbf{v}}_{\mathrm{CM}}$

- Velocity of the center of mass of the system:

$$\vec{\mathbf{v}}_{\mathrm{CM}} = \frac{d\vec{\mathbf{r}}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i m_i \frac{d\vec{\mathbf{r}}_i}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

$$M\vec{\mathbf{v}}_{\mathrm{CM}} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\mathrm{tot}}$$

- Using Newton's second for a system of many particles

F = ma

the acceleration of the center of mass of the system:

F= $M\vec{\mathbf{a}}_{\mathrm{CM}}$

$$\vec{\mathbf{a}}_{\mathrm{CM}} = \frac{d\vec{\mathbf{v}}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i m_i \frac{d\vec{\mathbf{v}}_i}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i$$

$$M\vec{\mathbf{a}}_{\mathrm{CM}} = \sum_i m_i \vec{\mathbf{a}}_i = \sum_i \vec{\mathbf{F}}_i$$

all internal force vectors cancel in pairs and we find that the net force on the system is caused only by external forces.

$$\sum \vec{\mathbf{F}}_{\mathrm{ext}} = M\vec{\mathbf{a}}_{\mathrm{CM}}$$

- Let us integrate this equation over a finite time interval:

$$\int \sum \vec{\mathbf{F}}_{\text{ext}}\, dt = \int M\vec{\mathbf{a}}_{\text{CM}}\, dt = \int M \frac{d\vec{\mathbf{v}}_{\text{CM}}}{dt}\, dt = M \int d\vec{\mathbf{v}}_{\text{CM}} = M\,\Delta\vec{\mathbf{v}}_{\text{CM}}$$

$$\Delta\vec{\mathbf{p}}_{\text{tot}} = \vec{\mathbf{I}}$$

where I is the impulse imparted to the system by external forces and ptot is the momentum of the system.

Finally, if the net external force on a system is zero so that the system is isolated, $\Delta\vec{\mathbf{p}}_{\text{tot}} = 0$

$$M\vec{\mathbf{v}}_{\text{CM}} = \vec{\mathbf{p}}_{\text{tot}} = \text{constant} \quad \left(\text{when } \sum \vec{\mathbf{F}}_{\text{ext}} = 0\right)$$

Example 9.14 The Exploding Rocket AM

A rocket is fired vertically upward. At the instant it reaches an altitude of 1 000 m and a speed of $v_i = 300$ m/s, it explodes into three fragments having equal mass. One fragment moves upward with a speed of $v_1 = 450$ m/s following the explosion. The second fragment has a speed of $v_2 = 240$ m/s and is moving east right after the explosion. What is the velocity of the third fragment immediately after the explosion?

$$\Delta \vec{\mathbf{p}} = 0 \quad \rightarrow \quad \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$$

$$M\vec{\mathbf{v}}_i = \frac{M}{3}\vec{\mathbf{v}}_1 + \frac{M}{3}\vec{\mathbf{v}}_2 + \frac{M}{3}\vec{\mathbf{v}}_3$$

$$\vec{\mathbf{v}}_3 = 3\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2$$

$$\vec{\mathbf{v}}_3 = 3(300\hat{\mathbf{j}}\text{ m/s}) - (450\hat{\mathbf{j}}\text{ m/s}) - (240\hat{\mathbf{i}}\text{ m/s})$$

$$= \boxed{(-240\hat{\mathbf{i}} + 450\hat{\mathbf{j}})\text{ m/s}}$$