Chapter 9

Linear Momentum and Collisions

**Linear Momentum and Its Conservation**

Consider two particles with masses m1 and m2 that can interact with each other and are isolated from their surroundings (isolated system).

If a force from particle 1 acts on particle 2 (F12), from Newton’s third law, there must be a second force applied by particle 2 on particle 1 (F21), where F12 and F21 are equal in magnitude and opposite in direction,

**F**21 + **F**12 = 0

From Newton’s second law

m1**a**1 +m2**a**2 =0

this means,

$$m\_{1}\frac{dv\_{1}}{dt}+m\_{2}\frac{dv\_{2}}{dt}=0$$

where v1 and v2 are the velocities of particle 1 and 2 respectively. Assuming the mass is constant. We can rewrite the equation to be,

$$\frac{d(m\_{1}v\_{1})}{dt}+\frac{d(m\_{2}v\_{2})}{dt}=0$$

Or
$$\frac{d(m\_{1}v\_{1}+m\_{2}v\_{2})}{dt}=0$$

(m v) is defined to be the liner momentum of a particle (p), **which is a vector quantity** ; thus,

$$\frac{d(p\_{1}+p\_{2})}{dt}=0$$

This means that the total momentum of the system (p1+p2) is constant (the total momentum of an isolated system at all times equals its initial momentum):

**p**1i + **p**2i = **p**1f + **p**2f

Example 9.1:

A 60-kg archer stands at rest on frictionless ice and fires a 0.5-kg arrow horizontally at 50 m/s. With what velocity does the archer move across the ice after firing the arrow?

Problem 2 (p282)

A 0.1-kg ball is thrown straight up into the air with an initial speed of 15 m/s. Find the momentum of the ball (a) at its maximum height and (b) halfway up to its maximum height.

**Impulse and Momentum**

As mentioned in the previous section, the liner momentum of a particle can be related to the force acting on it as follows (assuming a single force acts on the particle and the mass of the particle is constant):

$$F=ma=m\frac{dv}{dt}=\frac{d(mv)}{dt}=\frac{dp}{dt}$$

This means



We can find the change in the momentum of a particle when the force (which may vary with time) acts over some time interval (ti – tf) by integrating this expression:



The right hand side of this equation is defined to be the impulse (I) of the force F over the time interval (ti – tf) :

 

The impulse is a vector quantity and it equals the change in the momentum of the particle caused by that force:

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As the force may vary with time we can use the time averaged force:



Thus,



Example 9.4

A car of mass 1500 kg collides with a wall. The initial and final velocities of the car are vi = - 15i m/s and vf = 2.6i m/s , respectively. If the collision lasts for 0.15 s, find the impulse caused by the collision and the average force exerted on the car.

Problem 8 (p 283)

A ball of mass 0.15 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?

**Collisions in One Dimension**

* The law of conservation of linear momentum can be used to describe collisions between objects.
* A collision may entail physical contact between two macroscopic objects or a collision on an atomic scale such as that is between two charged particles (two protons) which never come into physical contact.
* Momentum of an isolated system is conserved in all collisions

A collision between two objects can be elastic or inelastic

* In an elastic collision, both momentum and kinetic energy are conserved (same before and after the collision); Thus,

 (Conservation of momentum)

 (Conservation of kinetic energy)

In the case of inelastic collision, the total the total kinetic energy of the system is conserved, although the momentum of the system is conserved, thus we can only use the equation describing the Conservation of momentum



If the collision is perfectly inelastic (when the colliding objects stick together after the collision) we can rewrite the previous equation to be (as both objects have the same final speed):



Example 9.6:

An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car.

If the smaller car were moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

Example 9.7

How can we determine the speed of the bullet (with mass m1) using the technique shown in the following figure?



Example 9.8

A block of mass m1 = 1.60 kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass m2 = 2.1 kg initially moving to the left with a speed of 2.50 m/s, as shown in the following figure . The spring constant is 600 N/m.

1. Find the velocities of the two blocks after the collision.
2. During the collision, at the instant block 1 is moving to the right with a velocity of 3.00 m/s, determine the velocity of block 2.
3. Determine the distance the spring is compressed at that instant.
4. What is the maximum compression of the spring during the collision?



**Two-Dimensional Collisions**

For two-dimensional collisions, the momentum of a system of two particles is conserved in each of the x and y directions. This means,





For instance, consider the glancing collision described in the following figure where particle 1 (blue ball) moves after collision at an angle θ with respect to the horizontal and particle 2 (which is initially at rest) moves at an angle φ with respect to the horizontal.



If we apply the law of conservation of momentum we obtain:

 (for x direction)

 (for y direction)

If the collision is elastic we can write the following equation:



Example 9.10:

A 1500-kg car traveling east with a speed of 25 m/s collides at an intersection with a 2500-kg van traveling north at a speed of 20 m/s. Find the direction and magnitude of the velocity of the wreckage after the Collision ( assuming that the vehicles stick together).

Example 9.12

In a game of billiards, a player wishes to sink a target ball in the corner pocket, as shown in the figure below. If the angle to the corner pocket is 35°, at what angle θ is the cue ball deflected?

(Assume that friction and rotational motion are unimportant and that the collision is elastic. Also assume that all billiard balls have the same mass m)



***Try problems (4,9,17,20,37) page (282-285)***