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104Math

List of Exercises:

Chapter 1.

[Parabola]

Find the elements of the parabola and sketch it in the exercises 1-9:

1. $y^2 = 3x$
2. $x^2 = -2y$
3. $x^2 = 4y$
4. $x^2 = -2y - 1$
5. $(x+1)^2 = -y - 3$
6. $(y-4)^2 = -8(x+2)$
7. $y^2 = 4y + 2x + 4$
8. $x^2 - 4x = y$
9. $x^2 + 8y + 6x + 1 = 0$

Find the equation of the parabola and sketch it in the exercises 10-18:

10. The vertex V is the origin and the focus F is (3,0).
11. The vertex V is the origin and the equation of the directrix is x=-5.
12. The vertex V is the origin and the equation of the directrix is y=2.
13. The vertex V is (1,2) and the focus F is (3,2).
14. The vertex V is (-1,1) and the equation of the directrix is x=2.
15. The focus F is (3,1) and the equation of the directrix is y=-3.
16. The focus F is (1,2) and the equation of the directrix is y=1.
17. The vertex V is (0.5,-1.5) and the equation of the directrix is x=-1
18. The vertex V is (1,-1) and the focus F is (1,3).

[Ellipse]

Find the elements of the ellipse and sketch it in the exercises 19-22:

19. $9y^2 + 25x^2 = 225$

20. $9y^2 + 4x^2 = 36$

21. $9y^2 + 25x^2 - 18y - 100x = 116$

22. $9y^2 + 4x^2 + 18y - 8x = 23$

Find the equation of the ellipse and sketch it in the exercises 23-26:

23. The foci are $F_1(2,0)$ and $F_2(-2,0)$, and it passes through the point $A(0,1)$.

24. The vertices are $V_1(3,0)$ and $V_2(-3,0)$, and the length of the minor axis is 4.

25. The foci are $F_1(2,3)$ and $F_2(-2,3)$, and it passes through the point $A(0,4)$.

26. The vertices are $V_2(-3,2)$ and $V_1(3,2)$, and the length of the minor axis is 4.

[Hyperbola]

Find the elements of the hyperbola and sketch it in the exercises 27-30:

27. $9y^2 - 25x^2 = 225$

28. $9y^2 - 16x^2 = 144$

29. $9y^2 - 25x^2 - 18y + 100x = 316$

30. $4y^2 - 8y - 9x^2 - 18x = 31$

Find the equation of the hyperbola and sketch it in the exercises 23-26:

31. The foci are $F_1(2,0)$ and $F_2(-2,0)$, and has one vertex on (1,0).

32. The vertices are $V_1(3,0)$ and $V_2(-3,0)$, and has one focus on (4,0).

33. The foci are $F_1(5,4)$ and $F_2(-5,4)$, and the distance between the vertices is 8.

34. The vertices are $V_1(2,4)$ and $V_2(2,-4)$, and the foci are $F_1(2,5)$ and $F_2(2,-5)$

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[Miscellaneous]

35. Find the elements of the conic section: $y = 2x^2 + 4x + 4$ and sketch it.

36. Find the equation of the parabola with focus $F(0,0)$ and its directrix with equation: $x = -2$.

37. Find the elements of the conic section: $4y^2 = -9x^2 + 18x + 27$ and sketch it.

38. Find the equation of the ellipse with foci $F_1(5,0)$ and $F_2(-5,0)$ and with vertices $V_1(8,0)$ and $V_2(-8,0)$.

39. Find the equation of the ellipse with foci $F_1(1,4)$ and $F_2(1,2)$ and the length of its major axis is 4.

40. Find the elements of the conic section: $4y^2 = 9x^2 + 18x + 45$ and sketch it.

41. Find the elements of the conic section: $x^2 - 2y^2 = 8$ and sketch it.

42. Find the equation of the hyperbola with foci $F_1(0,5)$ and $F_2(0,-5)$ and the distance between the vertices is 8.

43. Find the equation of the hyperbola with vertices $V_1(5,0)$ and $V_2(-5,0)$ and, with foci $F_1(8,0)$ and $F_2(-8,0)$.

44. Find the equation of the hyperbola with vertices $V_1(3,0)$ and $V_2(-3,0)$ and the equations of the asymptotes are $y = 2x$ and $y = -2x$.

45. Find the equation of the hyperbola with vertices $V_1(4,-4)$ and $V_2(4,4)$ and the distance between the foci is 10.

Chapter 2 . [Matrices and determinants]

Compute (if possible): $2BA$ and AB for the following matrices:

1) $A=\begin{pmatrix}1 & -2 & 3\\ 4 & 5 & -6\\ 2 & 0 & 1\end{pmatrix}$ $B=\begin{pmatrix}1 & -1\\ 2 & 3\\ 0 & 4\end{pmatrix}$;

2) $A=\begin{pmatrix}3 & 7\\ 2 & 2\\ 1 & -4\end{pmatrix}$ $B=\begin{pmatrix}2 & -1 & 3\\ -1 & 1 & 2\end{pmatrix}$;

3) $A=\begin{pmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 0 & 2 & 0\end{pmatrix}$ $B=\begin{pmatrix}1 & 2 & 1\\ 2 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}$;

4) $A=\begin{pmatrix}0 & 2 & 0\\ 3 & 2 & 3\\ 0 & 2 & 0\end{pmatrix}$ $B=\begin{pmatrix}1 & 2 & 1\\ 0 & 1 & 0\\ 1 & 2 & 1\end{pmatrix}$;

Evaluate the following multiplications:

1) $\begin{pmatrix}-4 & 3 & 8 & -5\end{pmatrix}\begin{pmatrix}7\\ -6\\ 0\\ 3\end{pmatrix}$ $\begin{pmatrix}-3 & 6 & -9\end{pmatrix}\begin{pmatrix}8\\ -3\\ 4\end{pmatrix}$ $\begin{pmatrix}5 & 3 & 7\end{pmatrix}\begin{pmatrix}2\\ 6\\ 1\end{pmatrix}$

2) $\begin{pmatrix}3 & -1\end{pmatrix}\begin{pmatrix}4 & 7 & -2 & 4 & 0\\ 2 & -6 & 8 & -3 & 7\end{pmatrix}$ $\begin{pmatrix}4 & -2\end{pmatrix}\begin{pmatrix}3 & 6\\ -1 & 5\end{pmatrix}$

3) $\begin{pmatrix}-3 & 6 & -1\end{pmatrix}\begin{pmatrix}5 & -2 & 7\\ -6 & 2 & 5\\ 9 & 0 & -1\end{pmatrix}$ $\begin{pmatrix}2 & 8 & 1\end{pmatrix}\begin{pmatrix}5 & 6\\ 3 & 1\\ 1 & 0\end{pmatrix}$

4) $\begin{pmatrix}9 & 1 & 4\end{pmatrix}\begin{pmatrix}6 & 2 & 1 & 5 & -8\\ 7 & 1 & -3 & 0 & 3\\ 1 & -5 & 0 & 3 & 2\end{pmatrix}$

5) $\begin{pmatrix}2 & 4 & 3 & -1 & 0\end{pmatrix}\begin{pmatrix}4 & 3 & 5 & -1 & 0\\ 1 & 3 & 5 & -2 & 6\\ 2 & -4 & 3 & 6 & 5\\ 3 & 4 & -1 & 6 & 5\\ 2 & 3 & 1 & 4 & -6\end{pmatrix}$

6) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -2 \\ 3 & 5 & 0 \\ 0 & -1 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ $\quad\begin{pmatrix} 2 & 0 & 1 \\ 1 & 5 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ $\quad\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

7) $\begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix}$ $\quad\begin{pmatrix} 0 & 2 & -1 & 4 \\ 1 & 5 & 3 & 0 \\ -2 & 0 & 1 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ -1 \\ 3 \end{pmatrix}$

8) $\begin{pmatrix} 4 & 2 & -1 \\ -2 & 1 & 6 \end{pmatrix}\begin{pmatrix} 5 & -1 & 2 & 3 \\ 1 & 3 & 5 & 2 \\ 0 & 1 & -2 & 4 \end{pmatrix}$ $\quad\begin{pmatrix} 3 & 1 & 4 \\ 1 & -3 & 5 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ -1 & 5 \\ 2 & -6 \end{pmatrix}$

9) $\begin{pmatrix} 5 & 1 \\ -2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 0 & 5 \end{pmatrix}$ $\quad\begin{pmatrix} 2 & 4 \\ 0 & -1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & -2 & 3 & -2 \\ 1 & 3 & -4 & 1 \end{pmatrix}$

10) $\begin{pmatrix} 4 & 1 & -2 & 3 \\ 0 & 2 & 1 & 5 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 0 & 2 \\ 3 & 1 \\ 0 & -2 \end{pmatrix}$ $\quad\begin{pmatrix} -1 & 0 & 3 & 1 \\ 2 & 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 0 \\ 3 & 0 & -1 & 2 \\ -1 & 4 & 3 & 0 \\ 2 & 0 & -2 & 1 \end{pmatrix}$

11) $\begin{pmatrix} 3 & 1 \\ -2 & 0 \\ 1 & -3 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}$ $\quad\begin{pmatrix} 1 & 0 & -2 \\ 2 & 4 & 1 \\ 0 & -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 & 1 & -2 \\ 0 & 2 & 1 & 3 \\ 1 & 0 & -3 & 2 \end{pmatrix}$

12) $\begin{pmatrix} 2 & 1 & 0 & -1 \\ 1 & 2 & -1 & 1 \\ 0 & -2 & 4 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 & -1 \\ 3 & 1 & -1 & 3 \\ 2 & 1 & 0 & 3 \\ 3 & 5 & 1 & 0 \end{pmatrix}$

Evaluate the following determinants:

1) $\begin{vmatrix} 9 & 8 \\ 7 & 5 \end{vmatrix}$ $\quad\begin{vmatrix} 3 & 2 \\ 1 & -4 \end{vmatrix}$ $\quad\begin{vmatrix} -2 & 5 \\ 3 & -3 \end{vmatrix}$

2) $\begin{vmatrix} 0 & 5 \\ -3 & -4 \end{vmatrix}$ $\quad\begin{vmatrix} 8 & 3 \\ -1 & 2 \end{vmatrix}$ $\quad\begin{vmatrix} -4 & 5 \\ 7 & -2 \end{vmatrix}$

3) $\begin{vmatrix} 7 & -6 \\ -5 & 4 \end{vmatrix}$ $\quad\begin{vmatrix} -\frac{2}{3} & \frac{2}{5} \\ -\frac{1}{3} & \frac{4}{5} \end{vmatrix}$ $\quad\begin{vmatrix} \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{4} \end{vmatrix}$

4) $\begin{vmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix}$ $\begin{vmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \\ 3 & -2 & 4 \end{vmatrix}$ $\begin{vmatrix} -3 & 1 & 2 \\ 0 & -1 & 5 \\ 6 & 0 & 1 \end{vmatrix}$

5) $\begin{vmatrix} -1 & 0 & 3 \\ 2 & 0 & -2 \\ 1 & -3 & 4 \end{vmatrix}$ $\begin{vmatrix} 5 & 1 & 2 \\ -3 & 2 & -1 \\ 4 & -3 & 5 \end{vmatrix}$ $\begin{vmatrix} 1 & 0 & 3 \\ 4 & 4 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

6) $\begin{vmatrix} 2 & 1 & 3 \\ 0 & -2 & 4 \\ 0 & 1 & 5 \end{vmatrix}$ $\begin{vmatrix} 1 & 5 & 4 \\ -3 & 6 & -2 \\ -1 & 5 & 3 \end{vmatrix}$ $\begin{vmatrix} 4 & 3 & 1 & 0 \\ -1 & 2 & -3 & 5 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -3 & 5 \end{vmatrix}$

7) $\begin{vmatrix} -1 & 3 & 0 & 2 \\ 2 & -1 & 1 & 0 \\ 5 & 2 & -2 & 0 \\ 1 & -1 & 3 & 1 \end{vmatrix}$ $\begin{vmatrix} 2 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ 1 & 3 & 2 & 1 \\ 3 & 1 & 1 & -2 \end{vmatrix}$ $\begin{vmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & 3 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \end{vmatrix}$

Chapter 3 . [Linear Systems]

Solve by one of the following methods (Cramer's Rule, Gauss elimination, and Gauss-Jordan Elimination) the following linear systems:

1) $\begin{cases} x-2y+z=0 \\ -x+y=1 \\ 4x+z=7 \end{cases}$ **;** $\begin{cases} 4x+2y=3 \\ -4x+y=6 \end{cases}$ $\begin{cases} x-y=10 \\ x+y=4 \end{cases}$ $\begin{cases} 2x+y=11 \\ 3x-y=4 \end{cases}$

2) $\begin{cases} 4x-5y=-34 \\ 2x-3y=-22 \end{cases}$ $\begin{cases} 3x-2y=-15 \\ 5x+6y=3 \end{cases}$ $\begin{cases} 2x-3y=5 \\ 3x+3y=10 \end{cases}$

3) $\begin{cases} 7x-4y=81 \\ 5x-3y=57 \end{cases}$ $\begin{cases} 2x-3y=3 \\ 4x+5y=39 \end{cases}$ $\begin{cases} x+4y=11 \\ 5x-2y=11 \end{cases}$

4) $\begin{cases} 5x-2y=3 \\ 2x+3y=5 \end{cases}$ $\begin{cases} 3x-2y=1 \\ 2x+y=10 \end{cases}$ $\begin{cases} 3x+4y=85 \\ 5x+4y=107 \end{cases}$

5) $\begin{cases} x+y+z=18 \\ x-y+z=6 \\ x+y-z=4 \end{cases}$ $\begin{cases} 2x+y=-3 \\ x+3y=19 \end{cases}$ $\begin{cases} 3x+y=9 \\ x+2y=8 \end{cases}$

6) $\begin{cases} x+y+z=35 \\ x-2y+3z=15 \\ y-x+z=-5 \end{cases}$ $\begin{cases} x+y=35 \\ x+z=40 \\ y+z=45 \end{cases}$ $\begin{cases} x+y+z=12 \\ x-y=2 \\ x-z=4 \end{cases}$

7) $\begin{cases} x-2y+2z=5 \\ 5x+3y+6z=57 \\ x+2y+2z=21 \end{cases}$ $\begin{cases} x+y+z=90 \\ 2x-3y=-20 \\ 2x+3z=145 \end{cases}$ $\begin{cases} x+2y+3z=14 \\ 2x+y+2z=10 \\ 3x+4y-3z=2 \end{cases}$

8) $\begin{cases} x-y=5 \\ y-z=-6 \\ 2x-z=2 \end{cases}$ $\begin{cases} 2x-4y+3z=10 \\ 3x+y-2z=6 \\ x+3y-z=20 \end{cases}$ $\begin{cases} 3x+y=5 \\ 2y-3z=-5 \\ x+2z=7 \end{cases}$

9) $\begin{cases} 2x-y-z-w=0 \\ x-3y+z+w=0 \\ x+y-4z+w=0 \\ x+y+w=36 \end{cases}$ $\begin{cases} x+y+2z+w=18 \\ x+2y+z+w=17 \\ x+y+z+2w=19 \\ 2x+y+z+w=16 \end{cases}$

10) $\begin{cases} x+y=a+b \\ y+z=b+c \\ z+w=a-b \\ w-x=c-b \end{cases}$ $\begin{cases} x+2y=5 \\ y+2z=8 \\ z+2u=11 \\ 2x+u=6 \end{cases}$ $\begin{cases} 3x-2y-z+w=-3 \\ -x-y+3z+2w=23 \\ x+3y-2z+w=-12 \\ 2x-y-z-3w=-22 \end{cases}$.

11) $\begin{cases} x+y+z+w=4 \\ x-2y+z-w=-1 \\ x+y+z+2w=5 \\ 2x-y+z-w=1 \end{cases}$ $\begin{cases} 2x-3y+z-w=-6 \\ x+2y-z=8 \\ 3y+z+3w=0 \\ 3x-y+w=0 \end{cases}$ $\begin{cases} x+y=4 \\ x-2y+z=-1 \\ x+2y+z=5 \\ 2x-4y+2z=1 \end{cases}$

Chapter 4 . [Integrals]

Compute the following integrals:

1) $\int (3x^2+1)\sin(x^3+x+1)\,dx$; $\int \frac{x+3}{(x-3)(x-2)}dx$; $\int \frac{5}{x^2+1}dx$;

2) $\int (x^2+1)\ln x\,dx$; $\int x^2 \sin x\,dx$; $\int \frac{x}{(x+2)^2}\,dx$; $\int x\sin x\,dx$

3) $\int \frac{dx}{x^2+9}$; $\int x^2\cos x\,dx$; $\int \frac{x+1}{(x-1)^2}\,dx$; $\int x^2 \ln x\,dx$; $\int x\ln(x)\,dx$;

4) $\int \frac{dx}{(x-3)(x-2)}$; $\int (x+1)\cos(x^2+2x)\,dx$; $\int \frac{x-2}{(x-2)(x-3)}dx$;

5) $\int \frac{x-1}{(x+2)^2(x+1)}dx$; $\int (2x+1)\sin x dx$; $\int \frac{\sin x}{\cos x}\,dx$;

6) $\int (3x^2+1)\sin(x^3+x+1)\,dx$; $\int \frac{x+3}{(x-3)(x-2)}dx$; $\int \frac{5}{x^2+1}dx$;

7) $\int (x^2+1)\ln x\,dx$; $\int \frac{2x+1}{(x+1)(x-4)}dx$; $\int \frac{2x}{x^2+1}dx$.

Chapter 5 . [Applications of Integrals]

Find the area of the regions determined by the curves:

1) $x+y=2, y=2, y=2x-4$;

2) $x+y=2, y=2, y=2x-4$;

3) $y=0,\ x=1, x=2,\ y=4x^2$;

4) $x=2, y=0, y=-x^2$;

5) $y=0, y=2x+2, y=-x+1$;

6) $x=2, y=0, y=x^2$;

7) $y = 0, y = -x + 6, y = \sqrt{x}$;

8) $x = 0, x = 1, \quad y = 0, \ y = x^2 - 4$;

9) $x + y = 2, y = 2, y = 2x - 4$;

10) $x = \frac{1}{2}, \quad x = 0, \quad y = x, \quad y = x^3$

11) $y = x, \quad y = x^4$

12) $x = 0, \quad x = \frac{\pi}{2}, \quad y = 0, \quad y = \cos x$

13) $y = 4, \quad y = 0, \quad x = 0, \quad x = y^2 - 4y$

14) $x = y - 2, \quad x^2 = y$

15) $x + y = 6, \quad y = x^2 + 4$

16) $y = 4, \quad y = -1, \quad y = x - 6, \quad y^2 = -x$

17) $y = -\frac{1}{5}x + 7, \quad y = 2 + |x - 1|$

18) $x = 1, \quad x = 0, \quad y = \sqrt{x}, \quad y = \frac{1}{2}x$

19) $y = \frac{1}{2}x, \quad y = \sqrt{x}$

20) $x = 2 - y^2, \quad x = y^2$

Find the volume of the solid of revolution generated by rotation about one of the coordinate axises of the region *R* limited by the following curves:

1. $x + y = 1, x = 1, y = 2x + 1$ (rotated about the y-axis);
2. x = 0, x=1, y=1, and $y = x^2 + 2$ (rotated about the x-axis);
3. $x=0$, $y=-4x-4$ and $y = x^2$ (rotated about the y-axis);

4. x = 0, x=1, and $y = x^2 + 2$ (rotated about the x-axis);

5. $y = 0,\ x = 1,\ y = 5\sqrt{x}$ (rotated about the y-axis);

6. $y = 0, \quad x = 3, \quad x = 1, \quad y = \frac{1}{x}$ (rotated about the x-axis);.

7. $y = 2, \quad y = x^2$ (rotated about the y-axis);

8. $y = 0, \quad y = x^2 - 4x$ (rotated about the x-axis);

9. $2y = x, \quad y^2 = x$ (rotated about the y-axis);

10. $y = 4 - x^2, \quad y = x^2$ (rotated about the x-axis);

11. $y = 0, \quad x = 1, \quad y = x^3$ (rotated about the y-axis);

12. $x = 2, \quad y = -2x + 3, \quad y = 2x - 1$ (rotated about the y-axis);.

13. $x = 0, \quad y = 1, \quad y^2 = x$ (rotated about the x-axis);

14. $y = 0, \quad x = 1, \quad y = x^2$ (rotated about the x-axis);

15. $y = 0, \quad x = 9, \quad x = 4, \quad y = \sqrt{x}$ (rotated about the x-axis);

16. $y = 0, \quad y - x + 2 = 0, \quad x = y^2$ (rotated about the y-axis);

17. $y - x + 2 = 0, \quad x = y^2$ (rotated about the x-axis);

Chapter 6. Partial Differentiation

In exercises 1 - 10 find all first and second partial derivatives of the given functions:

1. $w = 3x^3y^2$.

2. $f(x,y) = 2x^4y^3 - xy^2 + 3y + 1$.

3. $f(x,y) = 4e^{x^2y^3}$ at (0,1).

4. $w = \cos(x^5y^4)$.

5. $f(p,q) = \sqrt{p^2 + q^2}$ at (-1,0).

6. $w = \frac{x}{y} - \frac{y}{x}$.

7. $w = x^2\cos(2/y)$.

8. $f(x,y,z) = x + \sqrt{x^2 + y^2}$.

9. $w = y\ln(x^2 + z^4)$ at (2,0,1).

10. $w = xyze^{xyz}$.

In the following exercises use the chain rule to find $\frac{dw}{dt}$

11. $w = 3x^2y^3; \quad x = t^4, \quad y = t^2$.

12. $w = 3\cos x - \sin xy; \quad x = \frac{1}{t}, \quad y = 3t \;\; at \;\; t = 1$.

13. $w = \ln(2x^2 + y); \quad x = \sqrt{t}, \quad y = t^{2/3}$.

14. $w = e^{1-xy}; \quad x = t^{1/3}, \quad y = t^3$.

15. $w = x^2 - y\tan x; \quad x = 2t, \quad y = t + \frac{\pi}{2} \;\; at \;\; t = \pi$.

In the following exercises use the chain rule to find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$

16. $w = x\sin y; \quad x = s^2 + t^2, \quad y = st$.

17. $w = x^2 + 2xy, \quad x = s\ln t, \quad y = 2s + t \;\; at \;(0,1)$.

18. $w = x\ln y; \quad x = 3s + t, \quad y = st \;\; at \;(1,1)$

19. $w = x^2 \cos y; \quad x = s^2 t, \quad y = s - 1$.

20. $w = xy + yz; \quad x = 2s - t, \quad y = s - 2t, \quad z = -2s + 2t \;\; at \;(1,0)$.

In the following exercises find $\frac{dy}{dx}$

21. $x^3 - 3xy^2 + y^3 = 5$.

22. $x - \sqrt{xy} + 3y = 4$.

23. $2x^3 + x^2 y + y^3 = 1$.

24. $6x + \sqrt{xy} = 3y - 4$.

25. $x^{2/3} + y^{2/3} = 4$.

In the following exercises find $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$

26. $x\sin y + z^2 = 2xyz$.

27. $2xz^3 - 3yz^2 + x^2 y^2 + 4z = 0$.

28. $xz^2 + 2x^2 y - 4y^2 z + 3y = 2$.

29. $xe^{yz} - 2ye^{xz} + 3ze^{xy} = 1$.

30. $yx^2 + z^2 + \cos xyz = 4$.

Chapter 7. Introductory Differential Equations

In the following exercises solve the given differential equations:

1. $\frac{dy}{dx} = \frac{y}{x}$ $\quad y(1)=1.$

2. $y' = \frac{x^3}{(1+x^4)y}$.

3. $\sqrt{1+x^2}\,y' + x(1+y) = 0$.

4. $x\sec x\tan y - \frac{dy}{dx}\cdot\frac{1}{\cos x} = 0 \quad y(0) = \frac{\pi}{2}$.

5. $e^{-y}\sin x - y'\cos^2 x = 0$.

6. $y' = 1 - y + x^2 - yx^2$.

7. $y' + 3y = e^{-2x}$.

8. $y' + 2y = x \quad y(0)=1.$

9. $y' + y = \cos(e^x)$.

10. $x\frac{dy}{dx} + 2y = x^3 \quad y(2)=1.$

11. $2\frac{dy}{dx} + 4y = 1$.

12. $xy' + y = \sin x \quad y(\pi/2) = 1$.

13. $x\frac{dy}{dx} - 2y = x^3\sec x\tan x \quad y(\pi/3) = 2$.

14. $xy' - 3y = x^2 \quad x > 0$.

15. $\frac{dy}{dx} + y - \frac{1}{1+e^x} = 0$.

16. $\frac{dy}{dx} - xy = x \quad y(0)=3.$