

الاختبار 361السؤال الثالث

دالة $w = f(x, y)$ في متغيرين مشتقتها الجزئية من الرتبة الثانية متصلة

$$x = 2u + v, \quad y = u - v$$

$$\frac{\partial^2 w}{\partial u^2} + \frac{\partial^2 w}{\partial v^2} = 5 \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial y} + 2 \frac{\partial^2 w}{\partial y^2}$$

$$\frac{\partial^2 w}{\partial u^2} = 4 \frac{\partial^2 w}{\partial x^2} + 4 \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2}$$

نأى

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} = 2 \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}$$

$$\frac{\partial^2 w}{\partial u^2} = \frac{\partial}{\partial u} \left(\frac{\partial w}{\partial u} \right)$$

وله بنا:

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial v} = \frac{\partial w}{\partial x} - \frac{\partial w}{\partial y}$$

نأى

$$= \frac{\partial}{\partial x} \left(2 \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \right) \frac{\partial x}{\partial u} + \frac{\partial}{\partial y} \left(2 \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \right) \frac{\partial y}{\partial u}$$

$$\frac{\partial^2 w}{\partial v^2} = \frac{\partial}{\partial v} \left(\frac{\partial w}{\partial v} \right)$$

$$= \frac{\partial}{\partial x} \left(\frac{\partial w}{\partial x} - \frac{\partial w}{\partial y} \right) \frac{\partial x}{\partial v} + \frac{\partial}{\partial y} \left(\frac{\partial w}{\partial x} - \frac{\partial w}{\partial y} \right) \frac{\partial y}{\partial v}$$

$$= \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y \partial x} \right) \cdot 1 + \left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w}{\partial y^2} \right) \cdot (-1)$$

سأز في هاستقات جزئية من الرتبة الثانية

متصلة، فأز: $\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x}$

$$\frac{\partial^2 w}{\partial v^2} = \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2}$$

$$\frac{\partial^2 w}{\partial u^2} + \frac{\partial^2 w}{\partial v^2} = 5 \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial y} + 2 \frac{\partial^2 w}{\partial y^2}$$

بالتالي:

