King Saud University

College of Sciences

Department of Mathematics

Final Examination	Math 106	Semester I	1439-1440	Time: 3H

Exercise 1 : (2+2)

1. Approximate $\int_0^4 \sqrt{x^3+8}\,dx$ using Simpson's rule with $n = 4$

2. If $F(x) = (2+\sin(x))^{e^x}$, find $F'(x)$

Exercise 2 : (3+3+3)

1. Evaluate $\displaystyle\int \frac{(3^x+1)^2}{3^x}dx$.

2. Find $\displaystyle\int \frac{dx}{\sqrt{2^{2x}-1}}$.

3. Compute $\displaystyle\int \frac{dx}{\sqrt{x}\sqrt{1+x}}$.

Exercise 3 : (3+3+3)

1. Compute $\displaystyle\int \frac{dx}{x\sqrt{4-x^6}}$

2. Evaluate $\displaystyle\int \frac{dx}{(x^2-1)^{\frac{3}{2}}}$

3. Find $\displaystyle\int \frac{4x^2}{(x-1)^2(x+1)}dx$.

Exercise 4 : (3+3+3)

1. Does the integral $\displaystyle\int_0^{+\infty} (1+2x)e^{-x}\,dx$ converge? Find its value if it does.

2. Sketch the region bounded by $y=(x-1)^2$, $y=3-x$ and the $x-$axis and find its area.

3. Sketch the region bounded by $x=y^2+2$ and $x=4-y^2$ and set up an integral for the volume obtained by revolving the region about the line of equation $x=-1$.

Exercise 5 : (3+3+3)

1. Find the length of the curve given by: $x=\dfrac{t^3}{3}$, $y=\dfrac{2}{9}t^{\frac{9}{2}}$, $t\in[0,1]$.

2. Sketch the region inside the polar curve $r=3+3\cos(\theta)$ and to the left of the $y-$axis and find its area.

3. Find the area of the surface obtained by revolving the curve of equation $r=8\cos(\theta)$, $\theta\in[0,\frac{\pi}{2}]$ about the $y-$axis.

Solution of the final examination	Math 106	Semester I	1439-1440

Exercise 1 :

1.

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	$2\sqrt{2}$	1	2.8284
1	1	3	4	12
2	2	4	2	8
3	3	$\sqrt{35}$	4	23.6643
4	4	$6\sqrt{2}$	1	8.4853
				54.978

$\boxed{4}$

1,5

$$\int_0^4 \sqrt{x^3+8}dx \approx 18.326.$$ 0,5

2. $\ln F(x) = e^x \ln(2+\sin(x)),$ 0,5

$$F'(x) = F(x)\left(e^x \ln(2+\sin(x)) + \frac{e^x\cos(x)}{2+\sin(x)}\right).$$ 1,5

Exercise 2 :

$\boxed{9}$

1. $$\int (3^x + 3^{-x} + 2)dx = \frac{1}{\ln 3}3^x - \frac{1}{\ln 3}3^{-x} + 2x + c.$$ 1 + 2

$= \frac{1}{\ln 3}[3^x + 2x\ln 3 - 3^{-x}]$

2. $$\int \frac{dx}{\sqrt{2^{2x}-1}} \overset{t=2^x}{=} \frac{1}{\ln 2}\int \frac{dt}{t\sqrt{t^2-1}} = \frac{1}{\ln 2}\sec^{-1}(2^x) + c.$$ 2 + 1

3. $$\int \frac{dx}{\sqrt{x}\sqrt{1+x}} \overset{x=t^2}{=} \int \frac{2dt}{\sqrt{1+t^2}} = 2\sinh^{-1}(\sqrt{x}) + c.$$ 2 + 1

or $\cosh^{-1}(2x+1)$

Exercise 3 :

1. $$\int \frac{dx}{x\sqrt{4-x^6}} \overset{x^3=2t}{=} \frac{1}{6}\int \frac{dt}{t\sqrt{1-t^2}} = -\frac{1}{6}\text{sech}^{-1}\left(\frac{x^3}{2}\right) + c.$$ 2 + 1

2.

$\boxed{9}$

$$\int \frac{dx}{(x^2-1)^{\frac{3}{2}}} \overset{x=\sec(\theta)}{=} \int \frac{\cos(\theta)}{\sin^2(\theta)}d\theta$$ 1

$$= -\csc(\theta) + c$$ 1

$$= -\frac{x}{\sqrt{x^2-1}} + c.$$ 1

3.

$$\int \frac{4x^2}{(x-1)^2(x+1)}dx = \int \frac{3}{x-1}dx + \int \frac{2}{(x-1)^2}dx + \int \frac{1}{x+1}dx$$

1,5 + 1,5 $\quad = 3\ln|x-1| + \ln|x+1| - \dfrac{2}{x-1} + c.$

Exercise 4 :

1. $\int (1+2x)e^{-x}dx = -(3+2x)e^{-x} + c$, the the integral 1,5

$\int_0^{+\infty} (1+2x)e^{-x}dx$ converges and its value is 3. 1,5

$= \lim_{t\to\infty} \int_0^t (1+2x)e^{-x}dx = \lim_{t\to\infty} \left[-(3+2x)e^{-x}\right]_0^t$

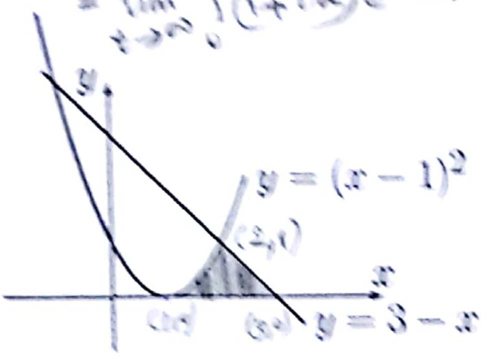
$= \lim_{t\to\infty} \left[-(3+2t)e^{-t} + 3\right] = 3$

1 point

1

9

2.

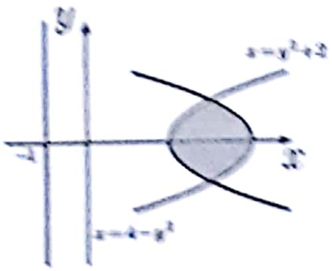


$$A = \int_1^2 (x-1)^2 dx + \int_2^3 (3-x)dx = \frac{5}{6}.$$ 1,5 + 0,5

or $A = \int_0^1 [(3-y) - (\sqrt{y}+1)]\,dy = \frac{5}{6}$

1

3.



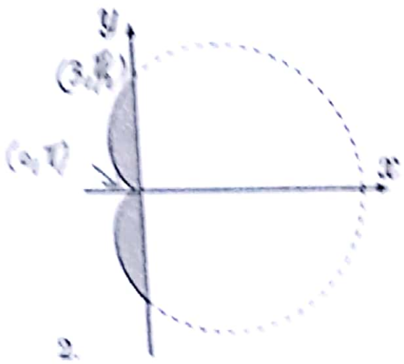
$$V = \pi \int_{-1}^{1} \left((1+4-y^2)^2 - (1+y^2+2)^2\right) dy.$$ 2

Exercise 5 :

1.

$$L = \int_0^1 \sqrt{t^4 + t^7}\,dt = \int_0^1 t^2\sqrt{1 + t^3}\,dt = \frac{2}{9}(2\sqrt{2} - 1). \quad 1+1+1$$

$\simeq 0.4063$



1

9

2.

$$A = \frac{2}{2}\int_{\frac{\pi}{2}}^{\pi} 9(1 + \cos(\theta))^2 d\theta = 9(\frac{3}{4}\pi - 2). \quad 1+1$$

$\simeq 3.2$

3.

$$A = 2\pi \int_0^{\frac{\pi}{2}} 8\cos^2(\theta)\sqrt{64\cos^2(\theta) + 64\sin^2(\theta)}\,d\theta = 32\pi^2.$$

$\simeq 315.5$

1+1+1