EXERCIES-SHEET-3

1) Solve the following differential equations:
(i) $xy^{''} - y^{'} + \frac{2}{x}y = 0, \quad x > 0$
(ii) $x^4 y^{'''} + x^3 y^{''} + x^2 y^{'} = 0, \quad x > 0$

Ans: (i) $y = c_1 x\cos(\ln x) + c_2 x\sin(\ln x)$, (ii) $y = c_1 + c_2 x\cos(\ln x) + c_3 x\sin(\ln x)$

(2) Use the substitution $x = e^t$ to reduce the Euler equation

$$4x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$$

to a second order constant coeffiecient equation and hence solve it.

Ans: $y = e^x\left(c_1\cos(\sqrt{3}\ln x) + c_2\sin(\sqrt{3}\ln x)\right)$

(3) Find the particular solution y_p for the following differential equations:
(i) $y^{(4)} - y = 4e^{-x}$
(ii) $y^{''} - 2y^{'} + y = 1 - e^x$
(iii) $y'' + \frac{1}{x}y^{'} - \frac{4}{x^2}y = \frac{1}{x^3}, \quad x > 0$

Ans: (i) $y_p = -e^{-x}$, (ii) $y_p = 1 + xe^x$, (iii) $y_p = -\frac{1}{3x}$

(4) Solve the system

$$\begin{aligned} \frac{dx}{dt} + y &= e^t \\ x - \frac{dy}{dt} &= e^{-t} \end{aligned}$$

Ans: $x(t) = e^{-t} - c_1\sin t + c_2\cos t + \sinh t$, $y(t) = c_1\cos t + c_2\sin t + \cosh t$