Q. #1. [Marks: 3+3+3+3=12]
(a) Determine whether the sequence \( \left\{ \left( \frac{n+1}{n} \right)^n \right\} \) converges or diverges and if it converges, find its limit.
(b) Determine the convergence or divergence of the series \( \sum_{n=1}^{\infty} \frac{1}{2^n-1} \).
(c) Find the interval of convergence and the radius of convergence of the power series \( \sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}} \).
(d) Find the first three non-zero terms of a Taylor series for the function \( f(x) = \cos x \) at \( x = \pi/3 \).

Q. #2. [Marks: 3+3+3+3=12]
(a) Evaluate the integral \( \int_0^4 \int_2^{\sqrt{y}} e^{x^3} \\, dx \\, dy \).
(b) Find the area of the portion of the surface given by the cone \( z^2 = 4x^2 + 4y^2 \) that is above the region in the first quadrant bounded by the line \( y = x \) and the parabola \( y = x^2 \).
(c) Use cylindrical coordinates to evaluate the integral \( \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 \\, dz \\, dy \\, dx \) \((a > 0)\).
(d) Find the mass of the solid enclosed between the two spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 4 \) with density \( \delta(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}} \).

Q. #3. [Marks: 4+4+4+4=16]
(a) Show that the following line integral is independent of path and find its value:
\( \int_{(0,0)}^{(\pi,\pi)} (x+y) \, dx + (x-y) \, dy \).
(b) Use Green’s theorem to evaluate the line integral
\( \int_C x y \\, dx + (x^2 + y^2) \, dy \),
where \( C \) is the closed curve determine by \( y = x \) and \( y^2 = x \) with \( 0 \leq x \leq 1 \).
(c) Use divergence theorem to evaluate the integral \( \int_S \vec{F} \cdot \vec{n} \\, dS \), where \( \vec{F} = 4x \vec{i} - 4y \vec{j} + z^2 \vec{k} \) and \( S \) is the surface of the region bounded by the cylinder \( x^2 + y^2 = 4 \) and the planes \( z = 0 \) and \( z = 3 \).
(d) Use Stokes’ theorem to evaluate \( \int_C \vec{F} \cdot d\vec{r} \), where \( C \) is the boundary of the portion of \( z = 4 - x^2 - y^2 \) above the \( xy \)-plane oriented upward and \( \vec{F}(x, y, z) = (x^2 e^x - y) \vec{i} + \sqrt{y^2 + 1} \vec{j} + z^3 \vec{k} \).